

# MODELLING STOCHASTIC POLITICAL RISK FOR CAPITAL

## BUDGETING

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### Abstract

In this paper we model political risk for international capital budgeting as the value of a hypothetical insurance policy that pays the holder any and all losses arising from political events. We address three important aspects of political risk that are widely acknowledged in the literature but either missing or incomplete in existing mathematical models: 1) loss causing political events arise from a wide range of sources, which are often mutually dependent; 2) the effect of a political event in terms of actual losses can vary depending on the economic, social and political conditions when it occurs; 3) the composition and the importance of the individual sources of political risk can change over time. Thus, the multivariate nature and dependency of loss causing political events are modeled as a conditional Poisson process that allows for dependency between the increments of the counting process. To account for the random effect of a loss causing political event due to the evolution of economic, social and political conditions, we model the overall economic, social and political climate as a stochastic loss index that represents the expected size of the jump if a political event occurs. To account for changes in the composition and importance of the sources of political risk, we employ a Bayesian updating process whereby the distribution of the conditional Poisson process is updated in time as new information arrives. We then put these elements together and follow Clark (1997) to measure the total cost of political risk as the value of a hypothetical insurance policy that pays any and all losses generated by a political event. Finally, we show how the model can be implemented in practice.

**Key words:** Capital budgeting, real options, conditional Poisson process, geometric

Brownian motion, Bayesian updating

**JEL Classification:** G31, D81, F21

## **1. Introduction**

Capital budgeting for foreign direct investment (FDI) is complicated by a wide range of diverse problems, designated by the comprehensive term “political risk”, that are rarely, if ever, encountered in domestic investment projects.<sup>1</sup> From a practical point of view, the most obvious sources of political risk are political instability and arbitrary government decisions reflected in political violence, taxes, asset destruction or expropriation, refusal to respect or enforce contracts, currency fluctuations and inconvertibility, etc. Other sources, however, include corruption, strikes, riots, sabotage, terrorism, kidnapping and war - to name only a few. In fact, “political risk” has become such a broad concept that its sources are virtually unlimited.

The common denominator of these diverse sources of political risk is that they are all random events and they all cause losses. However, loss levels can differ depending on the type of event as well as on the circumstances surrounding each event. For example, the loss due to a strike is likely to be different from a loss due to a decree that blocks repatriation of profits. On the other hand, the severity of a strike or a profit blocking decree and the ultimate loss they cause depends on the prevailing economic, social and political environment at the time they occur. Furthermore, the rate at which random political events occur or “arrive” is itself likely to be random. For example, expropriations are unlikely to occur with the same frequency as strikes and strikes are unlikely to occur with the same frequency as riots or currency devaluations. To further complicate the picture, the parameters of the distribution of the arrival rate can change over time as a result of changes in the domestic and international economic, financial and political environment. As an example of this, it is interesting to note that the number of expropriations, the most dramatic form of political

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<sup>1</sup> Although political risk exists, of course, even in domestic investment projects, it is conventional to discuss political risk in the context of foreign investment.

risk, fell from 336 between 1970 and 1975 to 87 between 1976 and 1979 to 15 between 1980 and 1985 and, finally, to 1 between 1986 and 1992 (Minor, 1994).<sup>2</sup>

Existing capital budgeting models that attempt to incorporate political risk in the investment analysis do not cope with these realities. They deal with one or at most two of the multiple risk sources and, consequently, cannot adequately deal with the multitude of risk sources that exist in reality and the fact that the rates at which the different types of political events occur can themselves be random variables with parameters that can change over time. More importantly, they fail to incorporate the fact that the size of the loss can vary according to the prevailing economic, social and political climate. In this paper we develop a model of political risk measurement for capital budgeting that addresses these issues. It includes a general process for handling multiple sources of political risk with random arrival rates, a method for updating these rates, and loss levels that change stochastically over time with the evolution of the value of the investment as well as with the economic, social and political environment.

The model we develop for valuing multivariate political risk follows Clark (1997) and Clark and Tunaru (2003) that model political risk as the value of a hypothetical insurance policy that pays any and all losses generated by a political event. The methodology features standard techniques of stochastic calculus developed in the financial literature on derivative pricing, combining a continuous process based on a geometric Brownian motion that accounts for ongoing change in loss levels and a jump process that accounts for the arrival of events causing explicit losses. The two processes are assumed to be independent. This type of modeling, employing a Poisson jump process with constant intensity, was used, among others, in Cummins and Geman (1995) for pricing catastrophe insurance, futures and call spreads. It is also common in the credit risk literature

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<sup>2</sup> In an interesting piece of empirical work, Davis (2001) estimates the probability of nationalization in South Africa

such as Jarrow and Turnbull (1995), Madan and Unal (1998) and Duffie and Singleton (1999) that model the timing of the default event as a Poisson process or a doubly stochastic Poisson process (Lando, 1994). It differs from these models, however, in that rather than considering a single type of investment ending event - default - it deals with a series of loss causing events emanating from different sources. We also exploit an idea commonly used in mathematical insurance: the assumption that the parameter of the Poisson jump process is a random variable. A Poisson process of this type with a random intensity rate is called a mixed Poisson process (Grandell, 1997) or a conditional Poisson process (Ross, 1983).

As an example of multivariate political risk modeled as a Poisson process with a random arrival rate, consider the following two types of events: public sector strikes and government decrees. The model proposed below takes into consideration the fact that strikes and decrees may have different *observed* rates of arrival that are just realizations of an *unobservable random rate* with some probability distribution. Thus, there will be some dependency between the interarrival times of events considered as a single class. For example, a series of decrees may be followed by a series of strikes. The conditional Poisson process is adapted to problems such as these since it allows for dependent increments. It is also adapted to Bayesian updating.<sup>3</sup> The importance of considering heterogeneous information arrival in financial markets has been emphasized in Asea and Ncube (1997), where a doubly stochastic Poisson jump process was used for the arrival of extraordinary financial and political announcements about shifts in fiscal and monetary policy.

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between 1990 and 1994, a period of political turmoil and threatened expropriation, as equal to 1.4%.

<sup>3</sup> The conditional Poisson process is a simplified version of a doubly stochastic Poisson process. The doubly stochastic Poisson process has the intensity parameter modelled as a stochastic process with positive range while the conditional Poisson process uses an intensity parameter that is a positive random variable. We use the conditional process because of practical considerations. First of all, it is much easier to estimate the parameters of the underlying probability distribution of a random variable than it is to estimate the parameters of a stochastic process. Secondly, using a random variable as the intensity of the Poisson process makes it possible to develop a Bayesian updating process for the estimation and application of the modelling.

Thus, the model we propose generalizes the model for political risk developed in Clark (1997) and Clark and Tunaru (2003) and refines it in several ways. First of all, we generalize the Poisson counting process modeling the arrival of loss causing political events to include multiple sources of political risk by using a general conditional Poisson process, as described above, that handles both discrete and continuous variables. Second, we incorporate the random element in the size of the jump when a loss causing political event occurs, which is due to the evolution of the overall economic, social and political environment surrounding the investment, by modeling it as Brownian motion. Third, we employ a Bayesian updating methodology capable of handling both discrete and continuous variables to account for the fact that the parameters of the conditional Poisson process can change with the evolution of the domestic and international economic, financial and political conditions.

The rest of the paper is organized as follows. Section 2 gives an overview of capital budgeting with political risk. Section 3 presents the model. Section 4 describes the updating methodology and gives an example and section 5 shows how the model can be implemented. Section 6 concludes.

## **2. An Overview of Capital Budgeting with Political Risk**

Political risk has a long and noble history in the theory and practice of foreign direct investment. Accounting for political risk in the capital budgeting process can be summarized in three steps. First, the risk must be identified; second, it must be assessed; and third, the assessment must be translated into consistent, concrete parameters compatible in theory and in practice with the discounted cash flow format of the modern capital budgeting process.<sup>4</sup>

Identifying the risk is often far from straightforward. In fact, as we have already mentioned, political risk is an extremely wide-ranging concept that touches most, if not all, aspects of the investment environment. Brewer (1981) says, 'Political risk often becomes a catchall term that refers to miscellaneous risks'. For Robbock and Simonds (1973), for instance, "political risk in international investment exists when discontinuities occur in the business environment, when they are difficult to anticipate, and when they result from political change". Root (1973) makes a distinction between transfer risks (potential restrictions on transfer of funds, products, technology and people), operational risks (uncertainty about policies, regulations, governmental administrative procedures which would hinder results and management of operations in the foreign country), and, finally, risks on control of capital (discrimination against foreign firms, expropriation, forced local shareholding, etc.). Other distinctions can also be made. Global political risk is related to the overall risk of a firm with several foreign subsidiaries whereas specific political risk is inherent to a particular investment in a given country. Micro-risk concerns a particular firm in a given country and depends on factors such as the nationality of the firm, its previous history in the country, its sector of activity, etc. Macro-risk, sometimes referred to as country risk, includes all events or measures likely to affect foreign investment in general. These events and measures can then be divided into two types: soft and hard. Soft measures attack the firm's cash flows indirectly and include actions such as blacklisting, protest movements and strikes. Hard measures attack the firm's cash flows directly in the form of expropriations, nationalizations, taxes and fines. Some authors make a further distinction between political risk and country risk where political risk refers to foreign direct investment and country risk refers to loans made by commercial banks to developing countries. Meldrum (2000) summarizes the definition of political risk as additional risks not present in domestic transactions that typically include risks arising from a variety of national differences in economic structures, policies, socio-political institutions, geography and currencies.

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<sup>4</sup> The discounted cash flow methodology, including internal rate of return, net present value, adjusted net present value,

For risk assessment, some authors such as Robock (1971) and Haendel *et al.* (1975), Kobrin (1979) or more recently Feils and Sabac (2000), focus on political risk as it affects the volatility of an investment's overall profitability both negatively and positively. Other authors such as Root (1972), Simon (1982), Howell and Chaddick (1994), Roy and Roy (1994) and Meldrum (2000) adopt a more practical stance and analyze risk as an explicit negative event that causes an actual loss or a reduction of the investment's expected return. Tests of political risk on investment outcomes reflect these two approaches. Kim and Mei (2001), Chan and Wei (1996), Cutler *et al.* (1989) and Bittlingmayer (1988) consider political risk with respect to stock market volatility. Other papers, such as Erb *et al.* (1995 and 1996), Cosset and Suret (1995), Bekaert (1995), and Bekaert and Harvey (1997) focus on losses and test political risk with respect to stock market performance. In this paper we adopt the second perspective and focus on political risk as an explicit, loss-causing event.

Traditional methods for integrating political risk analyses into the capital budgeting process involve using the analyses to estimate a risk premium or a cash flow adjustment factor that is then pasted onto the traditional NPV equation. Kobrin (1971), for example, suggests adjusting the discount rate. Stonehill and Nathanson (1968), Stobaugh (1969) and Shapiro (1978) suggest adjusting the expected cash flows. However, besides the fact that these methods fail to address the multivariate, stochastic nature of political risk, they also suffer from the absence of a sound, theoretical underpinning so that the risk premiums and adjustment factors must be determined *ad hoc*.

More recent methods use techniques borrowed from option pricing theory. Mahajan (1990), for example, models expropriation as a European style call option on an asset with a given maturity

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real option adjusted net present value, etc., is widely accepted as the appropriate format for project evaluation.

that pays no dividends and Clark (2003) models it as an American style call option on a dividend paying asset with an indeterminate maturity and a stochastic exercise price.<sup>5</sup> The approach in these papers is limited in that it does not recognize the exogenous, random nature of many types of political risk and the losses it can cause. Clark (1997) addresses this issue and models loss causing political events as an independent Poisson process. Clark and Tunaru (2003) extend the approach to two independent Poisson processes. The weakness of this approach is that both the arrival rate (rates in the case of Clark and Tunaru, 2003) and the size of the jump are treated as constant and common to all the potential risk sources. In the following section we address these points.

### 3. Modeling Political Risk

The expected instantaneous loss in Clark (1997) is equal to  $\lambda Y D(t) dt$ , where  $\lambda$  is the constant intensity parameter of the Poisson process,  $Y$  is the size of the jump, which is assumed to be constant and equal to 1, and  $D(t)$  is the dollar amount of the investment in the absence of political risk.<sup>6</sup> This section builds on this model and refines it in several ways. First of all, we use a conditional Poisson process to generalize the Poisson counting process that models the arrival of loss causing political events so that multiple, dependent sources of political risk are included. We also propose a Bayesian updating process whereby the parameters of the distribution of the intensity rate of the conditional Poisson process can change through time. We then model the size of the jump as a random variable that fluctuates as a result of the evolution of the investment's political environment. Finally, we relate the size of the jump to the investment's cash flows to determine the level of exposure to political risk.

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<sup>5</sup> Pointon and Hooper (1995) look at expropriation risk in the context of foreign bond pricing.

<sup>6</sup> The notation has been changed to correspond to the notation in this paper.



### 3.1 Modeling explicit political events as a conditional Poisson jump process

A conditional Poisson process can capture the fact that in practice explicit, loss-causing events arrive randomly and that the rate at which they arrive can itself be a random variable. Let  $\Lambda$  be a positive random variable. Then, the process  $\{N(t)\}_{t \geq 0}$  is called a conditional Poisson process if, given that  $\Lambda = \lambda$ ,  $\{N(t)\}_{t \geq 0}$  is a Poisson process with rate  $\lambda$ .<sup>7</sup> It should be noted that  $\{N(t)\}_{t \geq 0}$  itself is not a Poisson process since it has stationary increments but it does not have independent increments, as emphasized in Ross (1983).

We need to calculate the probability of a change in the state space in a small time interval.

If  $\Lambda$  has some probability distribution  $G$  then

$$P(N(t+dt) - N(t) = 1 \mid N(t) = n) = \int_0^{\infty} (\lambda dt) e^{-\lambda dt} dG^{(n)}(\lambda) \quad (1)$$

and therefore

$$\lim_{dt \rightarrow 0} \frac{P(N(t+dt) - N(t) = 1 \mid N(t) = n)}{dt} = \int_0^{\infty} \lambda dG^{(n)}(\lambda) = E(\Lambda \mid N(t) = n) \quad (2)$$

$$\text{Similarly it follows that } \lim_{dt \rightarrow 0} \frac{P(N(t+dt) - N(t) \geq k \mid N(t) = n)}{dt} = 0 \quad (3)$$

for any  $k \geq 2$ . The equations (1-3) show that

$$P(N(t_0 + dt) - N(t_0) = k \mid N(t_0) = n) = \begin{cases} E(\Lambda \mid \{N(t_0) = n\})dt + o(dt), & k = 1 \\ o(dt), & k \geq 2 \\ 1 - E(\Lambda \mid \{N(t_0) = n\})dt + o(dt), & k = 0 \end{cases} \quad (4)$$

Another way of interpreting this relationship is that, knowing that up to time  $t_0$  there were  $n$  losses, the probability that a loss causing political event will actually occur over the time interval  $dt$  is  $E(\Lambda \mid \{N(t_0) = n\})$ .

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<sup>7</sup> This process is also called a *weighted* Poisson process or a *mixed* Poisson process, see Grandell (1976) and Grandell (1997). Lundberg (1964), the first in depth treatment of this type of stochastic process, called them *compound* Poisson processes, which nowadays refers to a totally different process in actuarial mathematics.

To account for the fact that the estimates of the parameters of the probability distribution of  $\Lambda$  can change over time, we construct a learning process for estimating political risk. Following the above mathematical machinery and learning that up to the time  $t_0$  there were  $n$  events, based on equations (2 – 3) the estimated arrival rate of losses is equal to  $E(\Lambda|N(t_0) = n)$ . This quantity can be further calculated as

$$E(\Lambda|N(t_0) = n) = \frac{\int_0^{\infty} \lambda^{n+1} e^{-\lambda t_0} dG(\lambda)}{\int_0^{\infty} \lambda^n e^{-\lambda t_0} dG(\lambda)} \quad (5)$$

### ***3.2 Modeling the political environment and exposure to loss***

In the model of Clark (1997), the dollar loss exposure is equal to  $YD(t)$ , that is, the size of the jump  $Y$ , which is a constant, multiplied by  $D(t)$ , the dollar value of the investment, which evolves through time according to geometric Brownian motion. The geometric Brownian motion for  $D(t)$  reflects the random element associated with uncertain cash flows or asset values. It is unrealistic, however, to suppose that the severity of the loss, represented by the size of the jump, is constant. As we mentioned in the introduction, when an explicit, loss-causing event occurs, its effect will be more or less severe depending on the prevailing economic, social and political climate. Consequently, the size of the jump should be a random variable that reflects the expected size of the jump rather than a constant. Thus, we can think of  $Y$  as an index of the ongoing economic, social and political climate that evolves through time according to the ebbs and flows of the political, social and economic discourse. For example, suppose that there are two types of loss-causing events that could arrive with equal probabilities: a strike and a surtax on profits. If a strike occurs it will cause a 30% loss and if a tax occurs it will cause a 10% loss. In this case, we calculate  $Y$  as  $Y = 0.5 \times 0.3 + 0.5 \times 0.1 = 20\%$ . This figure can change over time. The social climate

could degenerate to the point where a strike would be long and bitter and cause a loss of 40% of asset value. On the other hand, relations with the government could improve to the point where a tax would likely generate a loss of only 5%. In this case,  $Y$  would rise to 22.5%. The probabilities of the two events could also change, for example, from 50-50 to 70-30, which would cause a further change in the value of  $Y$ .

With this in mind it is clear that both  $Y$  and  $D$  have a random element that determines at least partially how they will evolve. Furthermore, neither  $Y$  nor  $D$  can be negative. Geometric Brownian motion with drift can capture the partial randomness of a variable that cannot become negative so that<sup>8</sup>

$$dY(t) = \alpha Y(t)dt + \psi Y(t)ds(t) \quad (6)$$

$$dD(t) = \beta D(t)dt + \omega D(t)dw(t) \quad (7)$$

The parameters  $\alpha$  and  $\beta$  reflect the expected growth rates of  $Y$  and  $D$  respectively with  $\psi$  and  $\omega$  as the respective standard deviations. Although the parameter  $\alpha$  can theoretically be greater than, equal to, or less than zero, in practice it is hard to imagine a constantly deteriorating political climate and  $\alpha$  should be either zero, for countries with no expected long term trend in the political climate, or negative, for countries reverting to a relative level of no political risk at all.<sup>9</sup> The parameter  $\beta$  is greater than or equal to zero and implies a constant dividend yield.<sup>10</sup> The variables  $ds$  and  $dw$  are

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<sup>8</sup> Geometric Brownian motion is not exclusive. Other stochastic processes, such as mean reverting square root processes, could also be used.

<sup>9</sup> For example, from the perspective of a country in the European Union, other countries in the Union might be perceived as trending towards its own level of political risk, which, given that we restrict political risk to problems not encountered in domestic investments, would make it trending towards zero political risk.

<sup>10</sup> A constant dividend yield implies the infinite time frame we adopt in this paper (see footnotes 9 and 10). To see this, let

standard Wiener processes with zero mean and standard deviation of  $dt$ . The relationship between  $Y$  and  $D$  is reflected in  $dw(t)ds(t) = \rho dt$  where  $\rho$  is the instantaneous correlation between  $Y$  and  $D$ .

Let  $x(t) = Y(t)D(t)$  for any  $t$ . Using, Ito's lemma and equations (6) and (7) gives

$$dx(t) = (\alpha + \beta + \psi\omega\rho)x(t)dt + \sqrt{\omega^2 + \psi^2 + 2\omega\psi\rho}x(t)dz(t) \quad (8)$$

where  $dz(t) = \frac{\omega dw + \psi ds}{\sqrt{\omega^2 + \psi^2 + 2\omega\psi\rho}}$ . Equation (8) shows that the drift component of the dollar amount

at risk depends on  $\alpha$  and  $\beta$  as well as on the volatilities of  $Y(t)$  and  $D(t)$  and the correlation between them. The volatility of the dollar amount at risk depends on the individual volatilities and correlation as well. If we adopt the following notation,  $A = (\alpha + \beta + \psi\omega\rho)$  and  $\sigma = \sqrt{\omega^2 + \psi^2 + 2\omega\psi\rho}$ , we can write the evolution through time of the dollar amount at risk as

$$dx(t) = Ax(t)dt + \sigma x(t)dz(t) \quad (9)$$

where  $A$  is the expected rate of growth of the dollar amount at risk,  $\sigma^2$  is the variance of  $dx(t) / x(t)$ , and  $dz(t)$  is a Wiener process with zero mean and variance equal to  $dt$ . Equation (9) means that exposure to political risk (dollar amount at risk) is expected to change at a rate of  $A$

$D_t$  = value of the investment at time  $t$ ,  $dD = \beta Ddt + \omega Dd\zeta$

where  $\beta = R - \delta$  is the constant growth rate,  $\omega$  is the standard deviation of  $dD / D$  and  $d\zeta$  is a standard Wiener process

$\Delta_t$  = the instantaneous dividend at time  $t$ ,  $R$  = the instantaneous required rate of return on the investment

$\delta$  = the instantaneous dividend yield (proportional dividend),  $E$  = expectation operator

Using the discounted dividend model, we have

$$D_0 = E \int_0^T \Delta_t e^{-Rt} dt = \int_0^T \delta D_0 e^{(R-\delta)t} e^{-Rt} dt = \int_0^T \delta D_0 e^{-\delta t} dt = -D_0 e^{-\delta T} + D_0$$

The only way this can be true is if  $T = \infty$ .

with a standard deviation of  $\sigma$  times the random element in ongoing change represented by the Wiener process.

Now apply the Girsanov theorem so that the growth rate goes from the risk adjusted  $A$  to the risk neutral  $\varphi$  while preserving the variance structure of the asset. Equation (9) then becomes

$$dx(t) = \varphi x(t)dt + \sigma x(t)d\tilde{z}(t) \quad (10)$$

where  $d\tilde{z}(t)$  represents the new Wiener process under the risk neutral probability measure.

The shift to risk neutrality can be an advantage for practical applications of the model. In a risky world, the growth rate is equal to the difference between the risk adjusted required rate of return  $R_x$  and the dividend or convenience yield  $\delta_x$ . It may be easier to estimate  $\delta_x$  than  $R_x$ . If so, the risk neutral growth rate can be calculated as the difference between the riskless rate  $r$ , which can be observed, and the dividend rate  $\delta_x$ , which is known:  $\varphi = r - \delta_x$ . The relationship between the risk neutral and the risk adjusted growth rates is  $\varphi = A - B(R_m - r)$ , the risk adjusted growth rate minus the risk premium, where  $B(R_m - r)$  represents the CAPM's risk premium.

Let  $V(x(t), t)$  represent the value of a hypothetical insurance policy that pays the holder any and all losses arising from political risk. If there are no events the holder gets nothing. The typical direct investment involves setting up a subsidiary in the host country or purchasing all or part of an already existing company in the host country. Legislation in most countries is such that either there is no specified life span for a company or when the life span is specified, such as in France, it can be renewed indefinitely. Consequently, the typical direct investment can be viewed as a perpetual

claim.<sup>11</sup> Under this assumption, the derivative contract  $V$  can also be considered as a perpetual claim and therefore its value does not depend on time explicitly.<sup>12</sup> This means that  $V = V(x(t))$  and applying Ito's Lemma we get

$$dV = \sigma x(t) \frac{dV}{dx} d\tilde{z}(t) + \left[ \varphi x(t) \frac{dV}{dx} + \frac{1}{2} \sigma^2 x^2(t) \frac{d^2 V}{dx^2} \right] dt. \quad (11)$$

Taking expectations and recalling that  $d\tilde{z}$  is a Wiener process with zero mean, gives:

$$E(dV) = \varphi x(t) \frac{dV}{dx} dt + \frac{1}{2} \sigma^2 x^2(t) \frac{d^2 V}{dx^2} dt. \quad (12)$$

When a political event occurs at time  $t > t_0$ , the expected loss is  $E(\Lambda | N(t_0) = n)x(t)$ . After learning that there were  $n$  events prior to time  $t_0$ , the expected total return on the insurance policy on an infinitesimal interval of time is equal to  $E(dV)$  plus the expected cash flow  $E(\Lambda | N(t_0) = n)x(t) dt$  generated by a new explicit event. We make the usual assumption that the information that causes jumps is independent and uncorrelated with the market and, thus, yields the riskless rate of return that is assumed here to be constant  $r$ . Putting this together gives

$$rVdt = \varphi x(t) \frac{dV}{dx} dt + \frac{1}{2} \sigma^2 x^2(t) \frac{d^2 V}{dx^2} dt + E(\Lambda | N(t_0) = n)x(t)dt \quad (13)$$

This equation can be simplified and rewritten as a differential equation

$$\frac{1}{2} \sigma^2 x^2(t) \frac{d^2 V}{dx^2} + \varphi x(t) \frac{dV}{dx} - rV(t) + E(\Lambda | N(t_0) = n)x(t) = 0 \quad (14)$$

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<sup>11</sup> Many direct investments, of course, do have a specified contractual life span. When this is the case, the value of the insurance policy that we model in this section is a function of time as well as of the exposure to losses arising from political risk. The consequences of this on the value of the insurance policy depend on the particular conditions for terminating the investment on the specified date.

<sup>12</sup> In the case where the investment has a specified life span,  $V$  is a function of time as well as of the value of the investment. The consequences of this on the value of the option depend on the specific conditions for terminating the investment on the specified date. Mahajan (1990), for example, looks at the particular case where the government's option to exercise is limited to the project's termination date.

This is a differential equation of second order that should be solved for  $V$ . The solutions will depend on  $x$  and the current status given by  $t_0$  and  $n$ . The solutions of this equation also depend on the characteristics of the insurance policy in question reflected in the boundary conditions.

One solution based on the possibility of cashing in the insurance policy, described in more detail in appendix I, is

$$V(x(t)) = \frac{E(\Lambda \mid N(t_0) = n)x(t)}{r - \varphi} + A_1 x(t)^{\gamma_1} \quad (15)$$

where  $\gamma_1 > 1$  and  $A_1$  are known.

#### 4. The Bayesian updating process

Factors that influence political risk, such as economic and social conditions, legislation, financial innovations, etc. tend to evolve through time, which makes it necessary to update the analysis and re-evaluate the cost of political risk represented by the value of the hypothetical insurance policy. Bayesian updating is a well-known procedure that is particularly adapted to this type of a revision process with respect to the inference of the relevant parameters based on past and current information.<sup>13</sup> It leads to new parameter estimates that in turn give a different value for the political derivative.

As a learning process, knowing that  $n$  events are recorded up to the time point  $t_0$ , we are also interested in the expected time of the occurrence of the first explicit event after time  $t_0$ . The conditional distribution of  $\Lambda$  given that  $N(t_0) = n$  is given by

$$P(\Lambda \in (\lambda, \lambda + d\lambda) | N(t_0) = n) = \frac{e^{-\lambda t_0} \frac{(\lambda t_0)^n}{n!} g(\lambda) d\lambda}{\int_0^\infty e^{-\lambda t_0} \frac{(\lambda t_0)^n}{n!} g(\lambda) d\lambda} \quad (16)$$

where  $g$  is the p.d.f. of the distribution with c.d.f.  $G$ . The time  $T$  from  $t_0$  until the arrival of the next event has the following probability distribution

$$P(T \leq y | N(t_0) = n) = \frac{1 - e^{-\lambda y} \frac{(\lambda t_0)^n}{n!} e^{-\lambda t_0} g(\lambda) d\lambda}{\int_0^\infty \frac{(\lambda t_0)^n}{n!} e^{-\lambda t_0} g(\lambda) d\lambda} \quad (17)$$

and

$$E(T | N(t_0) = n) = \frac{\int_0^\infty \lambda^{n-1} e^{-\lambda t_0} g(\lambda) d\lambda}{\int_0^\infty \lambda^n e^{-\lambda t_0} g(\lambda) d\lambda} \quad (18)$$

One can make the assumption that the time scale is partitioned by the time points  $\{t_m\}_m$ , where  $m$  can be any positive integer. These time periods may coincide with some economic or political cycles that are known to affect the business environment such as the beef cycle in Argentina or the presidential election cycle in Mexico. The cover policies for the future period  $(t_m, t_{m+1}]$  can be calculated at the end of the time interval  $(t_{m-1}, t_m]$  during which  $n_m$  events are recorded. The distribution of the intensity  $\Lambda$  of the arrival of events can be updated in time when new information is collected from the arrival of new events.

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<sup>13</sup> It is important to remember that in the Bayesian updating, the absence of any events is information that is just as important as when events do occur.



Let  $g^{(m)}$  be the probability density function of the probability distribution  $G^{(m)}$ , updated at the end of the time period  $(t_{m-1}, t_m]$  following a recording of  $n_m$  events. From the Bayes' formula it follows that

$$g^{(m)}(\lambda) = \frac{\lambda^{n_m} e^{-\lambda(t_m - t_{m-1})} g^{(m-1)}(\lambda)}{\int_0^\infty \lambda^{n_m} e^{-\lambda(t_m - t_{m-1})} g^{(m-1)}(\lambda) d\lambda} \quad (19)$$

Since the denominator is just a normalizing constant, it can be seen that the density  $g^{(m)}$  at the end of the time period  $(t_{m-1}, t_m]$  depends only on the previous density  $g^{(m-1)}$ , the length of the time period  $t_m - t_{m-1}$  and the number of events that occurred  $n_m$ .

The advantage of taking a Bayesian approach is that the solution can be updated when new information is available. The value of the covering policy at the end of the period  $(t_{m-1}, t_m]$ , calculated as described in equation 15, is given by

$$V^{(m)}(x(t)) = \frac{E(A \mid N(t_m) - N(t_{m-1}) = n_m)x(t)}{r - \varphi} + A_1^{(m)}x(t)^{\gamma_1} \quad (20)$$

where  $A_1^{(m)}$  is calculated from the boundary conditions relative to the time interval  $(t_m, t_{m+1}]$ , and the conditional expectation is calculated relative to the probability  $G^{(m)}$ .

## 5. Implementing the Model

### 5.1 Estimating the dollar amount at risk

Implementing the model involves estimating the parameters for  $x(t)$ , the dollar amount at risk in the case of an explicit political event, and for the conditional Poisson

process,  $E(A | \{N(t_0) = n\})$ . In this section we deal with the parameters for  $x(t)$ . In the following section we deal with the conditional Poisson process.

The parameters for  $x(t)$  in equation 10,  $\varphi$  and  $\sigma$ , are detailed in equation 8. They depend on the growth rate  $\alpha$  and the standard deviation  $\psi$  of the political risk index  $Y(t)$  in equation 6, the growth rate  $\beta$  and standard deviation  $\omega$  of the project cash flows  $D(t)$  in equation 7, and the correlation between the two  $\rho$ .<sup>14</sup> For the project cash flows, the analysis is straightforward. The parameters  $\beta$  and  $\omega$  are standard inputs that figure prominently in the capital budgeting process and should be known by the firm. The index  $Y(t)$ , its growth rate and standard deviation must be estimated.

The political risk index itself may be a macro type of indicator that refers to foreign investment in general. However, in some cases a micro type of indicator associated with a particular project or the nationality of the investor may be more appropriate. For example, the *ceteris paribus* political risk surrounding a nuclear power generator can be far different from that of a factory manufacturing consumer goods for local consumption. By the same token, the *ceteris paribus* political risk for a factory manufacturing consumer goods for local consumption in Iran is likely to be different if it is French owned than if it is American owned. Thus,  $Y(t)$ ,  $\alpha$  and  $\psi$  must often be determined with respect to the investment under consideration in the context of traditional political risk analysis where an in-depth knowledge of local social, political and economic values plays the crucial role. The advantage of the foregoing model is that the questions that the analyst must answer are clearly defined: "What is the likely evolution

and standard deviation of the political environment associated with the investment expressed as a percentage of expected cash flows?" Thus, the first step is identifying the variables that will affect the political environment with respect to the investment. The second step is to estimate how these variables are likely to evolve. The third step is to calculate how changes in the political environment are likely to affect the dollar amount at risk.

On the macro level, many indices that reflect the political environment such as the consumer confidence index, the Free The World Economic Freedom Index, or The Heritage Foundation Index of Economic Freedom, among others, already exist. The advantage of these indices is that they are available for a wide range of countries over a relatively long time span so that a statistical relationship between the level of losses to political risk and the indices can easily be developed to determine  $Y(t)$ , its parameters and its correlation with  $D(t)$ . In the case where a macro-index is deemed inappropriate or inadequate, the analyst can use his knowledge to generate his own index and the associated parameters for  $\alpha$ ,  $\rho$  and  $\psi$ . For example, the same type of information contained in a special report could be applied to estimate a starting value for  $Y(t)$  along with the other relevant parameters. A more rigorous methodology would have the analyst construct a historical index that could be used to estimate the other parameters.

## ***5.2 Estimating and updating the parameters of the conditional Poisson process***

In order to estimate  $E(\Lambda \mid \{N(t_0) = n\})$ , we need to know the probability density function or the cumulative distribution function of  $\Lambda$ . Although  $\Lambda$  could be a discrete variable, in practice, given the nature of political risk, it is more likely to be a continuous variable. Thus, any distribution function for a variable with a positive range, such as the lognormal, the inverse

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<sup>14</sup> If the dividend or convenience yield on  $x(t)$  can be observed directly,  $\varphi$  can be calculated directly as  $\varphi = r - \delta_x$ , as

Gaussian, or the gamma, etc., is possible. This having been said, from a practical point of view, the gamma distribution is very convenient in that it lends itself to inputs by political analysis, as we shall see below, and all the updated distributions are gamma as well.<sup>15</sup> Once the distribution has been chosen, the parameters can be determined either by expert opinion supplied by the political analysts or by statistical methods such as Markov Chain Monte Carlo simulations. The updating procedure involves observing the number of events over the relevant interval and applying equation 19.

As an example of how the parameters of the conditional Poisson process can be estimated and updated, consider the case of an investment in Taiwan with a dollar amount at risk of \$1 million and an NPV in the absence of political risk of \$6 million. The risk-free rate is observed as  $r = 0.08$  and the analysts have determined that  $\varphi = 0$  and  $\sigma = 0.2$ . Loss causing political events can stem from local political conditions as well as from actions by the Republic of China that claims Taiwan as a renegade province. The possibilities of how this risk will manifest itself – blockades, saber rattling, war, intimidation, expropriation, discrimination against Taiwanese companies or those doing business with them, etc. – are simultaneous with random arrival rates and too numerous to be counted and measured independently. They are also likely to change over time as relations between the mainland and Taiwan mature and evolve. This situation can be captured by determining a continuous probability distribution and updating it over time. Suppose, then, that the random variable  $\Lambda$  is distributed according to a gamma distribution  $G(w, z)$  with the following p.d.f. parameterisation<sup>16</sup>

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mentioned in section 3.2 above.

<sup>15</sup> This system combines the Poisson and gamma distributions and is well known in decision theory and Bayesian analysis (Gelman et al., 1995, Bernardo & Smith, 1994). From a practical point of view, the gamma distribution is very convenient in that all the updated distributions are gamma as well.

<sup>16</sup> See the preceding footnote.

$$g(\lambda | w, z) = \frac{z^w}{\Gamma(z)} \lambda^{w-1} e^{-z\lambda} \quad (21)$$

It follows that the mean of this distribution is  $\frac{w}{z}$  and the variance is  $\frac{w}{z^2}$ .

Once the dollar amount at risk and the associated parameters have been determined, to start the political risk evaluation we only need to specify the beginning values of  $w$  and  $z$ , the parameters of the initial gamma distribution. To do this we require the input of the political risk analysts.<sup>17</sup> Suppose that based on an analysis of the China/Taiwan situation over the last five years, they estimate that loss causing events are very likely to come at a rate of 0.6 and that it would be very uncommon (2.5% probability) to have an arrival rate higher than 1.5. In fact, this is the only information needed. This subjective opinion can easily be quantified by a gamma distribution with  $w = 3$  and  $z = 5$  because the mean of this specific gamma distribution is 0.6 and 97.5% of the mass of the probability density function lies below 1.44 (in other words there is only 2.5% chance that the arrival rate is larger than 1.44), thereby reflecting the analysts' evaluation of the situation.

Suppose now that the time partition is given by a five-year cycle<sup>18</sup> and that there are no recorded events at the beginning of the budgeting process. Using equation 15 for a policy on a series of losses that cannot be cashed in with  $r = 0.08$ ,  $\varphi = 0$  and  $\sigma = 0.2$  gives the value of the insurance policy as \$7.5 million.<sup>19</sup> If one event is recorded over the following five-year period,  $w$  goes to 4 and  $z$  goes to 10. Using equation 19 to update the parameters and equation 20 to revalue the insurance policy, the value of the insurance policy falls to \$5 million. If 2, 0

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<sup>17</sup> In practice, this opinion could be obtained from an expert consultant, an analyst or a specialized agency.

<sup>18</sup> As mentioned above, the period itself should be determined by the country's characteristics. For example, it might be the electoral cycle as in Mexico, particular production peculiarities such as the beef cycle in Argentina, etc.

and 3 events respectively, occur in each of the next three five year periods, the *ceteris paribus* value of the insurance policy will be \$5, \$3.75 and \$4.5 million respectively. The details are summarized in Table 1.

Following Mahajan (1990) and Clark (1997), if the investment decision is taken at the beginning of the first time period, the NPV of the project adjusted for political risk is the unadjusted NPV minus the value of the insurance policy, which is  $\$6 - \$7.5 = -1.5$  million. The NPV adjusted for political risk is negative and the project is not undertaken. At the beginning of the second time period the adjusted NPV is positive,  $\$6 - \$5 = \$1$  million. Thus, if the investment decision is taken here, the investment will be accepted. Updating affects the investment decision.

**Table 1. Evolution of insurance solution in time and depending on the number of events occurred.**

| x(t) | r    | $\varphi$ | w | z  | $t_m$ | $n_m$ | V(x(t)) |
|------|------|-----------|---|----|-------|-------|---------|
| 1    | 0.08 | 0         | 3 | 5  | 0     | 0     | 7.5     |
| 1    | 0.08 | 0         | 4 | 10 | 5     | 1     | 5       |
| 1    | 0.08 | 0         | 6 | 15 | 10    | 2     | 5       |
| 1    | 0.08 | 0         | 6 | 20 | 15    | 0     | 3.75    |
| 1    | 0.08 | 0         | 9 | 25 | 20    | 3     | 4.5     |

## 6. Conclusion

<sup>19</sup> If the policy cannot be cashed in we rule out speculative bubbles so that the second boundary condition becomes  $V'(\infty) < \infty$ , which implies that  $A_1 = 0$ . The first boundary condition  $V(0) = 0$  is the same.

In this paper we have presented a model that measures political risk for international capital budgeting as the value of a hypothetical insurance policy that pays the holder any and all losses arising from political events. The model incorporates several features that address three important aspects of political risk that are widely acknowledged in the literature but either missing or incomplete in existing mathematical models. In this sense, it is very general. The multivariate nature and dependency of loss causing political events are modeled as a generalized, conditional Poisson process that allows for dependency between the increments of the counting process. To account for the fact that the composition and importance of the sources of political risk can change over time, we employ a Bayesian updating process whereby the distribution of the conditional Poisson process is updated as new information arrives. The random aspect of the amount actually lost when a political event occurs is captured in the geometric Brownian motions of the value of the investment in the absence of political risk and of the stochastic loss index that represents the expected size of the jump if a political event occurs.

Our approach is more flexible than known results and adapted to practical applications. We have shown how the model can be implemented with the inputs of the political risk analyst to estimate the parameters of the loss index and the stochastic arrival rate of loss causing political events. We have also used a mixture of Poisson and Gamma distributions to show how the analyst can interact in the Bayesian updating to improve the estimation process. He may decide to update the estimates of the parameters as more events unfold but he may also intervene subjectively in response to other signals or his own analysis. The latter can be done by modifying the parameters of the prior distribution.

We think that the most fruitful avenues of future research lie in applications of the model. To this end we are pursuing Bayesian techniques as a means of obtaining more accurate estimates of the conditional Poisson process and the parameters of its underlying distribution.



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## Appendix I – Solution when the policy can be cashed in

For a policy concerned with a series of losses, the differential equation to be solved is

$$\frac{1}{2} \sigma^2 x(t)^2 \frac{d^2 V}{dx^2} + \varphi x(t) \frac{dV}{dx} - rV + E(\Lambda \mid N(t_0) = n)x(t) = 0 \quad (\text{I.1})$$

and because  $r > \varphi$  the class of solutions of this equation is

$$V(x(t)) = \frac{E(\Lambda \mid N(t_0) = n)x(t)}{r - \varphi} + A_1 x(t)^{\gamma_1} + A_2 x(t)^{\gamma_2} \quad (\text{I.2})$$

where  $\gamma_1 > 1$  and  $\gamma_2 < 0$  are the roots to the quadratic equation in  $\gamma$ :

$$\frac{\sigma^2}{2} \gamma^2 + (\varphi - \frac{\sigma^2}{2}) \gamma - r = 0 \quad (\text{I.3})$$

The constants  $A_1, A_2$  depend on the boundary conditions. When exposure to political risk is zero the insurance policy has no value. Consequently  $V(0) = 0$  and this condition implies that  $A_2 = 0$ .

Since it is possible to cash in the insurance policy for a price related to its value, there will be a value of  $x$ , say  $x^*$ , where it will be optimal to cash the policy in. If  $\Omega(x^*(t))$  denotes the policy's cash in value then the value of the insurance policy cannot be higher than  $\Omega(x^*(t))$  because the policy will be cashed in when it reaches this level. The second boundary condition follows from the request that the value of the insurance policy is equal to the policy's cash-in value:

$$V(x^*(t)) = \Omega(x^*(t)) \quad (\text{I.4})$$

The smooth pasting condition that makes it possible to find both  $x^*$  and  $A_1$  is:

$$\frac{dV}{dx}(x^*(t)) = \frac{d\Omega}{dx}(x^*(t)) \quad (\text{I.5})$$

The solution can now be found analytically by solving the equations (I.4) and (I.5) simultaneously.