ON DERIVATIVES AND INFORMATION COSTS

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Information plays a central role in capital markets and in the process of asset pricing. The specific features of OTC markets require often an investment in information acquisition. Information costs can be defined in the context of Merton's (1987) model of capital market equilibrium with incomplete information. In this context, hedging portfolios can be constructed and analytic formulas can be derived using the Black and Scholes technology or the martingale method. This paper presents a simple framework for the valuation of exotic derivatives and OTC traded securities within a context of incomplete information. We incorporate information costs into a model, and then use this new model to price a variety of exotic options using the general context in Bellalah (2001). In each case, simple analytic formulae are derived.
From a pedagogical viewpoint, we illustrate the methodology and propose simple analytic formulas for pay-on-exercise options, power derivatives, outperformance options, guaranteed exchange-rate contracts in foreign stock investments, equity-linked foreign exchange options and quantos in the same context. These formulae are potential explanations of smiles and skews found in options price data. Our methodology can be applied for the valuation of several OTC and real options in the presence of incomplete information.

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Introduction

An important question in financial economics is how frictions affect equilibrium in capital markets since in a world of costly information, some investors will have incomplete information.
The trading of financial derivatives on organized exchanges has exploded since the begining of 1970's. The trading on "over-the-counter" or OTC market has exploded since the mid-1980's. Since the publication of the pionnering papers by Black-Scholes (1973) and Merton (1973), three industries have blossomed : an exchange industry in derivatives, an OTC industry in structured products and an academic industry in derivatives research. Each industry needs a specific knowledge regarding the pricing and the production costs of the products offered to the clients. Derivative instruments provide lower-cost solutions to investor problems than will competing alternatives. These solutions involve the repackaging of coarse financial products into their constituent parts to serve the investor demands. The "commoditisation" of instruments and the increased competition in the over-the-counter (OTC) market reduce profit margins for different players. The inevitable result is that products become more and more complex requiring more and more expenses in information acquisition. The problems of information, liquidity, transparency, commissions and charges are specific features of these markets.

Investors can spend time and resources to gather information about the stockmarket and other derivatives and OTC markets. For example, they may read newspapers, participate in seminars, subscribe to newsletters, join investment clubs, etc. Information in financial economics can be viewed as a commodity purchased in the market or produced in the household using both time and money as inputs.
Information costs correspond to the costs of collecting information and analyzing data. [2]

Merton (1987) adopts most of the assumptions of the original CAPM (Sharpe (1964) and Lintner (1965)) and relaxes the assumption of equal information across investors. Besides, he assumes that investors hold only

[2] Once the information is collected, the costs have already been incurred, and it is optimal to benefit from arbitrage opportunities whether or not they cover the original cost of collecting information.

securities of which they are aware. This assumption is motivated by the observation that portfolios held by actual investors include only a small fraction of all available traded securities. [3] The main distinction between Merton's model and the standard CAPM is that investors invest only in the securities about which they are "aware". This assumption is referred to as incomplete information. However, the more general implication is that securities markets are segmented.

Merton's (1987) model will be used as a key element in the valuation of derivative assets in this paper. This is because of the importance of information costs in the valuation process of financial assets.
The Merton's λ can be seen as a proxy for changes in the bid-ask spread.[4]

Kadlec and McConnell (1994) conclude that Merton's λ reflect also the elasticity of demand and that it may proxy for the adverse price movement aspect of liquidity.(footnote 19, page 629).
Peress (2000) presents a model to explain differences in households' portfolios by differences in private information. There is a main difference between Merton's (1987) model and the model in Peress (2000). In both models agents spend time and resources to gather information about the security's payoff, but in Merton's model investors are not all aware of the existence of the security but, if they are, they have information of the same quality. Hence, investors differ in the breadth of their cognizance.

All the above considerations about incomplete information must be accounted for in the pricing of options along the Black-Scholes (1973) lines and in the process of financial innovation. According to Merton (1998) :
"The most influential development in terms of impact on finance practice was the Black-Scholes model for option pricing... This sucess in turn increased

[3]In Merton's model, the expected returns increase with systematic risk, firm-specific risk, and relative market value. The expected returns decrease with relative size of the firm's investor base, referred to in Merton's model as the "degree of investor recognition". The model shows that an increase in the size of the firm's investor base will lower investors'expected return and, all else equal, will increase the market value of the firm's shares.

[4]In the study of Kadlec and McConnell (1994), the change in the parameter $\Delta\lambda$ is consistent with the predictions of Merton's model. Merton's λ may proxy for some aspects of liquidity that is not captured by the bid-ask spread.

the speed of adoption for quantitative financial models to help value options and assess risk exposures. pp 324".

In fact, the conceptual framework used to derive the option formula is used to price and evaluate the risk in financial and nonfinancial applications. New financial products and market designs improved computer and telecommunications technology. Innovations have improved efficiency by expanding opportunities for risk sharing, and reducing agency costs and information costs. This reveals the importance of information costs in the pricing of derivatives. The information cost corresponds for example for an institution to the costs of collecting information, analysing it, elaborating financial models, paying analysts, traders, etc. Therefore, we think that a risk-less hedge in a Black-Scholes economy must at least return the equivalent of a shadow cost of information plus the opportunity cost of funds. Even, if this cost is small for a specialized financial institution, it must be accounted for in the pricing of derivatives via a hedging portfolio as in the Black-Scholes theory.

Merton (1998) and Perold (1992) show that the cost of implementing financial strategies for institutions using derivatives can be one tenth to one twentieth of the cost of executing them in the underlying cash market securities. Hence, roughly speaking, if the information cost in using derivatives for a given asset is 5 %, then the information cost for the derivative is about one tenth of 5 %.

Using the concept of shadow costs of incomplete information, we have shown in Bellalah (1990, 1999 a, b) and Bellalah and Jacquillat (1995) how to account for these costs in the valuation of standard options. The proposed models account for information uncertainty and have the potential to explain the smile effect, which is a well-known anomaly in Black-Scholes type models. In this paper, we use a different approach and arbitrage arguments to derive the formulas for several OTC financial derivatives in a Black and Scholes (1973) economy with costly information. Valuation equations are proposed for pay-on-exercise options, power derivatives, outperformance options, guaranteed exchange-rate contracts in foreign stock investments, and equity-linked foreign exchange options and quantos. The approach can be used for the valuation of several other derivatives.

The structure of the paper is as follows.
Section 1 develops the main concepts regarding the pricing of options and commodity options within information uncertainty. Section 2 proposes simple formulas for the valuation of standard options in the presence of shadow costs of incomplete information. Section 3 presents a simple formula for the valuation of pay-on-exercise options. Section 4 presents some formulas for the pricing of power derivatives in the same context. Section 5 develops some simple formulas for valuation and hedging outperformance options. Section 6 shows how to price guaranteed exchange-rate contracts in Foreign stock investments within information uncertainty. Section 7 presents some formulas for the valuation and hedging of equity-linked foreign exchange options and quantos.

1. Arbitrage, information costs and option pricing

The valuation of simple or complex exotic options can be done in a Black and Scholes (1973) context. We explain how the arbitrage principle can be implemented in the presence of information uncertainty.

1.1. Arbitrage and information costs

Our definition of information costs or shadow costs of incomplete information is based on Merton's (1987) model.[5]

[5] Merton's model may be stated as follows :

$$\bar{R}_S - r = \beta_S[\bar{R}_m - r] + \lambda_S - \beta_S\lambda_m$$

where :

- $\bar{R}_S$: the equilibrium expected return on security S,
- $\bar{R}_m$: the equilibrium expected return on the market portfolio,
- r: the riskless rate of interest,
- $\beta_S = \frac{\mathrm{cov}(\tilde{R}_S/\tilde{R}_m)}{\mathrm{var}(\tilde{R}_m)}$: the beta of security S,
- λ_S: the equilibrium aggregate "shadow cost" for the security S. It is of the same dimension as the expected rate of return on this security S,
- λ_m: the weighted average shadow cost of incomplete information over all securities.

Arbitrage involves simultaneously at least two transactions in different markets giving the investor a riskless profit. Consider for example a stock traded on both the New York Stock Exchange and the Paris Bourse. Suppose that the stock is worth 200 dollars in New York and 201 euro in Paris.The exchange rate is one dollar for an euro. An arbitrageur could enter simultaneously in two transactions : buy the stock in New York and sell it in Paris. He would realize a profit of one euro per share bought. This arbitrage opportunity is attractive in the absence of transaction costs and information costs. This profitable opportunity is eliminated if informed arbitrageurs buy the stock in New York and sell it in Paris. The market forces will cause an equivalence between the prices in Paris and New York by acting on exchange rates. The actions of arbitrageurs eliminate the major disparity between the prices of the stock in different currencies.

Large international investment houses face low transaction costs. However, they suffer "shadow" costs to get informed about an arbitrage opportunity, to analyze market data, to implement models, to use information and communication technologies, to pay skills acting in these markets, etc. Arbitrage is implemented only if these shadow costs of "incomplete information" justify the deals. If these costs are less than one euro in the previous example, arbitrage is implemented. Hence, arbitrage is justified only if the gains from it cover at least all the necessary costs including the opportunity cost of funds (the interest rate). Arbitrage opportunities are detected only if we are informed about their existence. Therefore, an investor can never implement an arbitrage opportunity if he is not informed about it. Besides, the information cost must be less than the profit from arbitrage. If not, arbitrage will not be implemented and this can lead to an "inefficient" price in the market place.

As it appears in Merton's (1998) paper, his main contribution to the Black-Scholes option pricing theory was to demontrate the following result : in the limit of continuous trading, the Black-Scholes dynamic trading strategy designed to offset the risk exposure of an option would provide a perfect hedge. Hence, when trading is done without cost, the Black-Scholes dynamic strategy using the option's underlying asset and a risk-free bond would exactly replicate the option's payoff. In the absence of a continuous trading, which represents an idealized prospect, replication with discrete trading intervals is at best only approximate. In this case, the derivation of an option pricing model is completed by using an equilibrium asset pricing model. This approach is used in the original Black-Scholes model who derived their formula using the standard CAPM.

As it appears in the work of Black (1989), Scholes (1998) and as Merton (1998) asserts :
"Fisher Black always maintained with me that the CAPM-version of the option model derivation was more robust because continuous trading is not feasable and there are transaction costs". Using Merton's (1987) model, this implies an expected return equal to the riskless rate plus information costs.

1.2. Information costs and securities

In the standard Modigliani-Miller arbitrage theory, the assumption of the absence of frictions in financial markets allows an investment to yield a certain rate. Or in practice, arbitrage is not done instantaneously and investors can borrow or lend at presumed different rates.
Arbitrage can not be implemented without a minimal information about a given opportunity. Detecting this oportunity needs some expenses in information. Information costs may then justify the implementation of an arbitrage strategy. A rational investor is not able to invest in a given operation if he does not know about it. Therefore, we will have a Merton's (1987) incomplete information model which applies additional discount rate (shadow cost of incomplete information) to the asset's future cash flows. In this context, the unique cost of arbitrage in equilibrium would be the risk-free rate plus the information cost.
In this context, a trading strategy shaped by real-world information costs should incorporate an investment in well-known, visible stocks, and an investment delegated to professional money managers.

1.3. Information costs and derivatives

In the Black-Scholes theory, the investor can borrow or lend money at a risk-free rate and arbitrage takes place instantaneously. The main question is which investor implements arbitrage operations ?

Investors must first find the arbitrage opportunity. Then, they must collect the necessary funds and information to implement the strategy. Finally, they can implement the strategy based on some models for the pricing of financial assets. All this to say that arbitrage needs costly information.

For the ease of notation, let's denote the information cost by λ. A clear analysis of arbitrage implies the presence of an information cost for each asset and market. Hence, there are information costs for stocks, S, λ_S , for bonds, B, λ_B, etc. For a portfolio of N assets, there is an information cost for each asset and market. Since most of the option pricing models are based on an arbitrage argument, we expect information costs to appear in all arbitrage operations in bond markets, foreign exchange markets, equities transactions, derivatives, etc. While most traders are aware of the Black-Scholes theory, the arbitrage mechanism assumed cannot work in a real options market in the same way that it does in a supposed frictionless market. Figlewski (1989) studies the disparity between options arbitrage in theory and in practice.

One of the most important "imperfections" of real markets are transaction costs. The first main point is that hedging in not implemented in a continuous time framework.
The second point is that transaction costs are different from the costs of collecting information or information costs.
The third point, is that in less liquid markets, it is not always possible to implement an arbitrage strategy as described in the Black-Scholes theory.
The fourth point is that the appropriate hedge must account for some of the costs of arbitrage.
In a standard Black-Scholes approach in which the hedge is implemented instantaneously, the force of arbitrage drives the option price to its theoretical value. The results in Feglewski (1989) ignore information costs from the simulations since he uses only the trasaction costs which justify the band of arbitrage.

2. The extended Black and Scholes approach for standard options in the presence of shadow costs

In this section, we present valuation equations for standard options under incomplete information using the Black and Scholes partial differential approach (and the martingale approach).
The work of Markowitz (1952), Sharpe (1964) and Lintner (1965) on the

capital asset pricing model provided the general equilibrium model of asset prices under uncertainty. This model represents a fundamental tool in measuring the risk of a security under uncertainty.

The first work of Black and Scholes was to test the standard CAPM by developing the concept of a zero-beta portfolio. A zero-beta-minimum variance portfolio can be implemented by buying low beta stocks and selling high beta stocks. If the realized returns on this portfolio are different from the interest rate, this would be a violation of the predictions of the original CAPM. The arbitrage argument in Modigliani and Miller (see Miller (1988)) provided a general model of corporate finance by showing that the value of the firm is independent of how it financed its activities.
As it appears in Scholes (1998), his first work on option valuation was to apply the capital asset pricing model to value the warrants. The expected return of the warrant could not be constant for each time period if the beta of the stock was constant each period. This leads to the use of the CAPM to establish a zero-beta portfolio of common stocks and warrants. The portfolio is implemented by selling enough shares of common stock per each warrant held each period in order to create a zero-beta portfolio. In the context of the CAPM, the expected return on the net investment in the zero-beta portfolio would be equal to the riskless rate of interest. We will reproduce the same methodology here by applying the CAPMI of Merton (1987).

As it appears in Merton's (1998) paper, his main contribution to the Black-Scholes option pricing theory was to demontrate the following result : in the limit of continuous trading, the Black-Scholes dynamic trading strategy designed to offset the risk exposure of an option would provide a perfect hedge. Hence, when trading is done without cost, the Black-Scholes dynamic strategy using the option's underlying asset and a risk-free bond would exactly replicate the option's payoff.

Following the earlier work of Black and Scholes (1973), Black (1989) and Scholes (1998), consider the valuation of a warrant (or a derivative security) $C(S, t)$ where C is the derivative price, S is the current underlying asset price and T is the time to maturity. Using Taylor-series expansion of $C(S, t)$, ignoring terms of second order with respect to time over a short time interval, this gives

$$\Delta C(S,t) = \frac{\partial C}{\partial S}\Delta S + \frac{\partial C}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2 \quad (1)$$

In this expresion the term Δ corresponds to the change symbol. In this context, it is possible to use the simple model of capital market equilibrium with incomplete information (CAPMI) to describe the relation between the expected return on the warrant (or the option) and the expected return on the common stock and the market.
If the change in the option price as a function of its underlying price and time is replaced in the CAPMI, this shows how to create a zero-beta portfolio. This portfolio would have an expected rate of return equal to the constant interest rate plus the shadow cost of incomplete information.

For the ease of exposition, we follow the same steps as in Scholes (1998) and consider the returns on two investment strategies.
The first strategy consists in the purchase of the warrant or the option C and some riskless bonds using $(\frac{\partial C}{\partial S}S - C)$.
Buying the amount $(\frac{\partial C}{\partial S}S - C)$ in riskless bonds is equivalent to buying $(\frac{\partial C}{\partial S}S)$ in these bonds and selling C worth of these riskless bonds. The second strategy consists in buying $\frac{\partial C}{\partial S}$ of stocks or $\frac{\partial C}{\partial S}S$.
The initial cost is the same in both strategies and is equal to $\frac{\partial C}{\partial S}S$.
For the first strategy, the return on C is $\frac{\partial C}{\partial S}\Delta S + \frac{\partial C}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2$.
The return on $\frac{\partial C}{\partial S}S$ must be $\frac{\partial C}{\partial S}S(r + \lambda_S)\Delta t$ where r refers to the riskless rate. This is because of the sunk cost paid before investing in S since the investor requires the additional return λ_S before trading in this market.
The return on C must be $C(r + \lambda_C)\Delta t$ because of the sunk cost paid before investing in C where the investor requires the additional return λ_C.
It must be clear that even if the duplication argument is used by comparing with an investment in the riskless rate, the investor requires the additional return to be compensated for his sunk costs before constructing his portfolio of options and their underlying assets. The fact that the investor observes a return $(r + \lambda_C)$ or $(r + \lambda_S)$ does not cause an arbitrage opportunity for the bond markets since investors do not implement the replication strategy if they are not remunerated for the sunk costs by requiring an additional return. Even if they appear as sunk costs, information costs are regarded as continuous costs. In fact, by adding the information costs to the rate

of return on an investment, I assume a continuous information cost that is incurred throughout the hedging process. For the second strategy, the return on $\frac{\partial C}{\partial S}S$ must be $\frac{\partial C}{\partial S}\Delta S$.
The only uncertain term in both strategies is ΔS. As Δt goes to zero, the term $(\Delta S)^2$ converges (in the sense of mean squared convergence) $S^2\sigma^2 dt$ where σ is a constant proportional varianceof the underlying asset and dt is an infinitesimal change in time.
The term $(\Delta S)^2$ involves a form of variance which for a small time interval approaches $S^2\sigma^2$.
Since both strategies show the same risk and initial investment, then using the arbitrage argument, the returns must be the same over short times intervals. Equating the returns on the first strategy with those on the second strategy and substituting for ΔS^2, this gives the following differential equation

$$\frac{1}{2}\frac{\partial^2 C}{\partial S^2}S^2\sigma^2 + \frac{\partial C}{\partial S}S(r+\lambda_S) - (r+\lambda_C)C + \frac{\partial C}{\partial t} = 0 \tag{2}$$

The initial condition for a warrant or a call is $C(S,t^*) = max[S-K,0]$. where t^* is the maturity date, K is the strike price and $(t^*-t) = T$.
The number of units of the underlying asset needed to create a zero-beta portfolio is given by $\frac{\partial C}{\partial S}$.
Note also that the valuation depends only on the variability of returns and not on the expected return on the underlying asset.
In fact, in any case, the above valuation equation must apply because we hedge out risk of the underlying asset when constructing the zero-beta portfolio or the above replicating portfolio.
The main assumption leading to the above equation is that the expected return on the underlying asset over the next period of time is given by the CAPMI in the presence of a zero-beta stock.
It is also important to realize that since the underlying asset is assumed to have a zero beta, than the warrant or the option would also have a zero beta.
Using the Black-Scholes (1973) or Merton (1973) technology, the price of a European call is :

$$C(S,T) = Se^{-(\lambda_c-\lambda_S)T}N(d_1) - Ke^{-(r+\lambda_c)T}N(d_2) \tag{3}$$

with:
$d_1 = [\ln(\frac{S}{K}) + (r+\frac{1}{2}\sigma^2+\lambda_S)T]/\sigma\sqrt{T}$

$d_2 = d_1 - \sigma\sqrt{T}$
and where $N(.)$ is the univariate cumulative normal density function. [6]

If we consider a commodity contract or a commodity option, then the absence of costless arbitrage opportunities implies the following relationship :[7]

$$F = Se^{(b+\lambda_S)T}$$

where F is the current forward price, T is the option's maturity date, b is the constant proportional cost of carrying the commodity and λ_S is the information cost on the spot asset. In this case, the dynamics of forward prices are given by:

$$dF/F = (\mu - b - \lambda_S)dt + \sigma dz$$

Bellalah (1999 a) shows that the valuation formula for commodity calls is given by :

$$C(S,T) = Se^{((b-r-(\lambda_c-\lambda_S))T)}N(d_1) - Ke^{-(r+\lambda_c)T}N(d_2) \tag{4}$$

with:

$$\begin{aligned} d_1 &= [\ln(\frac{S}{K}) + (b + \frac{1}{2}\sigma^2 + \lambda_S)T]/\sigma\sqrt{T} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}$$

When λ_S and λ_c are set equal to zero, this equation collapses to that in Barone-Adesi and Whaley (1987). When λ_S and λ_C are set to zero and the cost of carrying the commodity $b = r$, this formula is the same as that in Black and Scholes.

Using the martingale approach and the risk neutral probability Q defined in the seminal papers of Harrison and Kreps (1979) and Harrison and Pliska (1981), the option price can be calculated as

$$C_t = e^{-r(t^*-t)}E_Q[S_{t^*}|_t]$$

[6] I worked on the solution to this problem in 1987 and the result appeared in my doctoral dissertation in 1990.

[7] See Bellalah (1999 a)

where $_t$ is the available information at time t. In the presence of information costs and under the probability Q, we have:

$$dS/S = [r + \lambda_S]dt + \sigma_S dW$$

where W is a brownian motion under Q. In this context, $(e^{-(r+\lambda_S)t}S_t)$ and $(e^{-(r+\lambda_c)t}C_t)$ are martingales under Q. If at time t^*, $C_{t^*} = max[S_{t^*} - K, 0]$ then the call value is given by:

$$C_t = e^{-(r+\lambda_c)(t^*-t)}E_Q[max[S_{t^*} - K, 0]|_t]$$

So we deduce :

$$C_t = e^{-(r+\lambda_c)(t^*-t)}[e^{(r+\lambda_S)(t^*-t)} \times C_t^{BS}] \tag{5}$$

where C_t^{BS} is the standard Black and Scholes price with a modified riskless rate equal to $(r + \lambda_S)$. Therefore, C_t is equal to the standard Black and Scholes price with a new riskless rate equal to $(r + \lambda_S)$ multiplied by the discount factor $e^{-(\lambda_c - \lambda_S)(t^*-t)}$.

3. The valuation of Pay-On-Exercise Options

A pay-on-exercise option is defined with respect to its strike price K and a payment B. When compared to a standard option, the option contract requires that the holder pays the option writer an amount B when the option expires in the money. At maturity, the buyer of a pay-on-exercise call receives the payoff of a standard call when the underlying asset price is above K and pays the option writer an amount B. Since the holder of such options, must pay an amount B, this option is always worth less than a standard call. Hence, the buyer of a pay-on-exercise call is entitled to a positive net payoff only when the underlying asset price is greater than $K + B$. However, if the underlying asset price lies between K and $K + B$, the buyer has to make a net payment to the option seller. This option can be valued in a Black and Scholes economy in a presence of a forward payment. Let's consider a forward on the underlying asset with a strike price K and a delivery date T. The value of this forward at time 0 is :

$$Se^{-dt} - Ke^{-(r+\lambda_S)T}$$

where d stands for a continuous dividend yield. The forward price is written as :

$$S_F = Se^{(r+\lambda_S - d)T} \tag{6}$$

Recall that the value of a standard call in a Black-Scholes (1973) world is :

$$c_a = e^{-(r+\lambda_c)T}[S_F N(d_1) - K N(d_2)] \tag{7}$$

$d_1 = [\ln(S_F/K) + (\sigma^2 T/2)]/\sigma\sqrt{T}$
$d_2 = d_1 - \sigma\sqrt{T}$

In this context, the value of a pay-on-exercise call is given by the same formula where K is replaced by $K + B$.

$$c_a = e^{-(r+\lambda_c)T}[S_F N(d_1) - (K+B) N(d_2)] \tag{8}$$

$d_1 = [\ln(S_F/K) + (\sigma^2 T/2)]/\sigma\sqrt{T}$
$d_2 = d_1 - \sigma\sqrt{T}$

It is possible to determine the value of B for which the contract initial value is zero. In this case, B is given by :

$$B = e^{-(r+\lambda_c)T} \frac{S_F N(d_1)}{N(d_2)} - K$$

This quantity corresponds to the fixed payment in the contract. The value of a pay-on-exercise put can be determined by a direct application of the put-call parity relationship :

$$p - c = (K+B)e^{-(r+\lambda_c)T} - Se^{-dT}$$

4. Pricing Power derivatives

There are at least three categories of power options.[8]
The first category consists of calendar-year and monthly physical options. The monthly options have some specifications similar to the electricity futures contract introduced on the New York Mercantile Exchange (Nymex) in 1996. The call option gives the right to the holder to receive power in a given location during some specified business days.
The second category corresponds to daily power options. These options are specified for a given period and can be exercised every day during this period.

[8] Eydeland and Geman (1998) describe power options and discuss the difficulties in pricing these options.

These options are not very liquid and are difficult to hedge.
The third category corresponds to hourly options. These options give access to power during some specified blocks of hours. The market for these options is also thin. For each category, the option payoff at the maturity date is given by $max[S_T - K, 0]$ where S_T is the spot price of electricity and K is the strike price per MWh. In general, power prices are highly volatile under extreme weather conditions and power exhibits higher price risks when compared to other assets like currencies, grains, metals gaz or oil. The convenience yield is a key concept in the pricing of commodity contracts. The convenience yield reflects the benefit from owning a commodity less the cost of storage. The relationship between the spot price $S(t)$ and the futures price $F(t,T)$ for a contract which matures in T years in the presence of information costs is :

$$F(t,T) = S(t)e^{(r-y+\lambda_S)(T-t)} \tag{9}$$

where y is the convenience yield and λ_S reflects the information cost on the underlying asset.
The convenience yield is assimilated to a continuous dividend yield made to the owner of the commodity. The European call on a commodity is given by an adaptation of Merton's (1973) commodity option model in the presence of information costs. The value at time 0 is :

$$c = S_t e^{-(y+\lambda_c-\lambda_S)(T-t)} N(d_1) - K e^{-(r+\lambda_c)(T-t)} N(d_2) \tag{10}$$

with :
$d_1 = \frac{[\ln(\frac{S_t e^{-(y+\lambda_c-\lambda_S)(T-t)}}{K e^{-(r+\lambda_c)(T-t)}}) + \frac{1}{2}\sigma^2(T-t)]}{\sigma\sqrt{T-t}}$
$d_2 = d_1 - \sigma\sqrt{T-t}$

The main difficulty in pricing power options is due to the fact that electricity is non storable. Therefore, the concept of convenience yield is misspecified in this context. One way to avoid this problem is to use forward or futures contracts for which the dynamics do not show the convenience yield. Hence, the use of the dynamics of power futures contracts and the specification of a convenient forward volatility structure $\sigma(t,T)$ allows a convenient approach for the pricing of power derivatives. Eydeland and Geman (1998) propose the following approximation for power futures prices :

$$F(t,T) = p_0 + j(w(t,T), L(t,T))$$

where :
p_0 : the load price,
$w(t,T)$: the forward price of marginal fuel (gas, oil, etc.),
$L(t,T)$: the expected load or demand for date T conditional on information at time t.
$j(.)$ represents a "power stack" function of the form $j = we^{(aL+b)}$ where a and b are positive constants.
A similar approximation can be developped in the presence of information costs where the "power stack" function j is of the following form :[9]

$$j = we^{(aL+b+\lambda_p)}$$

The above analysis shows that information costs can be used in the pricing of all commodities and commodity options.

5. Valuing and Hedging Outperformance Options

An outperformance option on an asset A vs an asset B with a maturity date T and a face value of one dollar is a contract with the following payoff in dollars :

$$c = \max[A(T) - B(T), 0] \tag{11}$$

The holder of this option receives at the maturity date any positive excess return of the asset A over the asset B. It is possible to value outperformance options by assimilating them as options issued in an imaginary country on a foreign underlying asset whose value is denominated in a foreign currency. Following Derman (1992), we use the following notations :
$S_i(0)$: the value in currency i at time $t = 0$ of one dollar worth of an asset S, (a stock),
$S_i(t)$: the value in currency i at time t of the same amount of stock S,
$\sigma(S_i)$: the volatility of stock S in currency i,
d_S : the continuous dividend yield for the asset S,
$c_i^{AB}(t)$: the value in currency i at time t of an outperformance option with the above payoff,

[9] When $F(t,T)$ is driven by the geometric Brownian motion, it is possible to get solutions in a standard Black and Scholes framework for monthly and calendar year options. For daily options, the analysis is complicated because of the absence of a standard hedging strategy.

$BS(S, K, r, \lambda_c, \lambda_S, \sigma, T-t)$: the equivalent of the Black and Scholes formula with information costs for a stock with price S, a strike price K, a riskless rate r, information costs λ_c and λ_S, a volatility σ and a time to maturity $T-t$.
The payoff of an outperformance option in dollar can be written as :

$$c_{\$}^{AB}(T) = \max[A_{\$}(T) - B_{\$}(T), 0] \tag{12}$$

The valuation of outperformance options in B-share currency units can be done as in Derman (1992). It can be shown that the value of an outperformance option in B-shares is similar to that of a standard call. Consider an investor who lives in a country where the currency is the B-share whose value at $t = 0$ is one dollar. The value of one share of stock A in this country can be written as : $A_B(t) = \frac{A_{\$}(t)}{B_{\$}(t)}$.
Since the value of one share of stock B, denoted B_B is one, and the riskless rate is the B dividend rate, d_B, the payoff of the outperformance call given by equation (12) can be expressed in B-shares as :

$$c_B^{AB}(T) = \max[A_B(T) - 1, 0] \tag{13}$$

This payoff corresponds to a standard option on $A_B(t)$ with a unit strike price. The volatility of the asset A_B is equivalent to the volatility of the asset $A_{\$}$ expressed in B-shares. In the presence of a correlation ρ_{AB} between the returns of the two assets $A_{\$}(t)$ and $B_{\$}(t)$, the volatility of the asset A_B is given by :

$$\sigma(A_B) = \sqrt{\sigma^2(A_{\$}) + \sigma^2(B_{\$}) - 2\rho_{AB}\sigma(A_{\$})\sigma(B_{\$})} \tag{14}$$

The value of the outperformmance call in equation (13) is given by the modified Black and Scholes formula in the presence of information costs with :

$$c_B^{AB} = BS(A_B, 1, d_B, \lambda_c, \lambda_S, \sigma(A_B), T-t) \tag{15}$$

The value of the outperformance option in dollars at time t can be obtained from the last formula. To do this, we take the value of this option in B-shares and convert it to dollars using the cross-rate, $B_{\$}(t)$:

$$c_{\$}^{AB}(t) = c_B^{AB}(t) B_{\$}(t)$$

or :

$$c_{\$}^{AB}(t) = B_{\$}(t)BS(A_B, 1, d_B, \lambda_c, \lambda_S, \sigma(A_B), T-t) \tag{16}$$

6. Guaranteed Exchange-rate contracts in Foreign stock investments

Guaranteed Exchange-rate contracts are derivative assets which have a dollar payoff that is independent of the exchange rate prevailing at the maturity date. They protect against exchange risk investors who hold foreign stocks or indexes. However, a change in the expected covariance between the exchange rate and the foreign asset price may produce a change in the contract's value. A guaranteed exchange-rate forward contract on a foreign stock is an agreement to receive on a certain date the stock's prevailing price in exchange for a specified foreign-currency delivery price. The prices are converted to dollars at a prespecified exchange rate. The value at delivery of a guaranteed exchange-rate forward contract on a given stock is given by the difference between the stock's price and the delivery price in a foreign currency. This difference is converted to dollars at a fixed exchange rate. In the same context, a GER call (put) entitles its holder the right to receive (deliver) on a certain date the stock's prevailing price in exchange for a specified foreign-currency delivery price. The prices are converted to dollars at a prespecified exchange rate.

6.1. Valuing a Guaranteed Exchange-rate forward contract

We use the following notations :
T : the time to delivery,
S_0 : the stock price in German mark at time $T = 0$,
$S(T)$: the stock price in German mark at time T,
d : the continuous dividend yield for the stock S,
$X(T)$: the spot dollar value of the German mark at delivery T,
K : the stock's delivery price in German marks,
X_0 : the value of the mark in dollars applied to convert the GER payoffs to dollars,
$F(0)$: the value of the GER forward in dollars at time $t = 0$,
$S_F(T)$: the stock's GER forward price in marks,
$r_{\$}$: the U.S riskless interest rate,

r_g : the German riskless interest rate,
σ_s : the volatility of stock S in marks,
σ_x : the volatility of the mark's value in dollars,
σ_{xs} : the covariance between returns of the mark in dollars and the stock price in marks,
ρ_{xs} : the correlation coefficient $\frac{\sigma_{xs}}{\sigma_x \sigma_S}$.

Following Derman, Karasinski and Wecker (1990), the dollar value of the forward contract at the delivery date is $(S(T) - K)X_0$. The value of the GER forward contract at time 0 is given by the discounted value of this payoff where all investments earn the U.S riskless rate. The expected value of this payoff needs the knowledge of the probability distributions of the dollar value of the mark and the dollar value of the German stock in a way such that the expected returns on these investments are the U.S riskless rate.[10]

Consider an investor who buys one share of the stock at $S(0)X_0$ dollars for $S(0)$ marks converted at X_0. If the stock pays d, he can reinvest this and own e^{dT} shares at delivery. The dollar value of his position is $e^{dT}S(T)X(T)$. The expected value of this position is :

$$E[e^{dT}S(T)X(T)] = e^{(d+r_s+r_x+\sigma_{xs}+\lambda_S)T}S(0)X_0 \tag{17}$$

where λ_S refers to the information costs on the underlying stock S. It is possible to show that $r_\$ = d + r_s + r_x + \sigma_{xs} - \lambda_S$.
Since $r_\$ = r_x + r_g$ or $r_g = r_\$ - r_x$, and since $r_s = r_\$ - r_x - d - \sigma_{xs} + \lambda_S$, it follows that the expected growth rate for the stock value in marks is :

$$r_s = r_g - d - \sigma_{xs} + \lambda_S$$

The above analysis shows that the lognormal distribution for the mark has a mean :

[10] In a Black and Scholes economy, the distribution of $X(T)$ is lognormal with a mean growth rate r_x and a volatility σ_x. The distribution of $S(T)$ is lognormal with a mean growth rate r_s and a volatility σ_s. In this context, the fair dollar value of a GER contract can be obtained from the discounting of its expected dollar-valued payoff at the U.S riskless rate. The dollar-based investment in the mark and the stock must have a mean growth rate $r_\$$. If we fix r_x and calculate the expected growth rate in the German mark, it is possible to show that $r_\$ = r_x + r_g$. If we fix r_s and calculate the expected growth rate of an investment in the German stock, then the following strategy can be used.

$E[X(T)] = X_0 e^{(r_\$ - r_g)T}$ and a volatilty σ_x.
The lognormal distribution for the stock has a mean :
$E[S(T)] = S(0)e^{(r_g - d - \sigma_{xs} + \lambda_S)T}$ and a volatilty σ_s.

Let us denote by $d' = d + \sigma_{xs}$. Since, the dollar value of the forward contract at the delivery date is $(S(T) - K)X_0$, then its initial value can be calculated using the last equation :

$$F(0) = e^{-(r_\$ + \lambda_c)T} E[(S(T) - K)X_0] = e^{-(r_\$ + \lambda_c)T}[S(0)X_0 e^{(r_g - d' + \lambda_S)T} - KX_0]$$

Let $S_F(T)$ (the forward price for delivery at T) be the value in marks of the GER delivery price K that makes the forward contract's value $F(0) = 0$, i.e.

$$S_F(T) = S(0)e^{(r_g - d' + \lambda_S)T} \tag{18}$$

By comparing this last equation and $[S(T)] = S(0)e^{(r_g - d - \sigma_{xs} + \lambda_S)T}$ with $d' = d + \sigma_{xs}$, it is clear that the forward price is simply the expected value of the stock in this economy.

6.2. Valuing a Guaranteed Exchange-rate option

Consider the pricing of a GER option with a strike price of K German marks. The payoff of a European GER put struck at X_0K dollars is :

$$\max[X_0K - X_0S(T), 0]$$

where the underlying imaginary stock has a volatility σ_s and a mean growth rate $(r_g - d')$. The put value in dollars at time 0 is :

$$p_0 = e^{-(r_\$ + \lambda_c)T} E[\max[X_0K - X_0S(T), 0]]$$

The value at time 0 is :

$$p_0 = X_0 e^{-r_\$ T}[Ke^{-\lambda_c T}N(-d_2) - S(0)e^{(r_g - d' - \lambda_c + \lambda_S)T}N(-d_1)] \tag{19}$$

with :
$d_1 = [\ln(\frac{S_0}{K}) + (r_g - d' + \lambda_S + \frac{1}{2}\sigma_s^2)T]/\sigma_s\sqrt{T}$
$d_2 = d_1 - \sigma\sqrt{T}$

It is possible to verify that the value of GER forward is equivalent to a portfolio with a long call and a short put with the same strike price K in marks with a guranteed exchange rate X_0.

7. Equity-linked foreign exchange options and quantos

As shown in Garman and Kohlhagen (1983), the Black and Scholes (1973) formula for stock options applies as well to the valuation of options on currencies where the foreign interest rate replaces the dividend yield. When an investor wants to link a strategy in a foreign stock and a currency, he can use at least four different type of options : a foreign equity option struck in foreign currency, a foreign equity option struck in domestic currency, fixed exchange rate foreign equity options known also as quanto options or an equity-linked foreign exchange option. These different types of options are analysed and valued in the this section.

7.1. The foreign equity call struck in foreign currency

When an investor in a foreign equity is not interested in the risk born from the drop in the exchange rate, he may invest in a foreign equity call struck in foreign currency. This option has the following pay-off :

$$C_1^* = X^* max[S^{'*} - K', 0]$$

where $S^{'*}$ is the equity price in the currency of the investor's country and K' is a foreign currency amount. The spot exchange rate expressed in domestic currency of a unit of foreign currency, X^*, stands in front of the pay-off to show that the latter must be converted into domestic currency.
The domestic currency value of this call option is given by :

$$C_1 = S'Xe^{-(d+\lambda_c-\lambda_S)T}N(d_1) - K'Xe^{-(r^*+\lambda_c)T}N(d_1 - \sigma_{S'}\sqrt{T}) \qquad (20)$$

where :

$$d_1 = [\ln(\frac{S'e^{-(d+\lambda_c-\lambda_S)T}}{K'e^{-(r^*+\lambda_c)T}})]/\sigma_{S'}\sqrt{T} + \frac{1}{2}\sigma_{S'}\sqrt{T}$$

where $\sigma_{S'}$ is the volatility of S' and r^* is the foreign risk-free rate.
This option can be easily hedged by an amount Δ_S in stocks and B' units of

foreign cash with :

$$\Delta_S = e^{-(d+\lambda_c-\lambda_S)T} N(d_1)$$
$$B' = -K' e^{-(r^*+\lambda_c)T} N(d_1 - \sigma_{S'}\sqrt{T})$$

7.2. The foreign equity call struck in domestic currency

When an investor wants to be sure that the future pay-off from the foreign market is meaningful when converted in his own currency, then the foreign equity option struck in foreign currency is appropriate. This option has the following pay-off for a call :

$$C_2^* = max[S^{'*} X^* - K, 0]$$

where K is the domestic currency amount.

For the foreign option writer, the pay-off is given by :

$$C_2^{'*} = max[S^{*'} - KX^{'*}, 0]$$

where $X^{'} = \frac{1}{X}$. X corresponds to the exchange rate quoted at the price of a unit of domestic currency in terms of the foreign currency. This pay-off corresponds to that of an option to exchange one asset (K units of our currency) for an other asset (a share of stock). The value of this option in the presence of information costs is given by :

$$C_2^{'*} = S' e^{-(d+\lambda_c-\lambda_S)T} N(d_2) - KX' e^{-(r+\lambda_c)T} N(d_2 - \sigma_{S'X'}\sqrt{T}) \qquad (21)$$

with :

$$d_2 = [\ln(\frac{S' e^{-(d+\lambda_c-\lambda_S)T}}{KX' e^{-(r+\lambda_c)T}})]/\sigma_{S'X'}\sqrt{T} + \frac{1}{2}\sigma_{S'X'}\sqrt{T}$$

$$\sigma_{(S'X')} = \sqrt{\sigma_{S'}^2 + \sigma_{X'}^2 - 2\rho_{S'X'}\sigma_{S'}\sigma_{X'}}$$

where $\rho_{S'X'}$ is the correlation coefficient between the rates of return on S' and X'. If we multiply this formula by the exchange rate and substitute $1/X$ for X', we get the domestic value of this option in the same context, i.e :

$$C_2 = S'Xe^{-(d+\lambda_c-\lambda_S)T}N(d_2) - Ke^{-(r+\lambda_c)T}N(d_2 - \sigma_{S'X}\sqrt{T}) \qquad (22)$$

with :
$d_2 = [\ln(\frac{S'Xe^{-(d+\lambda_c-\lambda_S)T}}{Ke^{-(r+\lambda_c)T}})]/\sigma_{S'X}\sqrt{T} + \frac{1}{2}\sigma_{S'X}\sqrt{T}$

$$\sigma_{(S'X)} = \sqrt{\sigma_{S'}^2 + \sigma_X^2 - 2\rho_{S'X}\sigma_{S'}\sigma_X} = \sigma_{(S'X')}$$

This option can again be easily hedged by an amount $\Delta_{S'}$ in stocks and B' units of foreign cash with :

$$\Delta_{S'} = e^{-(d+\lambda_c-\lambda_S)T}N(d_2)$$

$$B' = -Ke^{-(r+\lambda_c)T}N(d_2 - \sigma_{S'X}\sqrt{T})$$

This formula is equivalent to that of Black and Scholes in the presence of information costs with $S'X$ replacing S and $\sigma_{S'X}$ replacing σ. It is as if the Black and Scholes risk-neutral pricing approach were applied to the underlying asset $S'X$. This allows the derivation of simple rules that can be applied for the valuation and the hedging of the two following options.

7.3. Fixed exchange rate foreign equity call

When an investor wants to capture upside returns on a foreign investment and desires to hedge away all exchange risk by fixing in advance a rate that allows him to convert the payoff into domestic currency, this links a foreign equity option with a currency forward. This desired payoff corresponds to a fixed exchange rate foreign equity call, known as a Quanto with the following pay-off :

$$C_3^* = \bar{X}max[S^{'*} - K', 0] = max[S^{'*}\bar{X} - K, 0]$$

where $\bar{X}$ is the rate at which the conversion will be made. This payoff can be written in reciprocal units as :

$$C_3^{'*} = \bar{X}X^{'*}max[S^{'*} - K', 0]$$

The pay-off can be expressed in the following form :

$$C_3^{'*} = \bar{X}X^{'}e^{v}max[S^{'}e^{u} - K', 0]$$

where u and v stand for the natural logarithm of one plus the returns of S' and X'.

Following the methodology in Reiner (1992) and using the joint distribution for u and v, the value of this option in foreign currency in the presence of information costs is :

$$C_3' = \bar{X}X'[S'[\frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)T}}{e^{-(r^*+\lambda_c)T}})e^{-(\rho_{S'X}\sigma_{S'}\sigma_X)T}N(d_3) - K'e^{-(r+\lambda_c)T}N(d_3-\sigma_{S'}\sqrt{T})] \tag{23}$$

with :

$$d_3 = [\ln(\frac{S'e^{-(d+\lambda_c-\lambda_S)T}}{K'e^{-(r^*+\lambda_c)T}}) - \rho_{S'X}\sigma_{S'}\sigma_X T]/\sigma_{S'}\sqrt{T} + \frac{1}{2}\sigma_{S'}\sqrt{T}$$

The domestic value of this option is :

$$C_3 = \bar{X}[S'[\frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)T}}{e^{-(r^*+\lambda_c)T}}]e^{-(\rho_{S'X}\sigma_{S'}\sigma_X)T}N(d_3) - K'e^{-(r+\lambda_c)T}N(d_3-\sigma_{S'}\sqrt{T})] \tag{24}$$

with :

$$d_3 = [\ln(\frac{S'e^{-(d+\lambda_c-\lambda_S)T}}{K'e^{-(r^*+\lambda_c)T}}) - \rho_{S'X}\sigma_{S'}\sigma_X T]/\sigma_{S'}\sqrt{T} + \frac{1}{2}\sigma_{S'}\sqrt{T}$$

This option can be hedged in an unusual form by an amount Δ_S in stocks, B' in foreign cash and B in domestic currency where :

$$\Delta_S = \frac{\bar{X}}{X}\frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)T}}{e^{-(r^*+\lambda_c)T}}e^{-\rho_{S'X}\sigma_{S'}\sigma_X T}N(d_3)$$

$$B' = -\Delta_{S'}S'$$

7.4. An equity linked foreign exchange call

When an investor desires foreign equity exposure and wants to place a floor on the exchange component, he uses a strategy that combines a currency option with an equity forward. This strategy creates an equity linked foreign exchange call with the following payoff :

$$C_4^* = S'^* max[X^* - K, 0]$$

Note that this contract is the complement of the previous one. This payoff can be written also as :

$$C_4^{'*} = S^{'*} max[1 - KX^{'*} - K, 0] = KS^{'*} max[\frac{1}{K} - X^{'*}, 0]$$

The foreign value of this call option is given by :

$$C_4^{'*} = S' e^{-(d+\lambda_c-\lambda_S)T} N(d_4) - KS'X'[\frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)T}}{e^{-(r^*+\lambda_c)T}}] e^{-(\rho_{S'X}\sigma_{S'}\sigma_X)T} N(d_4 - \sigma_X\sqrt{T}) \tag{25}$$

with :

$$d_4 = [\ln(\frac{Xe^{-(r^*+\lambda_c)T}}{Ke^{-(r+\lambda_c)T}}) + \rho_{S'X}\sigma_{S'}\sigma_X T]/\sigma_X\sqrt{T} + \frac{1}{2}\sigma_X\sqrt{T}$$

The domestic value of this call option is :

$$C_4 = S'Xe^{-(d+\lambda_c-\lambda_S)T} N(d_4) - K[S' \frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)T}}{e^{-(r^*+\lambda_c)T}}] e^{-(\rho_{S'X}\sigma_{S'}\sigma_X)T} N(d_4 - \sigma_X\sqrt{T}) \tag{26}$$

This option can again be hedged in an unusual form by an amount Δ in stocks, B' in foreign cash and B in domestic currency with:

$$\Delta_S = C_4 \frac{C_4}{S'X}$$

$$B = -B'X$$

$$B' = \frac{KS'}{X} \frac{e^{-(r+\lambda_c)(d+\lambda_c-\lambda_S)}}{e^{-(r^*+\lambda_c)T}} e^{-(\rho_{S'X}\sigma_{S'}\sigma_X)T} N(d_4 - \sigma_X\sqrt{T})$$

$$e^{-(d+\lambda_c-\lambda_S)T} N(d_4)$$

$$B' = -Ke^{-(r+\lambda_c)T} N(d_4 - \sigma_{S'}\sqrt{T})$$

The following Table summarizes the main results with respect to the Black and Scholes formula in the presence of information costs.

$$Table - 1$$

The results for the different options using several models in the presence of information costs : Black-Scholes (B-S), Garman-Kohlhagen (G-K), Foreign-equity/Foreign-strike (FE/FS), Foreign-equity/Domestic-strike (FE/DS), Fixed-rate-foreign-equity,(FR/FE), Equity-linked-foreign-exchange,(FL/FE), Equity-linked-foreign-exchange (EL/FE).

$Type$	$Asset$	$Strike$	$Rate$	D.T.A	σ
B-S	S	K	$(r+\lambda_c)$	$(d+\lambda_c-\lambda_S)$	σ_S
G-K	X	K	$(r+\lambda_c)$	$(r^*+\lambda_c)$	σ_X
FE/FS	$S'X$	$K'X$	$(r^*+\lambda_c)$	$(d+\lambda_c-\lambda_S)$	$\sigma_{S'}$
FE/DS	$S'X$	K	$(r+\lambda_c)$	$(d+\lambda_c-\lambda_S)$	$\sigma_{S'X}$
FL/FE	$S'\bar{X}$	$K'\bar{X}$	$(r+\lambda_c)$	$\frac{-(r+\lambda_c)(d+\lambda_c-\lambda_S)}{(r^*+\lambda_c)})e^{\rho_{S'X}\sigma_{S'}\sigma_X}$	$\sigma_{S'}$
EL/FE	$S'X$	KS'	$\frac{-(r+\lambda_c)(d+\lambda_c-\lambda_S)}{(r^*+\lambda_c)}e^{\rho_{S'X}\sigma_{S'}\sigma_X}$	$(d+\lambda_c-\lambda_S)$	σ_X

D.T.A : distributions to the underlying asset.

Conclusion

Information plays a central role in the pricing of financial assets. Merton (1987) provides a simple model of capital market equilibrium with incomplete information. Merton's (1987) model shows that asset returns are an increasing function of their beta risk, residual risk, and size and a decreasing function of the available information for these assets. This model offers some insights on how information affects securities prices and the process leading to financial innovations.

As it appears in several papers, information affects the securities industry and the prices of financial assets. Therefore, it is important to develop models which account for the effects of information costs on asset prices. Differences in information are important in financial and real markets. They are used in several contexts to explain some puzzling phenomena like the 'home equity bias', the 'weekend effect', "the smile effect", etc.

When explaining the process of financial innovation, Scholes (1998) argues that innovations will continue because of the insatiable demand for

lower-cost. Information and financial technology will expand and so will the circle of understanding of how investors use this technology. As information and financial technology become more easily available, this will spur financial innovation.
In fact, in a world of information asymmetries, derivatives and OTC derivatives can provide lower-cost solutions to financial contracting problems and these solutions enhance economic efficiency.

When explaining the future directions of applications of the derivative technology, Merton (1998) asserts that the low-cost availability of the Internet does not solve the "principal agent" problem. Therefore, he believes that the trend will shift toward more integrated financial products and services which will integrate human capital considerations, hedging and income tax planning into the asset allocation decisions.

The concept of information costs is used in this paper for the pricing of futures contracts, commodity options and several standard OTC options in the presence of information uncertainty as in Bellalah (2001). The main ideas in Merton's (1987) model, Scholes (1998) and Merton (1987) Nobel lectures are used in this work. The extension of their arguments regarding option technology and information costs allows us to price several OTC derivatives in light of the recent suggestions in the work of Merton (1998) and Scholes (1998). The main intuition behind our extensions is the lack of transparency and liquidity in some OTC markets which is reflected in the search of costly information.
The problems of liquidity and transparency require an investment in information acquisition. The costly arbitrage concept is used in the pricing of financial assets in the presence of shadow costs of incomplete information. This allows the derivation of some equations for the pricing of derivatives.

Figlewski (1989) among others conclude that the impact of market imperfections is large and may be larger than many researchers have realized. In this context, the standard arbitrage cited in the literature becomes a weak force to drive actual option prices toward their theoretical values. Hence, option arbitrage must account for some of the market imperfections and at least the information costs considered in this study.
Our analysis extends the standard Black-Scholes context and offers some pos-

sible answers to the following issues. The first point is that since transaction costs are different from the costs of collecting information or information costs, it is possible to account for these costs in our analysis.
The second point is that in less liquid markets, it is not always possible to implement an arbitrage strategy as described in the Black-Scholes theory.
The third point is that the appropriate hedge must account for some of the costs of arbitrage.

Using the above main points, we provide simple analytic formulas for the pricing of pay-on-exercise options, power derivatives, outperformance options, guaranteed exchange-rate contracts in foreign stock investments, equity-linked foreign exchange options and quantos in the presence of information costs. Our models can be extended to the valuation of all other known OTC derivatives in the same context. This analysis can also be extended to the valuation of real options and in capital budgeting decisions as in Bellalah (2000 a,b, c). We are actualy collecting data to make empirical tests of some of our formulas.

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