Stock Market Dynamics in a

Regime Switching Asymmetric Power GARCH Model

by

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and

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Abstract

This paper analyzes the dynamics of Asian stock index returns through a Regime-Switching Asymmetric Power GARCH model (RS-APGARCH). The model confirms some stylized facts already discussed in former studies but also highlights interesting new characteristics of stock market returns and volatilites. Mainly, it improves the traditional regime-switching GARCH models by including an asymmetric response to news and, above all, by allowing the power transformations of the heteroskedasticity equations to be estimated directly from the data. Several mixture models are compared where a first-order Markov process governs the switching between regimes.

Keywords: Power GARCH, Regime Switching, Volatility, Stock Market.

JEL Classification: C13, C52, G15.

I. Introduction

Since the availability of high frequency financial data, a growing body of empirical studies, starting with Fama and French (1989), has investigated the predictability of mean and volatility of asset returns. Volatility of financial returns is indeed a central parameter for many financial decisions including the pricing and hedging of derivative products and risk management. Most of the volatility models presented in the empirical literature are based on the observation that volatility is time-varying and that periods of high volatility tend to cluster. The autoregressive conditional heteroskedasticity (ARCH) models, as introduced by Engle (1982) and extended to Generalized ARCH (GARCH) in Bollerslev (1986) have proven to be a useful means for empirically capturing these stylised facts.

Although such approaches provide an improvement in fit compared with constant variance models, recent evidence from financial market data seems to suggest that persistence in variance, as measured by GARCH models is so substantial that it sometimes implies an explosive conditional variance. To account for this apparent empirical regularity, Engle and Bollerslev (1986) introduce the Integrated-GARCH (I-GARCH) process, in which shocks to the variance do not decay over time. However, Lamoureux and Lastrapes (1990) show that one potential source of misspecification of ARCH/GARCH models is that the structural form of conditional means and variances is relatively inflexible and is held fixed throughout the entire sample period. As explained in Timmermann (2000) if the variance is high but constant for some time and low but constant otherwise, the persistence of such high- and low- volatility homoskedastic periods already results in volatility persistence. GARCH models, that cannot capture the persistence of such periods, put all the volatility persistence in the persistence of individual shocks, biasing thus upward our assessment of the degree to which conditional variance is persistent.

Although the ad hoc introduction of deterministic shifts into the variance process represents one possibility to allow for periods with different unconditional variances, the most promising approach to modelling these nonlinearities is by endogenizing changes in the data generating process through a Markov regime-switching model as introduced in Hamilton (1989). The model relies on different coefficients for each regime to account for the possibility that the economic mechanism generating the asset returns may undergo a finite number of changes over the sample period. In order not to rule out within-regime heteroskedasticity, Gray (1996) extends Hamilton's (1989) model to accommodate within-regime GARCH effects with a so-called Regime-Switching GARCH model (RS-GARCH). RS-GARCH models have the attractive feature of incorporating significant nonlinearities, while remaining tractable and easy to estimate. Although they represent a suitable framework to investigate how the volatility dynamics is affected by the states of the economy, surprisingly few improvements of the single-regime ARCH/GARCH literature have been adapted and tested in their regime-switching counterparts.

In particular, under classical GARCH models, shocks to the variance persist according to an autoregressive moving average (ARMA) structure of the squared residuals of the process. However, it is not necessary to impose a squared power term in the second moment equation as in Bollerslev (1986). Taylor's (1986) and Schwert's (1989) class of GARCH models, for instance, relate the conditional standard deviation of a series to lagged absolute residuals and past standard deviations. More recently, Ding, Granger and Engle (1993) suggest an extension of the GARCH family models that analyses a wider class of power transformations than simply taking the absolute value or squaring the data as in the traditional heteroskedastic models. Known as the Power GARCH (PGARCH) models, this addition to the GARCH family has been shown to be superior in fit to its less sophisticated counterparts (see Brooks, Faff, McKenzie, and Mitchell (2000) for an empirical investigation in a single-regime framework). Nesting the major two classes of GARCH models (namely, Bollerslev's and Taylor-Schwert's) the PGARCH specification also provides an encompassing framework which facilitates comparison.

An important contribution of the current paper is to highlight whether and to what extent these more flexible models improve both the fit and our understanding of asset returns dynamics when the assumption of a single regime is relaxed in favor of a regime-switching model. To this end, we introduce a new Regime-Switching Asymmetric Power GARCH (RS-APGARCH) model to analyze empirically Asian stock index returns. Our findings shed light on several interesting stylized facts about the relationships between both the dynamics of the conditional mean and variance and the state of the economy. It is shown that the RS-APGARCH model proposed in this paper is able to match some empirical regularities of stock index returns that could not be captured with the traditional regime-switching models already introduced in the literature, let alone using a single-regime GARCH model.

Another important novelty of our approach compared to the classical literature on regime-switching processes regards the choice of the underlying conditional distributions. Indeed, a regime-switching model relies on a mixture of conditional distributions where the parameters are either held constant - Hamilton (1989) - or rendered time-varying - Gray (1996) -. Following the traditional literature on mixture of distributions (see Kon (1984) or Ané and Labidi (2001)) most Markov regime-switching models adopt conditional Gaussian mixtures. Since our analysis focuses on recent years where stock markets have undergone important shocks (both economical and political), financial assets have experienced periods of extreme volatility. In order to capture a higher degree of kurtosis in asset returns, we follow Perez-Quiroz and Timmermann (2001) and introduce in our RS-APGARCH model a mixture of a Gaussian distribution and a Student-t density. In such a mixture, outliers or extreme returns will be modeled as drawn from a fat-tailed t-distribution with few degrees of freedom whereas the moderate returns will be generated by the conditional Gaussian density. With this additional characteristic our model enables us to differentiate the effect of the states of the economy on the dynamics of asset returns far beyond the mere difference of parameter values and/or conditional mean and variance structure: it allows for higher order conditional differences through conditional densities that exhibit very different probabilistic structures. We then test the necessity of introducing two leptokurtic densities in the model.

Finally, another contribution of this paper arises from the APGARCH structure used on the volatility of each regime. Ding and Granger (1996) show that the power term transformation of this model can be related the long run temporal dependency in the volatility also called the long memory property of the volatility. Using APGARCH models in a regime-switching framework we are thus able to investigate whether the degree of temporal dependency changes with the states of the economy.

The remainder of the paper is organized as follows. Section 2 describes the new Regime-Switching Asymmetric Power GARCH model introduced in this paper. The data and a preliminary empirical investigation motivating the use of our model are presented in Section 3. Section 4 contains the main empirical findings and the goodness-of-fit tests while Section 5 concludes.

II. The RS-APGARCH Model

One purpose of this paper is to investigate the impact of a general variance equation specification in a regime-switching context. Hence, without implying that the mean equation has no interest, we follow a classical approach and simply assume an autoregressive structure for the mean, that is:

$$R_t = a_0 + \sum_{i=1}^{l} a_i R_{t-i} + \varepsilon_t \ , \qquad (1)$$

where R_t is the stock market index return, $\mu_t = a_0 + \sum_{i=1}^{l} a_i R_{t-i}$ is the conditional mean and ε_t is the error term in period t. We use the t-statistics associated with the $a_i's$ in a single-regime version of the model to determine the optimal number of lags l to include in its regime-switching counterparts. Ljung-Box statistics were also used on the final specification to ensure that no significant higher order serial correlation is found in the series.

The error term in the mean equation (1) may be decomposed as

$$\varepsilon_t = \sigma_t e_t , \qquad (2)$$

where $e_t \sim (0,1)$. The standardized error term e_t is usually assumed to be normally distributed:

$$e_t \sim N(0,1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e_t^2}{2}\right). \qquad (3)$$

However, to capture conditional kurtosis in the error term, a Student-t density with few degrees of freedom ν is sometimes introduced:

$$e_t \sim t\,(0,1,\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}} \left(1+\frac{e_t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \qquad (4)$$

where $\Gamma(.)$ is the gamma function.

Studies on the predictability of asset returns also allow for nonlinear effects by explicitly modeling time dependence in the second conditional moment:

$$\sigma_t = \Psi\left(\{\varepsilon_{t-i}\}_{i=1}^{p}, \{\sigma_{t-i}\}_{i=1}^{q}\right), \qquad (5)$$

where $\Psi(\,.\,)$ is some time-invariant function.

Although such models allow for a wide range of nonlinear dynamics, they still assume a single structure for the conditional mean and variance and leave no room for the economic mechanism generating prices to undergo substantial changes according to the states of the economy. Recent economic theories, however, provide evidence of strong asymmetries in stock returns with regard to the underlying economic state. Following the popular Markov-switching approach introduced by Hamilton (1989), we use a latent regime indicator s_t to allow the mean and variance of a stationary series to take different structures according to the states of the economy. The mixture model presented in this paper follows the usual practice and assumes two states. The mean and variance equations now become state-dependent as outlined below:

$$\begin{cases} R_{s_t\, t} = a_{s_t\, 0} + \sum_{i=1}^{l} a_{s_t\, i} R_{t-i} + \varepsilon_t \\ \varepsilon_t \sim (0, \sigma^2_{s_t\, t}) \\ s_t = 1, 2 \end{cases} . \qquad (6)$$

The regime indicator s_t is then parameterized as a first order Markov process where

$$\begin{cases} P(s_t = 1 / s_{t-1} = 1, F_{t-1}) = P \\ P(s_t = 2 / s_{t-1} = 1, F_{t-1}) = 1 - P \\ P(s_t = 2 / s_{t-1} = 2, F_{t-1}) = Q \\ P(s_t = 1 / s_{t-1} = 2, F_{t-1}) = 1 - Q \end{cases} . \qquad (7)$$

If we denote $\eta(R_t / s_t = j, F_{t-1}; \theta_j)$ the distribution of R_t in regime $s_t = j$ conditional on information F_{t-1}, the unconditional returns R_t follow a mixture of distributions:

$$R_t / F_{t-1} \sim \begin{cases} \eta(R_t / s_t = 1, F_{t-1}; \theta_1) & w.p. \quad p_{1t} \\ \eta(R_t / s_t = 2, F_{t-1}; \theta_2) & w.p. \quad 1 - p_{1t} \end{cases} , \qquad (8)$$

where $p_{1t} = P(s_t = 1 / F_{t-1}; \theta)$ is the conditional probability of being in state 1 at time t given the information at time $t-1$ and $\theta = (\theta_1, \theta_2)$ is the set of parameters. The probability p_{1t} is called 'ex ante regime probability' because it is based solely on information already available at time $t-1$ and it forecasts the prevailing regime in the next period. These conditional state probabilities can be obtained recursively using the total probability theorem:

$$p_{1t} = P(s_t = 1 / F_{t-1};\, \theta) = \sum_{j=1}^{2} P(s_t = 1 / s_{t-1} = j, F_{t-1};\, \theta)\, P(s_{t-1} = j / F_{t-1}; \theta)\,. \qquad (9)$$

Finally, using Bayes' theorem, the conditional state probability p_{1t} can be written as:

$$p_{1t} = P(s_t = 1 / F_{t-1}; \theta) = \begin{array}{l} P \left[\dfrac{\eta(R_{t-1} / s_{t-1} = 1; F_{t-2};\ \theta_1)\ p_{1t-1}}{\eta(R_{t-1} / s_{t-1} = 1; F_{t-2}; \theta_1)\ p_{1t-1} + \eta(R_{t-1} / s_{t-1} = 2; F_{t-2}; \theta_2)\ (1 - p_{1t-1})} \right] \\ +\ (1-Q) \left[\dfrac{\eta(R_{t-1} / s_{t-1} = 2; F_{t-2}; \theta_2)\ (1 - p_{1t-1})}{\eta(R_{t-1} / s_{t-1} = 1; F_{t-2}; \theta_1)\ p_{1t-1} + \eta(R_{t-1} / s_{t-1} = 2; F_{t-2}; \theta_2)\ (1 - p_{1-1t})} \right] \end{array} \qquad (10)$$

The model in equation (8) thus implies that the density $\phi(R_t / F_{t-1}; \theta)$ of the asset return R_t conditional on information F_{t-1} is obtained by summing the density functions conditional on the state, $\eta(R_t / s_t = j, F_{t-1}; \theta_j)$, using the respective probabilities as weights:

$$\phi(R_t / F_{t-1};\, \theta) = p_{1t}\ \eta(R_t / s_t = 1, F_{t-1}; \theta_1) + (1 - p_{1t})\ \eta(R_t / s_t = 2, F_{t-1}; \theta_2)\,. \qquad (11)$$

The parameters of this model can be obtained by maximizing the following log-likelihood function:

$$L(R_T, R_{T-1}, ..., R_1; \theta) = \sum_{t=1}^{T} \ln\left[\phi(R_t / F_{t-1}; \theta)\right]. \qquad (12)$$

Although mixtures of normal distributions can approximate a very broad set of density families, we introduce Student-t distributions to better capture the excess kurtosis observed on any sample of asset returns that contains outliers or extreme values. We first follow Perez-Quiros and Timmermann (2001) and model the two density functions conditional on state 1 and state 2 of the economy respectively according to equations (3) and (4). We then investigate whether a mixture of two conditional Student-t densities improves the fit.

To fully parameterize the model and make the estimation possible one still needs to specify the form of the second moment in equation (5) where $\Psi_{s_t}(.)$ will now be a function of the regime s_t. The first specification follows Gray (1996) to accommodate within regime heteroskedasticity using a classical GARCH(1,1) process in each regime:

$$\sigma^2_{s_t\, t} = \beta_{s_t\, 0} + \beta_{s_t\, 1}\, \varepsilon^2_{t-1} + \beta_{s_t\, 2}\, \sigma^2_{s_t\, t-1}. \tag{13}$$

The problem of recovering a first-order Markov structure in a regime-switching model that includes such GARCH terms is solved using the method introduced in Gray (1996), namely by averaging the error term and conditional variance at each time:

$$\varepsilon_{t-1} = R_{t-1} - E(R_{t-1} / F_{t-2}) = R_{t-1} - [\, p_{1t-1}\ \mu_{1\, t-1} + (1 - p_{1t-1})\ \mu_{2\, t-1}\,], \tag{14}$$

$$\sigma^2_{t-1} = E(R^2_{t-1} / F_{t-2}) - \left[E(R_{t-1} / F_{t-2})\right]^2 = p_{1t-1}\left[\mu^2_{1\, t-1} + \sigma^2_{1\, t-1}\right] + (1 - p_{1t-1})\left[\mu^2_{2\, t-1} + \sigma^2_{2\, t-1}\right] - \left[p_{1t-1}\ \mu_{1\, t-1} + (1 - p_{1t-1})\ \mu_{2\, t-1}\right]^2 \tag{15}$$

where $\mu_{s_t\, t} = a_{s_t\, 0} + \sum_{i=1}^{l} a_{s_t\, i}\, R_{t-i}$ is the conditional mean in state s_t.

Although the ARCH family of models has been extended well beyond the simple specification of the initial ARCH model of Engle (1982) and GARCH model of Bollerslev (1986), most of the popular additions to the family have attempted to refine both the mean and variance equations while still relating the second moment to lagged squared residuals and past variances. The only noticeable alternative was presented by Taylor (1986) and Schwert (1989) who specified a power term of unity in that he related the conditional standard deviation of a series to lagged absolute residuals and past standard deviations.

In fact, it is possible to specify any positive value as the power term in the second moment equation and the asset return will still exhibit volatility clustering. The preference given to squared terms or even a power of unity is inherited from the Gaussian framework traditionally invoked regarding the data. As is well known, if a data series is normally distributed then it can be fully characterized by its first two moments. In this context squared or absolute residuals (returns) can be used to proxy the volatility process. However, if we accept that the data are very likely to have a non-normal error distribution, then one must use higher order moments to adequately describe the series. In this instance, the superiority of the squared or absolute term is lost and other power transformations may be appropriate. Indeed, for non-normal data, by squaring the returns or taking their absolute value, one effectively imposes a structure which may potentially furnish sub-optimal modeling and forecasting performance relative to other power terms. Recognizing the possibility that such power terms may not necessarily be optimal, Ding, Granger and Engle (1993) proposed a class of models which allows an optimal power transformation term to be estimated directly on the data. The model, known as the Asymmetric Power GARCH (APGARCH) model, relies on the following dynamics:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{p} \left(\alpha_i \quad (\left| \varepsilon_{t-i} \right| + \gamma_i \ \varepsilon_{t-i} \ ) \right)^{\delta} + \sum_{i=1}^{q} \beta_i \ \sigma_{t-i}^{\delta} \ , \qquad (16)$$

where the γ_i's enable to introduce an asymmetric response to past shocks and δ corresponds to the optimal power transformation directly estimated on the data.

In this paper we investigate the relevance of this general asymmetric power transformation model in a regime-switching context by specifying another structure for the function $\Psi_{s_t}(.)$ of equation (5). More specifically, we assume that the volatility in each regime is driven by the following APGARCH(1,1) dynamics:

$$\sigma_{s_t\, t}^{\delta_{s_t}} = \beta_{s_t\, 0} + \left(\beta_{s_t\, 1} \ (\left| \varepsilon_{t-1} \right| + \gamma_{s_t\, 1} \quad \varepsilon_{t-1} \ ) \right)^{\delta_{s_t}} + \beta_{s_t\, 2} \ \sigma_{t-1}^{\delta_{s_t}} \ . \qquad (17)$$

The resulting model will be called the Regime-Switching APGARCH (RS-APGARCH) model. An important feature of the APGARCH dynamics is that the power term δ can be related to the long memory property of the process (see Ding and Granger (1996)). Hence, our RS-APGARCH model provides a convenient framework to study the long memory property of asset returns as a function of the current state of the economy. The empirical findings should thus highlight whether the different regimes characterizing the states of the economy exhibit different levels of long run dependency. In particular, can we identify a regime characterized by long memory and another one only exhibiting short memory?

III. Data and Preliminary Study

The empirical study focuses on the Pacific Basin area that has undergone important shocks over the past decades and may thus represent an excellent region to test the usefulness of a regime-switching model. In order to investigate the potential differences between developed and emerging markets, we select indices corresponding to a wide range of stock market sizes. More specifically, we examine four Asian stock market indices: the NIKKEI 225 Index (NKY) for Japan, the Hang Seng Index (HIS) for Hong Kong, the Singapore SES-ALL Index (SESALL) for Singapore and the Kuala Lumpur Composite Index (KLCI) for Malaysia. The daily percentage returns over the period March 1984 to September 2003 are measured as $R_t = 100 \times \ln(P_t / P_{t-1})$. The usual descriptive statistics of the four series are displayed in Table 1.

Insert somewhere here Table 1

We first observe that the indices present similar statistical characteristics. They all exhibit a significant negative skewness and a kurtosis larger than 3. Not surprisingly, all series fail to pass the Jarque-Bera normality test. The degrees of asymmetry and leptokurtosicity, however, vary a lot from one market to another, implying possible important distributional differences that should be reflected by discrepancies in the RS-APGARCH dynamics. It can also be noted that the percentage mean is close to zero in all cases and plots of the return series (not shown here) indicate that the return process of each stock market index is quite stable around its mean. This justifies, in some ways, that no extra care is brought to the definition of the conditional mean equations in this paper.

Insert somewhere here Figure 1 and Table 2

To understand the motivations behind the model introduced in this paper, we first study the correlation of the transformed returns. Figure 1 gives the autocorrelograms obtained for the returns, squared returns and absolute returns of each stock market index. Consistent with the efficient market theory, we find that the stock market returns themselves contain little serial correlation. Indeed, in agreement with Fama (1976) and Taylor (1986) we observe a significant autocorrelation at the first lag that indicates the presence of short memory in stock index returns. This dependence, however, dies away very fast and virtually all other lags show no significant autocorrelation at the usual confidence level. Although the autocorrelation of squared returns seems to persist a little longer, we observe that the squared transformation does not necessarily exhibit a long run dependence. On the contrary, in agreement with Ding and Granger (1996), we then find that the absolute returns all exhibit long memory. In three out of four cases the autocorrelation of absolute returns remains above the 95% confidence interval even for lags as long as 200. Table 2 provides the sample autocorrelations of R_t, R_t^2 and $|R_t|$ for lags 1 to 5 and 10, 20, 50, 100, 150 and 200. It enables to clearly assess differences in the rate of decay of the autocorrelation as the lag τ increases. Overall, the plots in Figure 1 and the numerical values of Table 2 clearly highlight the importance of the power transformation used to investigate and/or capture the existence of long memory in stock market returns.

Insert somewhere here Figure 2 and Table 3

The seminal papers of Fama (1965) and Mandelbrot (1967) have shown that large absolute (squared) returns are more likely than small absolute (squared) returns to be followed by a large absolute (squared) return. The existence of such a clustering effect together with the classical assumption of normality of asset returns suggest that the classical GARCH models provide a suitable time-varying structure to capture changes in volatility. The disparities between the long-run behavior of the correlograms for absolute and squared returns, however, suggest that no excessive trust should be put in the usual power transformations (squared power or power of unity) when trying to model volatility of stock market returns. Other power transformations may indeed convey more information about the volatility process outside the Gaussian world. To gauge the influence of the power transformation $|R_t|^d$ on the autocorrelation decay, Table 3 presents the values $\rho_\tau(d)=corr(|R_t|^d,|R_{t-\tau}|^d)$ for different lags τ and power transformations d. For all indices, it is found that the autocorrelation is positive at least up to order 50, proving the existence of the documented long memory in stock markets. It is also confirmed that the decay of this autocorrelation strongly depends on the power transformation

d. We then fix the lag τ and study how the autocorrelation $\rho_\tau(d)$ evolves as a function of the power transformation d. Figure 2 gives the plots of the calculated $\rho_\tau(d)$ for $\tau=1$ and $\tau=10$. The obtained graphs are similar in shape to those presented by Ding, Granger and Engle (1993) for the S&P 500 Index: for all indices, there exists a power transformation d for which the long memory is the strongest. Our results differ, however, from their empirical findings in that the power transformation yielding the maximum autocorrelation is index-specific and also sensitive to the number of lags.

Combining all these empirical facts, one easily understands why the use of an arbitrary power transformation in a GARCH equation may be misleading. The motivations to the introduction of Power GARCH models then become very clear: allowing the power δ of the heteroskedasticity equation to be estimated from the data, this general class of models is more likely to capture the stylized features of volatility. Coupling the flexibility of Power GARCH models with that of regime-switching models, one should obtain a general framework rich enough to accurately describe the specificity of the different stock markets.

IV. Empirical Findings

We now turn to the estimation of the different models underlying our discussion. They are all estimated using the GAUSS CML module. The standard errors are computed from the diagonal of the heteroskedastic-consistent covariance matrix - see White (1980) -. Due to the presence of strong nonlinearities in the regime-switching models, good starting values are important to obtain the convergence. Following a classical approach, we started by estimating the model with constant parameters for the means and variances and then augmented it by steps using simpler versions to determine the best starting values. Robustness to the starting values has, of course, been tested. Since the simplest versions are nested in the final model, such approach can also be used to assess the relevance of each additional parameter through a likelihood ratio test. To save space, the intermediate results are not reported in this paper. However, the final model corresponds to the best fit according to the likelihood ratio test. Ljung-Box statistics are also used to investigate any remaining autocorrelation in power transformations of the standardized residuals.

Single-Regime GARCH and APGARCH models

We begin by considering whether a single-regime model is sufficient to account for the conditional heteroskedasticity in the stock index returns. Beyond the now classical GARCH(1,1) model, an APGARCH(1,1) version is also estimated on each return series. The maximum likelihood estimates of both models are reported in Table 4. Virtually all the t-statistics on the coefficients are largely significant at the 5% level. Although not the center of our investigation, we tried several autoregressive specifications for the conditional means and found that a first-order autoregressive process best represents the conditional mean of each series. Regardless of the stock market and the selected model, the coefficient a_1 for the first-order lag is positive and significant. The presence of first-order autocorrelation in index returns has largely been documented in the literature and may be explained by an asynchronous response to news of the stocks composing the index.

Insert somewhere here Table 4

The APGARCH model introduces two additional parameters relative to the GARCH model presented in this section. First of all it includes an asymmetric term through the parameter γ_1. We observe that this parameter is negative and very significant for each market. Although the magnitude of the asymmetric response to past shocks seems to vary from one market to another, the inclusion of this term proves useful in all cases. Then, the novelty of this family of models is to endogenize the computation of the optimal power transformation δ to capture volatility clustering. The power term estimated for the APGARCH(1,1) model fitted to each of the four national indices are also presented in Table 4. The maximum power term is 2.4054 for Singapore and the minimum is 1.5544 for Japan. As argued in the introduction, the invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance. We use the likelihood ratio test (LRT) to assess whether the APGARGH structure really represents an improvement over the classical GARCH. The obtained LRT are respectively 172.11

for the NIKKEI 225 Index, 113.12 for the Hang Seng Index, 73.07 for the Singapore SES-ALL Index and 42.29 for the Kuala Lumpur Composite Index, all values are far beyond the usual 5% significance level. The flexibility brought by an endogenous power transformation is thus useful for each stock market return series.

In this single-regime framework, any conditional heteroskedasticity can only be driven by the ARCH (β_1) and GARCH (β_2) terms of the underlying model. Not surprisingly, the persistence of volatility, as measured by the sum $\beta_1 + \beta_2$, is 0.9744 on average for the GARCH(1,1) model, indicating a very strong level of volatility persistence. Despite the additional flexibility brought by the estimation of the optimal power transformation from the data, it can be observed that the obtained level of volatility persistence is very high with the APGARH(1,1) model - 0.9643 - and comparable to the level obtained with the classical GARCH(1,1) specification.

The existence of such a high persistence level, as explained by Lamoureux and Lastrapes (1990), may be the result of structural breaks in the dynamics representing each stock index return and should be better captured through a model whose structure is flexible enough to switch form according to the states of the economy. The interest of a regime-switching framework is particularly clear for the Singapore SES-ALL Index. Indeed, the Ljung-Box statistics show that neither the GARCH nor the APGARCH single-regime models are rich enough to suppress all tracks of heteroskedasticity in high-order transformations of the standardized residuals. Moreover, although not provided here, an analysis of the standardized residuals distribution reveals that the conditional normality assumed for the log-likelihood estimation of the parameters is violated for all indices, indicating misspecifications in the models. We thus move to the estimation of regime-switching versions of these models.

Regime-Switching GARCH and APGARCH models

We begin by considering whether the classical regime-switching GARCH model introduced by Gray (1996) is sufficient to account for the conditional heteroskedasticity in stock index returns. The parameter estimates for this model appear in Table 5. In many respects, the results are similar to past work. There is persistence of both regimes with P and Q both exceeding 0.9 and the regimes tend to be separated by different variances. Although the conditional mean parameters are not all significantly different from zero, they provide the interesting economic result that one regime corresponds to market decreases while the other regime models positive conditional returns. The regime corresponding to the negative conditional mean also corresponds to the period of high volatility with an unconditional variance several times bigger than the unconditional variance of the regime representing the "good state" of the economy.

Unlike the single-regime models, heteroskedasticity can now be driven by switches between regimes as well as within-regime volatility persistence. There are thus really two sources of volatility persistence. If one regime has low average variance and one has high average variance, and if the regimes are persistent, volatility will be persistent. The parameter estimates for P and Q suggest that regime persistence is an important source of volatility persistence. Additionally, if the effect of an individual shock takes a long time to die out, there is within-regime persistence. The reported parameter estimates indicate that within each regime, the GARCH processes are *much* less persistent than in a single-regime GARCH model. This is consistent with the findings of Lamoureux and Lastrapes (1990): the explosive variance often obtained with GARCH models may be caused by trying to use a single-regime model to capture a multi-regime process.

The Ljung-Box statistics relating to power transformations of the residuals, however, have not been reduced sufficiently in the case of the Hang Seng Index to accept the null hypothesis of no serial correlation in the residuals. In addition, the absence of strongly significant ARCH and GARCH parameters in most cases, casts doubt on the goodness-of-fit provided by this classical regime-switching GARCH model.

As argued in Section 2, there is no obvious reason why one should assume the conditional variance is a linear function of lagged squared residuals as in Bollerslev's GARCH, or the conditional standard deviation a linear function of lagged absolute residuals as in the Taylor-Schwert model. We have shown in the previous sub-section that the Asymmetric Power GARCH class of models provides a noticeable improvement in fit over GARCH processes when used in the single-regime context. We will thus investigate whether a regime-switching model may benefit from such an APGARCH structure and extend Gray's model to the RS-APGARCH model presented in Section 2.

The model was first estimated using conditional Gaussian densities. Following Perez-Quiros and Timmermann (2001), it is then re-estimated using their mixture of a Student-t and a Gaussian densities to incorporate a possible difference of leptokurtosicity between normal market conditions and periods of extreme fluctuations. Eventually, we test whether fat-tailedness is present regardless of the state of the economy using a mixture of two Student-t distributions. An LRT test was conducted to select the best model. To conserve space, Table 6 only presents details of the best specification, namely an RS-APGARCH(1,1) model relying on two conditional Student-t distributions.

As seen in the table, the values obtained for the degrees of freedom are quite small and indicate a strong level of within-regime leptokurtosicity. It is worth mentioning that the biggest two stock markets (namely, Japan and Hong Kong), exhibit more differential leptokurtosicity across regimes. Although we do not obtain, like Perez-Quiros and Timmermann (2001), that one regime is best described by a leptokurtic distribution while the other regime could be proxied by a Gaussian conditional density, it seems that the departure from normality (as measured by the degree of freedom of the Student-t density) is much weaker for the regime corresponding to the low average variance. The situation of smaller or emerging markets appears quite different however. Indeed, Singapore and Malaysia reveal less difference in across regime leptokurtosicity. Not surprisingly, the two parameters ν_{s_t}

are the closest in the case of Malaysia. Although both states of the economy can also be interpreted as bull and bear market situations, overall, the market always evolves with great fluctuations in the case of an emerging country.

Since Black (1976), the so-called leverage effect of stock market returns has been largely documented in the finance literature. It is known that stock returns are negatively correlated with changes in return volatility. That is, volatility tends to rise in response to "bad news" - i.e., excess returns lower than expected - and to fall in response to "good news" – i.e., excess returns higher than expected -. Since Nelson (1991), empirical studies have shown that it is crucial to include an asymmetric response of volatility to positive and negative shocks. Our RS-APGARCH model provides such an asymmetric term through the parameter $\gamma_{s_t 1}$ which is allowed to vary from one regime to another. It is found that a highly significant asymmetric effect is present in both regimes for each stock index. The level of asymmetry, however, differs according to the regime. In all cases, the regime presenting the highest level of leptokurtosicity also corresponds to the state of the economy where the asymmetric response to news is the smallest. Market participants thus seem to differentiate less between good and bad news when they are in an extremely volatile period. This could imply that the perception of risk differs during periods of large fluctuations and moments of apparent tranquility. In the latter situation, market participants have more time to refine their definition of risk by incorporating higher order moments (in particular a third order term for asymmetry).

In order not to impose a sub-optimal structure on the volatility process of each regime, our RS-APGARCH model also relies on a direct estimation of the optimal power transformation parameter δ_{s_t} for each regime. We first observe that when the power transformation is endogenized, the volatility persistence becomes much stronger than what is observed using a traditional regime-switching GARCH model. In Table 5, the level of within regime persistence was dramatically reduced and parameter estimates were often insignificant. On the contrary, we find that in the RS-APGARCH model, the persistence of previous shocks represents an important source of volatility persistence that comes in addition to the high persistence of both regimes (large values of P and Q). This means that the existence of such persistence cannot entirely be explained by structural changes in the parameter values as proposed in Lamoureux and Lastrapes (1990). There exists a strong within-regime clustering effect regardless of the state of the economy. However, in order to adequately capture this effect no arbitrary parameterization of the conditional variance or standard deviation should be used. Rather, the power transformation of the standardized residuals that best captures GARCH effects should be obtained endogenously from the data.

We also observe that with the regime-switching power GARCH model, virtually all coefficients produce highly significant t-statistics, first sign of a better model specification. To test more formally the significance of the power GARCH parameters δ_{s_t} as well as the asymmetry parameters $\gamma_{s_t 1}$, an LRT was constructed to compare Gray's regime-switching GARCH model with our RS-APGARCH model using Student-t conditional densities. The LRT statistic, which is distributed χ^2_6 under the null, is respectively 389.78 for the NIKKEI Index, 57.64 for the Hang Seng Index, 322.80 for the Singapore SES-ALL Composite Index and 350.88 for the Kuala Lumpur Composite Index. All values are significant at any usual confidence level indicating that i) leptokurtic conditional densities are required on each regime, ii) each regime responds asymmetrically to positive and negative shocks and iii) the flexibility brought by a direct estimation of the power GARCH term improves the fit.

One last important characteristic of our model is the relationship that exists between the power transformation δ and the existence of long memory in the underlying process (see Ding, Granger and Engle (1993)). The significant values obtained for the power terms δ_{s_t} for both regimes when modeling the dynamics of the four Asian index returns proves that the long memory property of stock returns is not simply due to structural breaks as was often argued (see Gourieroux and Jasiak (2001) or Granger and Hyung (1999)) but that there exists a long-run dependence within each regime.

V. Conclusion

This paper develops a new general class of regime-switching models called Regime-Switching Asymmetric Power GARCH model. It allows a free power term for the GARCH specification of each regime rather than assuming an absolute or squared term like most of the classical models. Since this type of APGARCH model has not yet been considered in a regime switching context, an important contribution of the current paper is to augment our understanding of whether and to what extent these types of more flexible models are statistically superior to their less sophisticated counterparts.

The empirical investigation uses four Asian stock market indices corresponding to various market situations and provides interesting conclusions about how to understand time variations in stock index returns. Most obviously, it seems that commonly used single-state specifications for stock index returns that adopt the same model for recessions and expansions are misspecified and can be strongly rejected against a two-state model. We also find that the APGARCH structure provides a considerable improvement over the classical GARCH structure in both regimes. The empirical results do indicate that all generalizations brought by our model are statistically and economically significant. More specifically, a variety of new stylized facts about the dynamics of stock index returns has emerged from this RS-APGARCH model.

We first recover, but with a higher level of significance however, a now classical result of the regime-switching literature: one regime could be regarded as modeling expansion periods while the second regime is clearly identified to recessions. Expansions are characterized by a positive conditional mean and a low-volatility regime while recessions exhibit a negative conditional mean and are always synonym of a much higher volatility level.

Even if a basic regime-switching model with constant parameters would result in a leptokurtic process, we tried several conditional distributions for each regime to investigate the within-regime level of leptokurtosicity. The classical conditional Gaussian densities are shown not to be sufficient, even in a two-state framework, to incorporate all the kurtosis of the underlying series. We find that, whatever the investigated stock market index, both regimes are best modeled with a conditional Student-t distribution. However, we do recover, to some extent, the interesting result obtained by Perez-Quiros and Timmermann (2001) on U.S. stock returns using a mixture of Gaussian and Student-t densities. Indeed, it seems that for developed markets, the estimated degree of freedom of one Student-t distribution is large enough to statistically accept the convergence to conditional normality in this regime. The second regime, however, exhibits a strong leptokurtosicity and captures all extreme returns.

Presenting both developed and emerging markets in this study, we are able to refine the result. Indeed, the “convergence” of one regime to normality is not obtained for emerging markets where the level of leptokurtosicity remains very strong and even comparable for recession and expansion cycles. This result should of course be tested on a larger sample of stock market indices. If it were confirmed, this would indicate that the degree of leptokurtosicity of the regime representing the good market condition (expansions) could be used to assess the level of development of a financial market.

The RS-APGARCH model also introduces the possibility of within-regime asymmetric response to news. It is found that the introduction of such parameters is strongly significant and that asymmetries are important in both states of the economy. Although the classical leverage effect of stock market returns is obtained for both regimes, the asymmetric response to news is consistently stronger in the low-volatility regime. Market participants thus seem to differentiate less between good and bad news during extremely volatile periods. The unusually high level of volatility in the latter periods could bias the market participants perception of news and reduce their ability of differentiating between good and bad news. When markets evolve more smoothly, however, the information process may be less noisy and market participants may recover their ability to assess the content of new information and to react accordingly. If confirmed by other empirical studies, this result should open a challenging avenue of research for microstructure models of agent behavior and price formation in financial markets.

Moreover, the improvements of the RS-APGARCH model are mainly due to the endogenous determination of the power transformation term used in the GARCH structure of each regime. Using Gray's RS-GARCH model, we find that the within-regime volatility persistence is consequently reduced relative to the single-state GARCH model. Such result gives credit to Lamoureux and Lastrapes (1990) thesis that structural breaks account for most of the volatility persistence observed with a single-state model through the regime persistence (very high P and Q). When the power GARCH term is introduced, however, we observe that not only the ARCH and GARCH parameters become statistically much more significant, but also that the within-regime heteroskedasticity increases strongly compared to the RS-GARCH level. This seems to indicate that the squared terms arbitrarily used in the traditional RS-GARCH models are sub-optimal and do not allow to fully capture the within-regime clustering effects.

Lastly, as explained in Ding and Granger (1996) the APGARCH class of model we use on both regimes has no memory in return themselves, but long memory in absolute returns and their power transformations. The estimated power terms δ_{s_t} are significant and different in both regimes, implying the existence of long memory in both states of the economy. Again, this shows that the existing long memory in stock returns does not only result from structural breaks as it has often been argued in the single-state literature: we do find the existence of within-regime long memory. Nevertheless, the values of δ_{s_t} obtained for all stock indices are too close from one regime to the other to conclude that one regime predominantly captures short-run dependencies while the other regime exhibits long memory.

VI. References

Ané T., and C., Labidi, 2001. Revisiting the Finite Mixture of Gaussian Distributions with Application to Futures Markets. *Journal of Futures Markets*, 21, 4, 347-376.

Black, F., 1976. Studies in Stock Price Volatility Changes. Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Association, 177-181.

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31, 307-327.

Brooks, R.D., Faff, R.W., McKenzie, M.D., and H., Mitchell, 2000. A Multi-Country Study of Power ARCH Models and National Stock Market Returns. *Journal of International Money and Finance*, 19, 377-397.

Ding, Z., Granger, W.J., and R.F., Engle, 1993. A Long Memory Property of Stock Market Returns and a New Model. *Journal of Empirical Finance*, 1, 83-106.

Ding, Z. and W.J., Granger, 1996. Modeling Volatility Persistence of Speculative Returns: a New Approach. *Journal of Econometrics*, 73, 185-215.

Engle, R.F., 1982. Autoregressive Conditional Heteroscedasticity with Estimates of U.K. Inflation. *Econometrica*, 50, 987-1008.

Engle, R.F., and T., Bollerslev, 1986. Modeling the Persistence in Conditional Variances. *Econometric Review*, 5, 1-50, 81-87.

Fama, E. F., 1965. The Behavior of Stock Market Prices. *Journal of Business*, 38, 34-105.

Fama, E.F., 1970. Efficient Capital Markets: a Review of Theory and Empirical Work. *Journal of Finance*, 25, 383-417.

Fama, E.F., and K.,R., French, 1989. Size and Book-To-Market Factors in Earnings and Returns. *Journal of Finance*, 50, 131-155.

Gourieroux, C. and J., Jasiak, 2001. Memory and Infrequent Breaks, Economics Letter, 70, 29-41.

Granger, C. W., and N., Hyung, 1999. Occasional Structural Breaks and Long Memory. Discussion Paper 99-14, University of California San Diego, June.

Gray, S., 1996. Modeling the Conditional distribution of Interest Rates as a Regime-switching Process. *Journal of Financial Economics*, 42, 27-62.

Hamilton, J.D., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57, 357-384.

Kon, S. J., 1984. Models of Stock Returns – A Comparison. *Journal of Finance*, 39, 1, 147-165.

Lamoureux, C.G., and W.D., Lastrapes, 1990. Persistence in Variance, Structural Change and the GARCH Model. *Journal of Business & Economics Statistics*, 8, n°2, 225-234.

Mandelbrot, B., 1967. The Variation of Some Other Speculative Prices. *Journal of Business*, 40, 393-413.

Nelson, D.B., 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59, 2, 347-370.

Perez-Quiros, G., and A., Timmermann, 2001. Business Cycle Asymmetries in Stock Returns: Evidence From Higher Order Moments and Conditional Densities. *Journal of Econometrics*, 103, 259-306.

Schwert, G.W., 1989. Why Does Stock Market Volatility Change Over Time? *Journal of Finance*, 44, 1115-1153.

Taylor, S., 1986. Modelling Financial Time Series. *John Wiley and Sons*, New York.

Timmermann, A., 2000. Moments of Markov Switching Models. *Journal of Econometrics*, 96, 75-111.

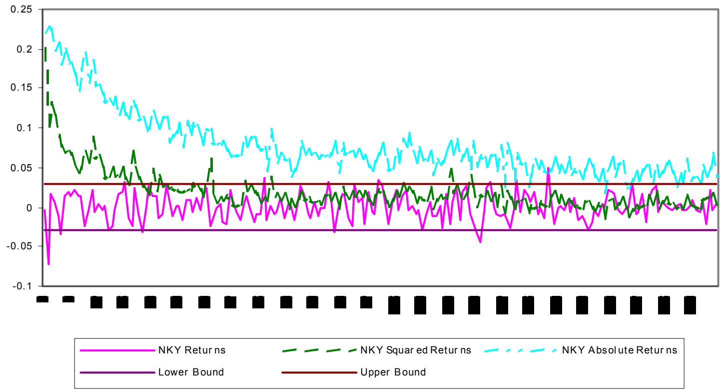
White, A., 1980. A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test For Heteroskedasticity, *Econometrica*, 48, 817-838.

Figure 1

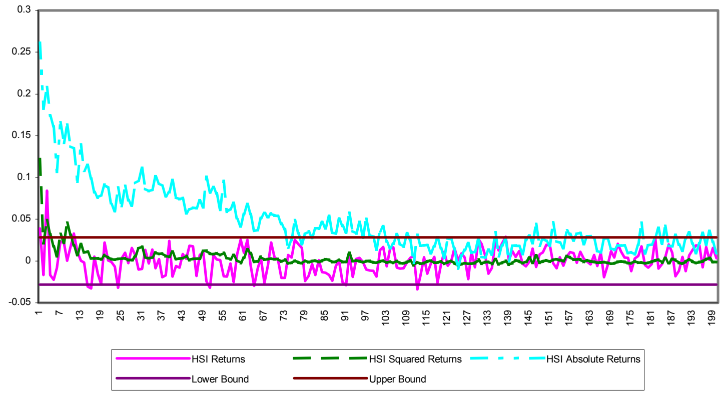
Autocorrelograms for the Returns, Squared Returns and Absolute Returns.

We plot the autocorrelograms of R_t, R_t^2 and $|R_t|$ for lag 1 to lag 200. The straight lines correspond to $\pm 1.96/\sqrt{T}$, the 95% confidence interval for the estimated sample autocorrelations. For all indices, in agreement with Fama (1976) or Taylor (1986), we find a significant first order autocorrelation for R_t, indicating the presence of short memory in stock index returns. However, this dependence dies out very fast and for virtually all other lags, the autocorrelation of stock index returns is not significant at the usual confidence level. Although the autocorrelation of squared returns seems to persist a little longer, we observe that the squared transformation does not necessarily exhibit a long run dependence. However, in agreement with Ding, Granger and Engle (1993), we find that the absolute returns all exhibit a very long memory. In three out of four indices, the autocorrelation of absolute returns remains above the 95% confidence interval for lags as long as 200. The Hang Seng stands out, however, with its faster long run dependence decay even for a power transformation of unity.

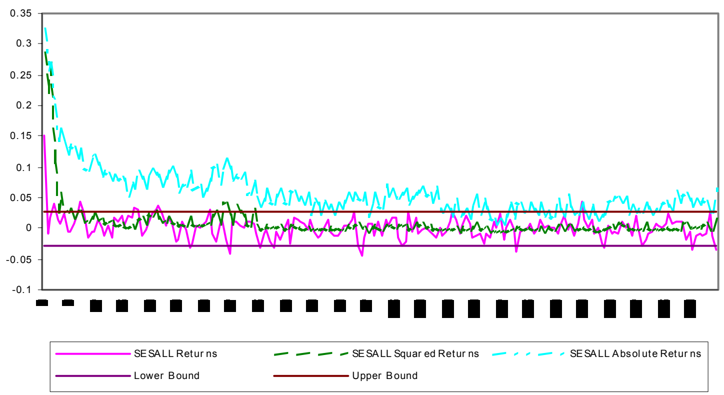
Autocorrelograms for the NKY Returns, Squared Returns and Absolute Returns



Autocorrelograms for the HSI Returns, Squared Returns and Absolute Returns



Autocorrelograms for the SESALL Returns, Squared Returns and Absolute Returns



Autocorrelograms for the KLCI Returns, Squared Returns and Absolute Returns

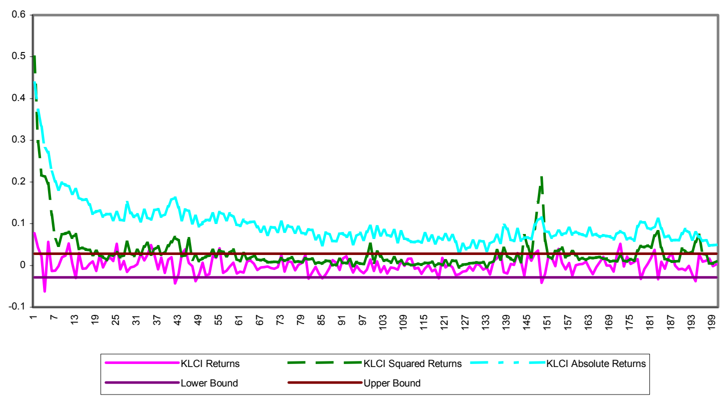
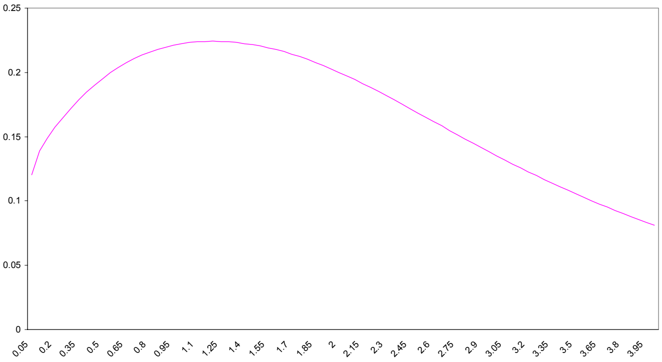


Figure 2

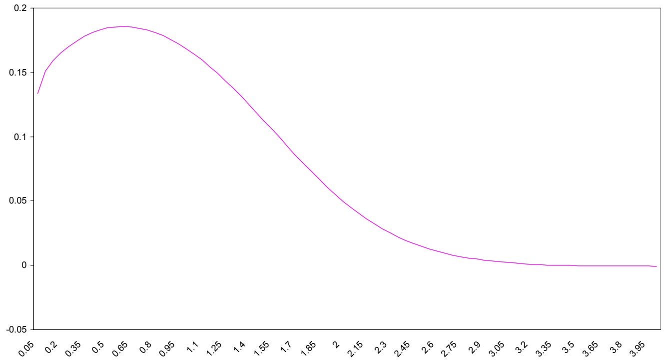
Autocorrelations of $|R_t|^d$ at lag 1 and 10 as a function of d.

Figure 2 studies the level of autocorrelation $\rho_\tau(d)=corr(|R_t|^d,|R_{t-\tau}|^d)$ at lags 1 and 10 as a function of the power transformation d. For all indices, there exists a power transformation d for which long memory is the strongest. These graphs are similar in shape to those presented for the S&P 500 Index in Ding, Granger and Engle (1993). Nevertheless, it seems that the optimal autocorrelation for a particular index is i) first sensitive to the index analyzed and ii) also sensitive to the number of lags.

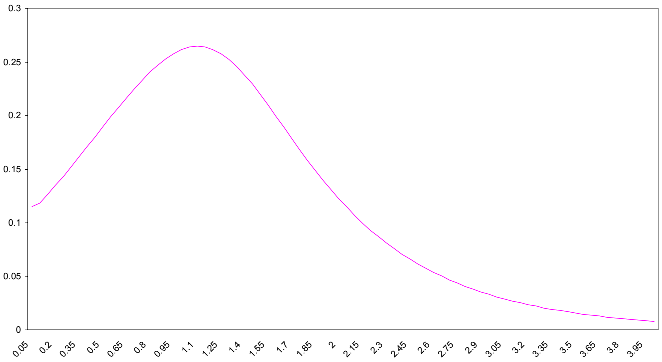
$\rho_d(1)$ for the NIKKEI 225 INDEX



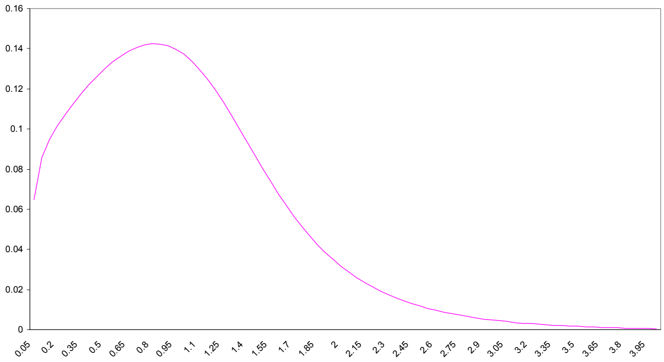
$\rho_d(10)$ for the NIKKEI 225 INDEX



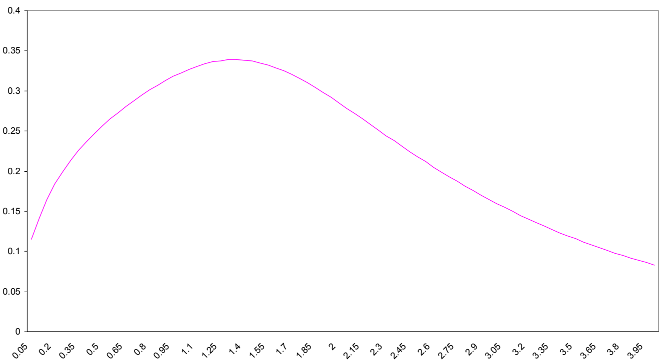
$\rho_d(1)$ for the Hang Seng Index



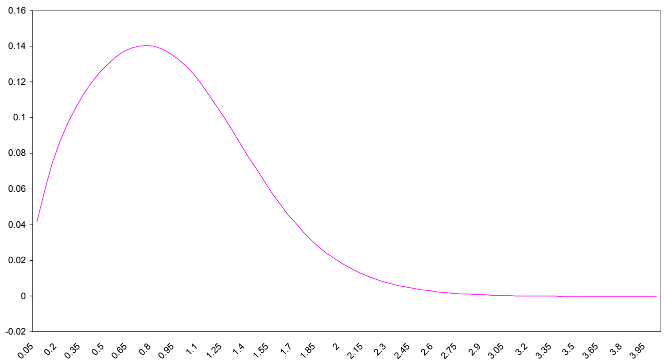
$\rho_d(10)$ for the Hang Seng Index



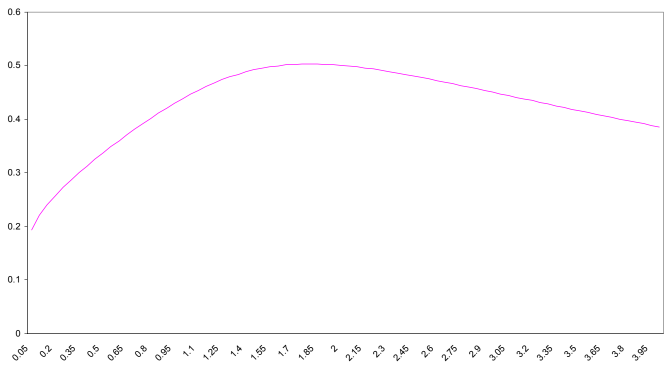
$\rho_d(1)$ for the Singapore SES ALL Index



$\rho_d(10)$ for the Singapore SES ALL Index



$\rho_d(1)$ for the Kuala Lumpur Composite Index



$\rho_d(10)$ for the Kuala Lumpur Composite Index

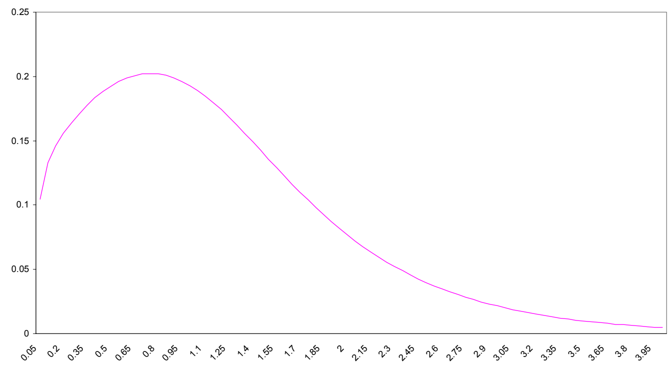


Table 1

Descriptive Statistics.

Table 1 summarizes the descriptive statistics for the four stock market indices used in this study, namely, the NIKKEI 225 Index (NKY) for Japan, the Hang Seng Index (HIS) for Hong Kong, the Singapore SES-ALL Index (SESALL) for Singapore and the Kuala Lumpur Composite Index (KLCI) for Malaysia. Daily percentage return series over the period March 1984 to September 2003 are measured as $R_t = 100 \times \ln(P_t / P_{t-1})$. The usual first and second order statistics are reported together with the number of observations T for each series. A 95% confidence interval for a test of index returns normality is given by $\pm 1.96 * \sqrt{6/T}$ for the sample skewness and $3 \pm 1.96 * \sqrt{24/T}$ for the sample kurtosis. All the series presented in this study exhibit both significant skewness and kurtosis. The Jarque-Bera statistic also presented in this Table 1 rejects the unconditional normality for all series. We use bold characters to indicate significance at the 5 percent level.

Descriptive Statistics for the Daily Stock Index Returns

	NKY	HSI	SESALL	KLCI
Mean	-0.0009	0.0507	0.0059	0.0118
Variance	1.9393	3.2054	1.6049	2.7501
Minimum	-16.1354	-40.5422	-24.9202	-24.1534
Maximum	12.4303	17.2471	12.9743	20.8174
Number of Values	4829	4834	4891	4806
Skewness	**-0.1134**	**-3.0882**	**-2.0180**	**-0.2124**
p-value	[<.001]	[<.001]	[<.001]	[<.001]
Kurtosis	**10.6913**	**70.8124**	**45.4443**	**34.5296**
p-value	[<.001]	[<.001]	[<.001]	[<.001]
JB	**11908.24**	**933517.38**	**370303.06**	**199024.77**
p-value	[<.001]	[<.001]	[<.001]	[<.001]

Table 2

Autocorrelations for the Returns, Squared Returns and Absolute Returns of All Indices.

Table 2 gives the sample autocorrelations of R_t, R_t^2 and $|R_t|$ for lags 1 to 5 and 10, 20, 50, 100, 150 and 200 computed for the four Asian stock market indices presented in this study. It shows that although the autocorrelation of R_t decreases very fast as the lag τ increases, the decay is much less pronounced for the autocorrelation of R_t^2. It also highlights that such decay is generally not obtained when absolute returns $|R_t|$ are used, confirming the importance of the power transformation of returns selected to compute autocorrelations when trying to capture long memory.

Autocorrelations for the Returns, Squared Returns and Absolute Returns of All Indices

						NIKKEY 225 INDEX					
Lag	1	2	3	4	5	10	20	50	100	150	200
NKY Returns	-0.0043	-0.0730	0.0156	0.0072	-0.0118	0.0223	-0.0282	-0.0246	0.0347	0.0503	0.0051
NKY Squared Returns	0.2001	0.1041	0.1307	0.1146	0.0914	0.0499	0.0392	0.0593	0.0222	0.0020	0.0028
NKY Absolute Returns	0.2223	0.2267	0.2241	0.1988	0.2062	0.1681	0.1352	0.0978	0.0663	0.0456	0.0371
						HANG SENG INDEX					
Lag	1	2	3	4	5	10	20	50	100	150	200
HSI Returns	0.0392	-0.0167	0.0843	-0.0165	-0.0222	0.0174	0.0220	-0.0244	-0.0187	0.0193	0.0032
HSI Squared Returns	0.1221	0.0218	0.0490	0.0322	0.0176	0.0314	0.0069	0.0122	-0.0020	0.0009	-0.0013
HSI Absolute Returns	0.2613	0.1827	0.2082	0.1740	0.1612	0.1371	0.0918	0.1007	0.0137	0.0237	0.0095
						SINGAPORE SES ALL INDEX					
Lag	1	2	3	4	5	10	20	50	100	150	200
SESALL Returns	0.1526	-0.0080	0.0189	0.0393	0.0147	0.0062	0.0041	0.0315	0.0102	0.0028	-0.0341
SESALL Squared Returns	0.2852	0.2223	0.2693	0.1097	0.0268	0.0182	0.0102	0.0075	0.0019	0.0020	0.0127
SESALL Absolute Returns	0.3240	0.2596	0.2672	0.1986	0.1461	0.1304	0.0905	0.0801	0.0364	0.0271	0.0665
						KUALA LUMPUR COMPOSITE INDEX					
Lag	1	2	3	4	5	10	20	50	100	150	200
KLCI Returns	0.0772	0.0427	0.0221	-0.0622	0.0562	0.0238	0.0249	0.0066	-0.0133	-0.0198	0.0032
KLCI Squared Returns	0.5006	0.2965	0.2166	0.2121	0.1969	0.0768	0.0245	0.0165	0.0068	0.0596	0.0104
KLCI Absolute Returns	0.4383	0.3726	0.3323	0.2800	0.2717	0.1929	0.1310	0.1021	0.0662	0.0827	0.0496

Table 3

Influence of the Power Transformation $\left| R_t \right|^d$ on the Autocorrelations.

To gauge the influence of the power transformation $\left| R_t \right|^d$ on the autocorrelation decay, Table 3 presents the values $\rho_\tau(d) = corr(\left| R_t \right|^d, \left| R_{t-\tau} \right|^d)$ for different lags τ and power transformations d. For all indices, it is found that the autocorrelation is positive at least up to order 50 confirming the existence of the documented long memory in stock markets. It is also confirmed that the decay of this autocorrelation strongly depends on the power transformation d. However, the power return transformations exhibiting the highest level of long run dependency does not seem to be identical for each index (unlike previous findings).

Autocorrelations for Different Power Transformations of the Absolute Returns

NIKKEI 225 INDEX

Lag	d = 0.125	d = 0.25	d = 0.5	d = 0.75	d = 1	d = 1.25	d = 1.5	d = 1.75	d = 2	d= 2.25	d = 2.5	d = 2.75	d = 3
1	0.1443	0.1651	0.1952	0.2134	0.2223	0.2241	0.2204	0.2122	0.2001	0.1850	0.1684	0.1515	0.1350
2	0.1756	0.1970	0.2227	0.2319	0.2267	0.2083	0.1788	0.1419	0.1041	0.0712	0.0462	0.0289	0.0177
5	0.1546	0.1744	0.1991	0.2095	0.2062	0.1898	0.1620	0.1270	0.0914	0.0611	0.0385	0.0234	0.0138
10	0.1548	0.1699	0.1842	0.1826	0.1681	0.1432	0.1117	0.0788	0.0499	0.0284	0.0146	0.0068	0.0026
50	0.0734	0.0852	0.0980	0.1013	0.0978	0.0901	0.0802	0.0696	0.0593	0.0497	0.0411	0.0336	0.0271

HANG SENG INDEX

Lag	d = 0.125	d = 0.25	d = 0.5	d = 0.75	d = 1	d = 1.25	d = 1.5	d = 1.75	d = 2	d= 2.25	d = 2.5	d = 2.75	d = 3
1	0.1219	0.1433	0.1892	0.2326	0.2613	0.2576	0.2196	0.1682	0.1221	0.0869	0.0615	0.0436	0.0310
2	0.1137	0.1354	0.1704	0.1907	0.1827	0.1419	0.0882	0.0462	0.0218	0.0096	0.0040	0.0015	0.0005
5	0.0953	0.1183	0.1526	0.1705	0.1612	0.1228	0.0747	0.0383	0.0176	0.0076	0.0030	0.0011	0.0002
10	0.0910	0.1075	0.1301	0.1418	0.1371	0.1135	0.0804	0.0513	0.0314	0.0191	0.0117	0.0072	0.0044
50	0.1021	0.1123	0.1198	0.1166	0.1007	0.0737	0.0451	0.0242	0.0122	0.0059	0.0028	0.0012	0.0005

SINGAPORE SES ALL INDEX

Lag	d = 0.125	d = 0.25	d = 0.5	d = 0.75	d = 1	d = 1.25	d = 1.5	d = 1.75	d = 2	d= 2.25	d = 2.5	d = 2.75	d = 3
1	0.1532	0.2003	0.2571	0.2961	0.3240	0.3385	0.3354	0.3155	0.2852	0.2512	0.2178	0.1871	0.1597
2	0.1016	0.1554	0.2075	0.2397	0.2596	0.2664	0.2601	0.2439	0.2223	0.1987	0.1753	0.1532	0.1328
5	0.0776	0.1132	0.1466	0.1572	0.1461	0.1163	0.0793	0.0479	0.0268	0.0143	0.0075	0.0038	0.0019
10	0.0671	0.0974	0.1309	0.1419	0.1304	0.1006	0.0650	0.0363	0.0182	0.0085	0.0037	0.0014	0.0004
50	0.0511	0.0728	0.0900	0.0921	0.0801	0.0580	0.0347	0.0174	0.0075	0.0027	0.0006	-0.0002	-0.0005

KUALA LUMPUR COMPOSITE INDEX

Lag	d = 0.125	d = 0.25	d = 0.5	d = 0.75	d = 1	d = 1.25	d = 1.5	d = 1.75	d = 2	d= 2.25	d = 2.5	d = 2.75	d = 3
1	0.2318	0.2726	0.3370	0.3916	0.4383	0.4739	0.4953	0.5031	0.5006	0.4914	0.4783	0.4633	0.4475
2	0.2049	0.2412	0.3021	0.3474	0.3726	0.3751	0.3582	0.3296	0.2965	0.2636	0.2331	0.2058	0.1817
5	0.1761	0.2067	0.2462	0.2678	0.2717	0.2606	0.2409	0.2185	0.1969	0.1770	0.1592	0.1431	0.1286
10	0.1399	0.1636	0.1922	0.2021	0.1929	0.1682	0.1359	0.1039	0.0768	0.0557	0.0399	0.0284	0.0201
50	0.0962	0.1123	0.1260	0.1214	0.1021	0.0754	0.0497	0.0298	0.0165	0.0084	0.0037	0.0011	-0.0002

Table 4

Parameter Estimates for Single-Regime GARCH and APGARCH Models.

Table 4 presents the parameter estimates for single-regime GARCH and APGARCH models. T-statistics based on heteroskedastic-consistent standard errors are presented in parenthesis. In addition, we provide for each index and each estimated model the Ljung-Box statistics for serial correlation up to 20 lags of different power transformations of the residuals. $LB^i(20)$ denotes for $i = 2,3,4$ the Ljung-Box statistic for the corresponding power of the residuals.

	NIKKEI 225 IINDEX			
	GARCH(1,1) Model		APGARCH(1,1) Model	
Mean Parameters:				
Constant: a_0	0.0716	(4.1387)*	0.0304	(2.1560)*
Lag Return of Order 1: a_1	0.0425	(2.4285)*	0.0432	(2.1818)*
Variance Parameters:				
Constant: β_0	0.0314	(1.9382)**	0.0306	(3.6867)*
ARCH Term: β_1	0.1402	(3.1155)*	0.1289	(4.0662)*
Asymmetric ARCH Term: γ_1			-0.4161	(-5.4392)*
GARCH Term: β_2	0.8508	(18.9066)*	0.8671	(24.0861)*
Power Transformation: δ			1.5744	(4.0153)*
Log-Likelihood:	-7774.80		-7688.74	
Ljung-Box Statistics				
LB^2 (20)	13.28		12.79	
LB^3 (20)	2.42		3.21	
LB^4 (20)	0.17		0.24	

	HANG SENG INDEX			
	GARCH(1,1) Model		APGARCH(1,1) Model	
Mean Parameters:				
Constant: a_0	0.1086	(5.5126)*	0.0597	(3.1587)*
Lag Return of Order 1: a_1	0.1163	(6.4254)*	0.1241	(6.9329)*
Variance Parameters:				
Constant: β_0	0.0972	(3.4105)*	0.1152	(3.7894)*
ARCH Term: β_1	0.1545	(5.2730)*	0.1315	(7.6900)*
Asymmetric ARCH Term: γ_1			-0.3885	(-5.1051)*
GARCH Term: β_2	0.8208	(29.7391)*	0.8215	(32.8600)*
Power Transformation: δ			1.8148	(6.1414)*
Log-Likelihood:	-8696.75		-8640.18	
Ljung-Box Statistics				
LB^2 (20)	129.91		67.99	
LB^3 (20)	254.82		137.83	
LB^4 (20)	270.12		119.68	

	SINGAPORE SES-ALL INDEX			
	GARCH(1,1) Model		APGARCH(1,1) Model	
Mean Parameters:				
Constant: a_0	0.0209	(1.2666)	-0.0023	(-0.1654)
Lag Return of Order 1: a_1	0.1710	(8.9528)*	0.1817	(9.3659)*
Variance Parameters:				
Constant: β_0	0.0657	(2.7489)*	0.0735	(1.8467)**
ARCH Term: β_1	0.1455	(4.1690)*	0.1309	(4.5609)*
Asymmetric ARCH Term: γ_1			-0.2058	(-4.2085)*
GARCH Term: β_2	0.8170	(21.2207)*	0.7993	(10.3940)*
Power Transformation: δ			2.4054	(3.2753)*
Log-Likelihood:	-7191.87		-7155.32	
Ljung-Box Statistics				
LB^2 (20)	11.04		6.23	
LB^3 (20)	0.73		0.19	
LB^4 (20)	0.10		0.04	

	KUALA LUMPUR COMPOSITE INDEX			
	GARCH(1,1) Model		APGARCH(1,1) Model	
Mean Parameters:				
Constant: a_0	0.0311	(1.2196)	0.0059	(0.2744)
Lag Return of Order 1: a_1	0.2042	(10.9197)*	0.2063	(10.9153)*
Variance Parameters:				
Constant: β_0	0.0937	(2.7478)*	0.0929	(2.3881)*
ARCH Term: β_1	0.1829	(5.6978)*	0.1851	(6.0097)*
Asymmetric ARCH Term: γ_1			-0.1609	(-2.4159)*
GARCH Term: β_2	0.7859	(22.7138)*	0.7929	(21.7829)*
Power Transformation: δ			1.6918	(4.0973)*
Log-Likelihood:	-7866.74		-7845.61	
Ljung-Box Statistics				
LB^2 (20)	4.86		4.33	
LB^3 (20)	0.07		0.07	
LB^4 (20)	0.03		0.02	

* denotes significance at the 5% confidence level while ** denotes significance at the 10% confidence level

Table 5

Parameter Estimates for the Regime-Switching GARCH Model.

The parameter estimates obtained for the Regime-Switching GARCH model are summarized in Table 5 together with the corresponding t-statistics based on heteroskedastic-consistent standard errors. The Ljung-Box statistics for serial correlation up to 20 lags of different power transformations of the residuals are also supplied.

NIKKEI 225 INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	-0.2202	(-0.3339)	0.1009	(1.6118)
Lag Return of Order 1: $a_{st\,1}$	0.0305	(0.4552)	0.0142	(0.2958)
Variance Parameters:				
Constant: $\beta_{st\,0}$	1.4908	(0.4744)	0.0508	(0.1318)
ARCH Term: $\beta_{st\,1}$	0.1502	(1.1117)	0.0646	(1.6479)
GARCH Term: $\beta_{st\,2}$	0.5299	(0.9325)	0.6476	(0.8008)
Transition Probabilities:				
P($s_t = i / s_{t-1} = i, F_{t-1}$) (i.e, *P* and *Q*)	0.9541	(4.8928)*	0.9744	(131.6756)*
Log-Likelihood:	-7680.10			
Ljung-Box Statistics				
LB^2 (20)	10.31			
LB^3 (20)	3.86			
LB^4 (20)	0.38			

HANG SENG INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	0.0096	(0.0671)	0.1037	(5.2639)*
Lag Return of Order 1: $a_{st\,1}$	0.1024	(3.0658)*	0.0657	(3.9107)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	0.8351	(0.9458)	0.2276	(3.4695)*
ARCH Term: $\beta_{st\,1}$	0.1176	(4.0975)*	0.0765	(3.9230)
GARCH Term: $\beta_{st\,2}$	0.7897	(6.9271)*	0.6953	(10.7631)*
Transition Probabilities:				
P($s_t = i / s_{t-1} = i, F_{t-1}$) (i.e, *P* and *Q*)	0.9947	(207.2291)*	0.9975	(906.8181)*
Log-Likelihood:	-8429.94			
Ljung-Box Statistics				
LB^2 (20)	238.02			
LB^3 (20)	349.29			
LB^4 (20)	393.35			

SINGAPORE SES-ALL INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	-0.0914	(-0.6551)	0.0152	(1.1603)
Lag Return of Order 1: $a_{st\,1}$	0.1905	(2.3460)*	0.1901	(6.6006)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	3.5639	(1.6967)**	0.2413	(0.9925)
ARCH Term: $\beta_{st\,1}$	0.1984	(3.1392)*	0.1426	(5.2619)*
GARCH Term: $\beta_{st\,2}$	0.1236	(0.8698)	0.31	(1.0930)
Transition Probabilities:				
P($s_t = i / s_{t-1} = i, F_{t-1}$) (i.e, *P* and *Q*)	0.8852	(4.1676)*	0.9744	(40.2644)*
Log-Likelihood:	-6910.58			
Ljung-Box Statistics				
LB^2 (20)	4.23			
LB^3 (20)	0.15			
LB^4 (20)	0.05			

KUALA LUMPUR COMPOSITE INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	-0.1227	(-1.0407)	0.0248	(1.6870)**
Lag Return of Order 1: $a_{st\,1}$	0.1491	(3.2203)*	0.2367	(11.6600)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	4.1922	(2.3269)*	0.4047	(7.1000)*
ARCH Term: $\beta_{st\,1}$	0.193	(4.2984)*	0.1918	(6.5238)*
GARCH Term: $\beta_{st\,2}$	0.4604	(2.5062)*	0.1736	(2.5491)*
Transition Probabilities:				
P($s_t = i / s_{t-1} = i, F_{t-1}$) (i.e, *P* and *Q*)	0.9127	(32.3652)*	0.9757	(112.1494)*
Log-Likelihood:	-7611.20			
Ljung-Box Statistics				
LB^2 (20)	7.11			
LB^3 (20)	0.15			
LB^4 (20)	0.04			

* denotes significance at the 5% confidence level while ** denotes significance at the 10% confidence level

Table 6

Parameter Estimates for the Regime-Switching APGARCH Model.

Table 6 gives the parameter estimates for the new Regime-Switching APGARCH model introduced in this paper. The conditional distributions of the error-terms are found to be best represented by Student-t densities for all indices and both states of the Markov-switching process. Again, t-statistics based on heteroskedastic-consistent standard errors and Ljung-Box statistics for serial correlation up to 20 lags of different power transformations of the residuals are provided.

NIKKEI 225 INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	-0,0561	(-2.4497)*	0,0939	(4.5582)*
Lag Return of Order 1: $a_{st\,1}$	-0,0238	(-1.5973)	0,1038	(3.6293)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	0,03	(3.6144)*	0,0499	(2.9702)*
ARCH Term: $\beta_{st\,1}$	0,0738	(6.4736)*	0,1447	(3.6175)*
Asymmetric ARCH Term: $\gamma_{st\,1}$	-0,5957	(-4.6905)*	-0,321	(-2.9102)*
GARCH Term: $\beta_{st\,2}$	0,9218	(75.5573)*	0,7847	(15.5079)*
Power Transformation: δ_{st}	1,1977	(4.7641)*	1,832	(6.5710)*
Student-t Parameters:				
Degree of Freedom: ν_{st}	8,3826	(7.1818)*	5,9497	(5.6357)*
Transition Probabilities:				
$P(s_t = i / s_{t-1} = i, F_{t-1})$ (i.e, *P* and *Q*)	0,9998	(9998)*	0,9997	(1999.4)*
Log-Likelihood:	-7485,21			
Ljung-Box Statistics				
LB^2 *(20)*	29,92			
LB^3 *(20)*	9,69			
LB^4 *(20)*	1,17			

HANG SENG INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	0,0928	(3.3623)*	0,0269	(0.4769)
Lag Return of Order 1: $a_{st\,1}$	0,115	(4.7325)*	0,0628	(2.8675)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	0,2981	(3.0294)*	0,1817	(1.9923)*
ARCH Term: $\beta_{st\,1}$	0,0999	(2.5615)*	0,0492	(2.2062)*
Asymmetric ARCH Term: $\gamma_{st\,1}$	-0,4648	(-2.5580)*	-0,5314	(-1.9925)*
GARCH Term: $\beta_{st\,2}$	0,6486	(6.4027)*	0,8788	(44.8367)*
Power Transformation: δ_{st}	2,1621	(4.4052)*	2,3176	(7.3364)*
Student-t Parameters:				
Degree of Freedom: ν_{st}	5,5105	(7.2164)*	9,1159	(5.0430)*
Transition Probabilities:				
$P(s_t = i / s_{t-1} = i, F_{t-1})$ (i.e, *P* and *Q*)	0,9981	(712.9285)*	0,9979	(712.7857)*
Log-Likelihood:	-8401,12			
Ljung-Box Statistics				
LB^2 *(20)*	28,65			
LB^3 *(20)*	30,68			
LB^4 *(20)*	9,21			

SINGAPORE SES-ALL INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	0,0235	(0.8103)	-0,0113	(-0.7483)
Lag Return of Order 1: $a_{st\,1}$	0,121	(4.0199)*	0,2207	(9.8088)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	0,1511	(0.8011)	0,1137	(3.3052)*
ARCH Term: $\beta_{st\,1}$	0,1336	(1.9334)**	0,2063	(3.7853)*
Asymmetric ARCH Term: $\gamma_{st\,1}$	-0,2416	(-2.9391)*	-0,1355	(-1.7217)**
GARCH Term: $\beta_{st\,2}$	0,7898	(4.6295)*	0,6291	(5.9914)*
Power Transformation: δ_{st}	1,9501	(3.4502)*	2,1718	(3.3157)*
Student-t Parameters:				
Degree of Freedom: ν_{st}	5,8611	(4.3762)*	4,8692	(5.8502)*
Transition Probabilities:				
$P(s_t = i / s_{t-1} = i, F_{t-1})$ (i.e, *P* and *Q*)	0,9991	(832.5833)*	0,9991	(1248.8750)*
Log-Likelihood:	-6749,18			
Ljung-Box Statistics				
LB^2 *(20)*	3,11			
LB^3 *(20)*	0,10			
LB^4 *(20)*	0,03			

KUALA LUMPUR COMPOSITE INDEX

	State 1		State 2	
Mean Parameters:				
Constant: $a_{st\,0}$	-0,007	(-0.1369)	0,0095	(0.4611)
Lag Return of Order 1: $a_{st\,1}$	0,1663	(5.1327)*	0,2169	(9.8144)*
Variance Parameters:				
Constant: $\beta_{st\,0}$	0,1834	(1.5621)	0,204	(4.9275)*
ARCH Term: $\beta_{st\,1}$	0,185	(5.6402)*	0,2159	(4.2500)*
Asymmetric ARCH Term: $\gamma_{st\,1}$	-0,2044	(-2.9926)*	-0,1789	(-2.6040)*
GARCH Term: $\beta_{st\,2}$	0,7766	(15.2274)*	0,5456	(5.4342)*
Power Transformation: δ_{st}	1,715	(5.9056)*	2,2523	(3.2254)*
Student-t Parameters:				
Degree of Freedom: ν_{st}	5,2699	(6.9680)*	4,8182	(7.7313)*
Transition Probabilities:				
$P(s_t = i / s_{t-1} = i, F_{t-1})$ (i.e, *P* and *Q*)	0,9982	(831.8333)*	0,9988	(1248.5)*
Log-Likelihood:	-7435,76			
Ljung-Box Statistics				
LB^2 *(20)*	3,26			
LB^3 *(20)*	0,04			
LB^4 *(20)*	0,01			

* denotes significance at the 5% confidence level while ** denotes significance at the 10% confidence level