CONSTANCY AND PERPETUITY:
Simplifying or Camouflaging?

Shyamasri Ghosh
The Institute of Policy analysis

Gilles Duteil
Université d'Aix-Marseille III

Dilip K. Ghosh
The University of the West Indies

CONSTANCY AND PERPETUITY:
Simplifying or Camouflaging?

Abstract:

This work examines the validity and the legitimacy of the assumption of constancy and/or perpetuity - often used in theory, practice and education in finance. Many of the major results in the existing literature such as weighted average cost of capital, Modigliani-Miller value invariance propositions, and others, which are based on these assumptions, are thrown into clear relief in an effort to understand and highlight the weakness or the general usefulness of those results.

CONSTANCY AND PERPETUITY:
Simplifying or Camouflaging?

The existing literature is replete with results derived under the assumption of constancy and/or perpetuity. The assumption of constant value over an infinite time horizon has made derivations easier, and the derived results have appeared neater. In this paper, we like to bring out some of these results, refocus on those findings, and examine if the underlying assumptions of constancy and perpetuity are simplifying or camouflaging. It can be shown that these postulates are clarifying at times, but at various instances quite confusing and even misleading. What is most intriguing is the fact some of these misleading conclusions (based on those simple and not necessarily realistic assumptions) have appeared as landmark results in the existing literature and thus dominated most of the textbooks in finance and in the practice in the world of finance. In the sections to follow, we present some of those results and show if they hold up beyond the postuates behind them.

I. Valuation of Cash Flows

First, consider the cash flows that are brought out in the basic course in finance, and note that the present value of such a series is determined correctly as follows:

$$P_0 = C_1/(1+k) + C_2/(1+k)^2 + ... + C_n/(1+k)^n \; 1, \tag{1}$$

where P_0 stands for the present value of the future cash flows, C_1, C_2, ..., C_n, k is the rate of discount, and n is the number of periods. If $C_1 = C_2 = ... = C_n = C$ (a *constant* value), then eq. (1) is reduced to:

$$P_0 = (C/k)\left[1 - 1/\{(1+k)^n\}\right]2. \tag{2}$$

When n 6 4,

$$P_0 = C/k \; 3. \tag{2.1}$$

If, however, there is a growth in cash flows such as $C_2 = C_1(1 + g)$, $C_3 = C_2(1 + g)$, ... $C_n = C_{n-1}(1 + g)$, and n 6 4, then obviously

$$P_0 = C_1/(k - g)\ 4. \tag{2.2}$$

This is the well-known Gordon dividend growth model if the cash flows are construed as dividend stream (see Gordon, 1959). The statement is often made that growth and value of a stock, for instance, are directly related: higher the growth rate, higher the value of the stock. It is also remarked that interest rate rate and asset value are inversely related: higher the interest rate, lower the asset value, and *vice versa*. Equation (2.2) verifies those remarks readily. Yet, if you look at Equation (1), you get the same results. So constancy and perpetuity assumptions are simplifying; the direct relation beteen asset price and growth rate of cash flows and the inverse relation between interest rate rate and asset value remain unscathed independent of the assumption of constancy and/or perpetuity.

II: Weighted Average Cost of Capital

The question is: are the assumptions of constancy and/ot perpetuity always simplifying? Let us look into some other cases where those postulates have been invoked in the analytical literature. The traditional view of the weighted average cost of capital (WAAC / ρ), enshrined in standard textbooks (e.g., [5], [10], etc.), is as follows:

$$\rho = k_E\left(\frac{E}{E+D}\right) + k_D\left(\frac{D}{E+D}\right)5 \tag{3}$$

in the absence of taxes; in the presence of taxes (t for tax rate), Equation (3) is modified as follows[1]:

$$\rho^t = k_E\left(\frac{E}{E+D}\right) + k_D(1 - t)\left(\frac{D}{E+D}\right)6 \tag{3.1}$$

Here K_E and K_D are the component cost of capital for equity (E) and debt (D), respectively, and ρ^t is the tax-adjusted (weighted average) cost of capital.

In an interesting paper, Arditti (see Arditti, 1973), however, first notes, and subsequently Arditti and Levy (see Arditti and Levy, 1977) further elaborate that the weighted average cost of capital in the absence of taxes (ρ), as in Equation (1), depends upon two ingredients: earnings before interest and taxes (X) is a constant value *ad infinitum* (/ level perpetuity), - that is, constancy of X, and this value X continuing through infinity. Let us examine how this claim is established.

Let X_j be the earnings before interest and taxes at year j, j = 1,2,...n, and hence:

$$V = \sum_{j=1}^{n} \frac{X_j}{(1+\rho)^j} \; 7 \qquad (4)$$

If we allow X_j = X □ j, then

$$V = X \sum_{j=1}^{n} \frac{1}{(1+\rho)^j} \; 8 \qquad (4.1)$$

whence:

$$\rho = \frac{X}{V}.\left(1 - \frac{1}{(1+\rho)^n}\right) 9 = \left[\left(\frac{X - k_D.D}{E}\right)\frac{E}{V} + \frac{k_D.D}{V}\right]\left(1 - \frac{1}{(1+\rho)^n}\right) 10 \qquad (1^*)$$

Again,

$$\square\, E = \sum_{j=1}^{n}\left(\frac{X - k_D D}{(1+k_E)^n}\right) - \frac{D}{(1+k_E)^n} \; 11 \qquad (5)$$

we get:

$$\frac{X - k_D D}{E} = \left[\frac{1}{1 - \left(\frac{1}{(1+k_D)^n}\right)}\right] 12 \left(1 + \frac{\frac{D}{S}}{(1+k_D)^n}\right) 13 \qquad (5N)$$

If the value of (4N) is substituted into (1^*), the following expression is easily obtained:

$$\rho = \; 14 \left[1 - \left(\frac{1}{(1+k_D)^n}\right)\right] 15 \left[\frac{1}{1 - \left(\frac{1}{(1+k_D)^n}\right)}\right] 16 \left(1 + \frac{\frac{D}{S}}{(1+k_D)^n}\right) k_E \frac{E}{V} \; 17$$

$$+ \left[1 - \left(\frac{1}{(1+\rho)^n}\right)\right] k_D \frac{D}{V} \; 18 \qquad (5O)$$

As n 6 4, ρ = $k_E \frac{E}{V} + k_D \frac{D}{V}$ 19. For n<4, the standard textbook version of the weighted average cost of capital is not true as the expression (5O) shows. Note that none of the textbooks in corporate finance has brought this fact to life, and practitioners in the financial world hardly ever recognize this reality in their capital budgeting decisions, in the determination of optimal budgets and the like.

More directly, as by definition, $X = X - K_D D + K_D D$, and X is a constant value running from period 1 into perpetuity,

$$V = \frac{X}{\rho} 20$$

or

$$\rho = \frac{X}{V} = \frac{X - k_D . D}{V} + \frac{k_D D}{V} 21 \qquad (6)$$

Rewrite Equation (5) as

$$\frac{X}{V} = \left(\frac{X - k_D . D}{E} \right) \frac{E}{V} + k_D \left(\frac{D}{V} \right) 22.$$

Note that

$$\left(\frac{X - k_D . D}{E} \right) \equiv k_E 23,$$

and thus

$$\rho = \frac{X}{V} = k_E \frac{E}{V} + k_D \frac{D}{V} 24.$$

It is thus shown that the textbook version of the weighted average cost of capital is essentially dependent upon the *constancy* of X, and that X is constant *perpetually*. One should note now that none of the textbooks in corporate finance has brought this fact to life, and practitioners in the financial world hardly ever recognize this reality in their capital budgeting decisions, in the determination of optimal budgets and the like.

In an interesting approach, based on the work of Reiley and Wecker (see Reiley and Wecker, 1973), Ang (see Ang, 1973) takes up the issue as to whether the weighted average cost of capital measures the true overall cost of capital, and demonstrates that the weighted average cost of capital is the true cost of capital only in the case of perpetual constancy of cash flows. Let us sketch his logic now. If debt capital (D) and equity capital (E) generate constant perpetual cash flows (I / interest payments) and (M / dividend payments), then $D = \sum_{j=1}^{\infty} \frac{I}{(1 + k_D)^j} = \frac{I}{k_D} 25$, and $E = \sum_{j=1}^{\infty} \frac{M}{(1 + k_E)^j} = \frac{M}{k_E} 26$, and the value of total capital (the market value of the firm), V = E + D =

$\frac{M}{k_E} + \frac{I}{k_D}$27. Note that the market value of the firm can be expressed as:

$$\text{V} = \sum_{j=1}^{\infty} \left(\frac{M+I}{(1+\rho)^j} \right) = \frac{M+I}{\rho} \text{28.} \qquad (7)$$

Obviously, then

$$\frac{M}{k_E} + \frac{I}{k_D} = \frac{M+I}{\rho} \text{29}$$

or

$$\rho = \frac{M+I}{\frac{M}{k_E} + \frac{I}{k_D}} \text{30.}$$

Since $\frac{M}{k_E} = E$ 31 and $\frac{I}{k_D} = D$ 32, we can rewrite:

$$\rho = \frac{X}{V} = k_E \frac{E}{V} + k_D \frac{D}{V} \text{33,} \qquad (8)$$

which is the weighted average cost of capital, already brought out here and almost everywhere else. Ang, however, moves further with the issue, and examines the validity of the result of Reiley and Wecker. In that effort, he postulates constant growth rate of dividend *à la* Gordon[2] (see Gordon, 1959), and re-expresses V as follows:

$$\text{V} = \sum_{j=1}^{\infty} \frac{I + D(1+g)^{j-1}}{(1+g)^j} \text{34} = \frac{I}{\rho} + \frac{D}{\rho - g} \text{35} \qquad (9)$$

From Equation (9), one gets the following quadratic equation:

$$\text{V} \cong \rho(\rho - g) - \text{I}(\rho - g) - \rho \cong \text{D} = 0, \quad \text{or}$$

$$\text{V} \cong \rho^2 - \rho(\text{V} \cong g + \text{I} + \text{M}) + \text{I} \cong g = 0, \qquad (10)$$

the solution of which is:

$$\rho = \frac{Vg + I + M +\_ \sqrt{(Vg + I + M)^2 - 4VIg}}{2V} \text{36} \qquad (11)$$

Note now that if g = 0 (which is the case of constant perpetual cash flows),

$$\rho = \left(\frac{I+M}{V}, 0) \right) \text{37,}$$

and that yields $\rho = \left(\frac{I+M}{V},0)\right)$38 = $k_E\frac{E}{V}+k_D\frac{D}{V}$ 39

as already pointed out through (7). However, let us examine g □ 0, situations involving nonconstant cash flows. From (11) we can easily deduce that g>0 (or g<0), the term under the radical sign is equal to $(Vg - I)^2 + I^2 + M^2 > 0)$, and hence true cost of capital is higher than weighted cost of capital. It is clear that constancy and perpetuity postulates in this case in point are more camouflaging than simplifying.

III. Modigliani-Miller Prpositions

Next, let us bring out the celebrated propositions of Modigliani and Miller (see Modigliani and Miller, 1958; Modigliani and Miller, 1963) on capital structure and cost of capital, which (expressed in Prposition I and Proposition II) are as follows:

Proposition I: $\rho = \frac{X}{V}$ 40,

where X measures the earnings before interest and taxes, V (/ E + D) is the value of the firm, and ρ the cost of capital. This proposition, expressed differently, states that the value of a firm is independent of its capital structure. Their second proposition reads as follows:

Proposition II: $k_E = \rho + (\rho - k_D)\cdot\frac{D}{E}$ 41

This proposition, as Modigliani and Miller (see Modigliani and Miller, 1958) have shown, hinges essentially on the validity of *Proposition I*. It is instructive now that we reexamine the proofs underlying those results. Note that Modigliani and Miller start off with the scenario that there are two firms with the same expected earnings X, and then they assume that firm 1 is financed entirely with equity capital while firm 2 has both debt and equity in its capital structure. Their proof then proceeds as follows[3]:

Consider an investor holding s_2 dollars' worth of the shares of company 2, representing a fraction α of the total outstanding stock, S_2.The return from this portfolio, denoted by Y_2, will be a fraction α of the income available for the shareholders of company 2, X_2,is, under all circumstances, the same as the anticipated total return to company 1. X_1, , we can hereafter replace X_2 and X_1 by a common symbol X. Hence, the return from the initial portfolio can be written as: $Y_2 = \alpha(X - rD_2)$. Now suppose the investor sold his αS_2 worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount αS_2 realized from the sale of his initial holding and borrowing an additional amount αD_2 on his own credit, pledging his new holdings in company 1 as collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)$ of the shares of earnings of company 1. Making proper allowance for the interest payments on his personal debt αD_2, the return from the new portfolio, Y_1, is given by:

$$Y_1 = \frac{\alpha(S_2 + D_2)}{S_1} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2 \; 42.$$

If one compares Y_1 with Y_2, one can see that as long as $V_2 > V_1$, Y_1 will be greater than Y_2, and in that case it pays owners of company 2's shares to sell their holdings, thereby depressing S_2 and hence V_2, and to acquire shares of company 1, thereby raising S_1 and thus V_1. Thus Modigliani and Miller prove that "levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account".

Note now that this elegant arbitrage argument establishing the value invariance (alternatively known as leverage indifference) proposition rests innocuously on the assumptions of *constancy* and *perpetuity* for X; without these assumptions no way they could express the value of a firm as $V_j = X_j/\rho$ (= X/ρ, finally in their proof under the assumption that $X_1 = X_2 = X$). Their *Proposition II* again uses the fact $V_j = X_j/\rho$ (= X/ρ). These neat and profound results are thus built on the *terra firma* of constancy and perpetuity.

IV. Asset Growth, Corporate leverage, Dividend Payout and Tax Rate

Another important result in the area of corporate leverage, dividend payout, tax rate and asset growth rate is given as follows[4]:

$$g^* = (1 - t)(1 - b)\{\rho + (\rho - k_D)\frac{D}{E}43\}. \tag{12}$$

where

g^* = growth rate of assets after taxes[5],

b = dividend payout ratio.

t = tax rate applicable to corporate income,

and other symbols carry their earlier connotations. Equation (12) is quite significant as it shows the relationship between asset growth and (i) corporate leverage (D/E), (ii) tax rate, and (iii) dividend payout. Note here also that this neat result expressed in Equation (11) rests on the assumption that D/E remains *constant*. Of course, it can be shown that if a firm attains steady-state equilibrium, not only debt equity ratio becomes constant, but this constancy will prevail perpetually. The question then obviously is: is a firm always in steady-state equilibrium? In the absence of steady-state condition the result in Equation (12) is not valid necessarily.

V. Conclusions

We find that in many major results in theoretical literature we use constancy and/or perpetuity assumptions, and in most practical applications, we take those results without questioning the legitimacy or adequacy of the measures such as weighted average cost of capital. We have cited only a limited number of cases to highlight the issue, but the literature has many such cases where more work is needed to have it properly rectified and flawless.

Endnotes

1. There is a serious debate on the expression of tax-adjusted weighted average cost of capital in the literature. Arditti (1973), and Arditti and Levy (1977) strongly argue that tax-adjusted cost of capital should be as follows: $\rho^t = k_E(1-t)\left(\frac{E}{E+D}\right) + k_D\left(\frac{D}{E+D}\right)$ 44, as opposed to $k_E\left(\frac{E}{E+D}\right) + k_D(1-t)\left(\frac{D}{E+D}\right)$ 45.

2. One may note the debate between Gordon (1959) and Miller and Modigliani (1961), and see the role of constancy and perpetuity in the dividend irrelevance proposition as well.

3. Note that Modigliani and Miller's r and i are our k_E and k_D, respectively.

4. See Gup (1983).

5. g^* is also the rate of growth of dividends and profits.

References

Ang, J., 1973, "Weighted Average Vs True Cost of Capital", *Financial Management*, Vol. 2, No. 3, Fall.

Arditti, F., 1973, "The Weighted Average Cost of Capital: Some Questions on its Definition, Interpretation, and Use", *The Journal of Finance*, Vol. 28, September.

Arditti, F. and H. Levy, 1977, "The Weighted Average Cost of Capital as a Cutoff Rate: A Critical Analysis of the Classical Textbook Weighted Average", *Financial Management*, Fall.

Gordon, M., 1959, "Dividends, Earnings and Stock Prices", *Review of Economics and Statistics*, May.

Gup, B., 1983, *Principles of Financial Management*, John wiley & Sons, Inc., New york.

Modigliani, F. and M. H. Miller, 1958, "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review*, June.

Modigliani, F. and M. H. Miller, 1963, "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review*, June.

Miller, M. and F. Modigliani, 1961, "Dividend Policy, Growth, and the Valuation of Shares", *The Journal of Business*, Vol. XXXIV, No. 4, October.

Riley, R. and W. Wecker, 1973, "On the Weighted Average Cost of Capital", *Journal of Financial and Quantitative Analysis*, Vol. 8, January.

Weston, F. and E. Brigham, 1975, *Managerial Finance*, Dryden Press.