FINITE DIFFERENCE APPROACH TO DETERMINATION OF CAPACITY OF SLUICED DRAINAGE SYSTEM

By C. A. W. Deane *B.Sc., D.H.E.*
LECTURER, DEPT OF CIVIL ENGINEERING, U.W.I.

SYNOPSIS

Starting from the basic differential equations for unsteady flow finite difference expressions have been developed for the determination of water levels and discharges from sluiced drainage systems. Limited comparisons available indicate that the results obtained closely approximates the actual behaviour of such systems. The method developed is of an iterative nature and is ideally suited for solution with a digital computer.

INTRODUCTION

The coastal plain of Guyana is about 270 miles long and on the average 25 miles wide (see Figure 1). Within a strip of 120 miles in length four large rivers - the Corentyne, Berbice, Demerara and Essequibo drain the greater part of the interior. Between the large rivers several small rivers, which discharge either directly into the sea or into the large rivers, drain the coastal plain. Many of these small rivers overflow their banks causing extensive flooding. In addition there are several large swamps which, because of the pattern of land holding, are not drained by any rivers.

The coastal plain is generally very flat, with average ground level slopes being of the order of 1 foot in 5 miles. In general, the land is low, the majority of the coastal plain being on average two feet below mean high tide level. Consequently extensive sea, river and swamp defence works have been constructed to protect inhabited areas from inundation.

Despite these formidable natural disadvantages, at present ninety percent (90%) of the total population of Guyana (approx. 650,000) live on the coastlands, and about 1 million acres is used for the cultivation of various crops and as pasture. This area accounts for approximately twenty-five percent (25%) of that country's gross domestic produce and about sixty-seven percent (67%) of its exports, the most important crops being sugar, rice, coconuts and ground provisions.

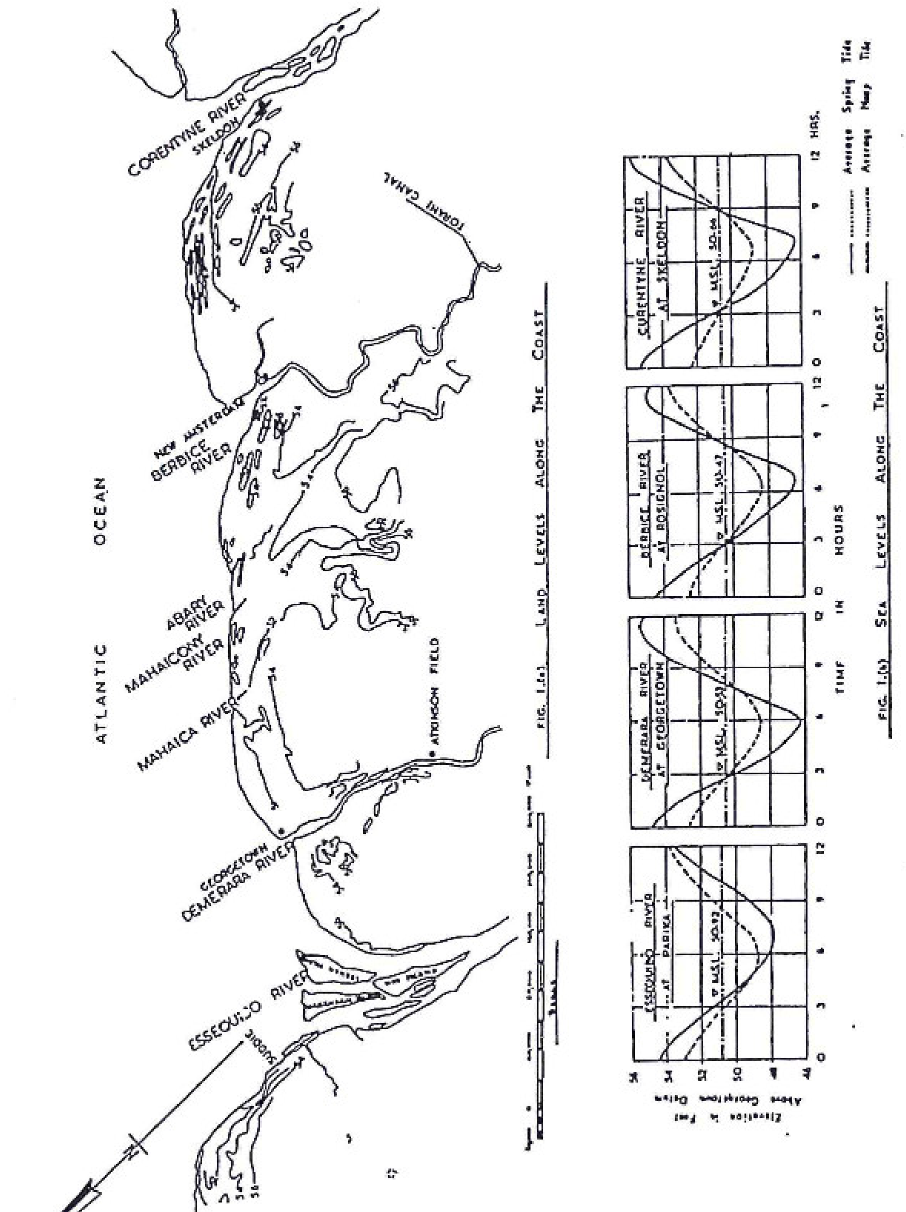


FIG. 1. — LAND AND SEA LEVELS ALONG THE COASTLANDS OF GUYANA

Recent soil surveys have revealed that 2¾ million acres of this coastal plain would be suitable for cultivation of a wide range of crops, as long as adequate drainage can be provided. The entire coastal plain experiences high intensities of rainfall, and the provision of artificial drainage is therefore very expensive.

Artificial drainage is provided either by pumped drainage or by gravity drainage through sluiced outlets into tidal waters at low stages of the tide. The capital and maintenance costs of pumped drainage systems are very high and the provision of such drainage systems so far has proved economic only for highly populated areas and for crops of great economic return. Consequently, the majority of the agricultural areas in Guyana are drained through sluiced outlets into tidal waters. In a previous paper[1], the author discussed some of the factors which influence the design of such agricultural drainage systems, and discussed briefly a method for designing the drains.

The method used for obtaining a preliminary design of the drains assumes steady flow throughout a tidal cycle, a condition which does not agree with the actual performance of drainage systems. Consequently, after the preliminary designs have been completed it is advisable to impose a design storm on the system and analyse its performance. This is essential as the designer should ensure that the crops planned for the area are not submerged for longer periods than was assumed in determining the design drainage coefficient.

The literature contains only a few references to methods of analysis of similar systems as developed in Holland[2], India[3] and France[4]. In all cases these methods were developed for low intensities of run-off, and for the simplified case of one main channel in which considerable storage could accumulate.

Independently of the above-mentioned papers, which were unavailable at that time, the author developed a method for the particular case of the Black Bush Polder which is fully described in the Appendix to the paper by Scott et al[5]. In addition to its applicability only to a drainage system with a long channel leading from the drainage area to the sluiced outlet, the method developed made the simplifying assumption that there was a low, even rate of run-off from the fields into the drains during a 24-hour period.

Considerable experience on the performance of drainage systems since the preparation of that method has led to the devleopment of the method described in this paper, which is intended to deal with the determination of the volume of water removed from a drainage system in a tidal cycle after imposition of a design storm.

THE HYDROLOGIC APPROACH

Several factors influence the hydrologic approach adopted in the analysis of the behaviour of such drainage systems. Among the most important of these are the drainage layout, rainfall intensity, crop husbandry and the soil conditions. These are described below.

THE DRAINAGE LAYOUT

Layout of the main and collector drains is governed largely by the topography of the land, the desirable method of irrigation and to a great extent by the pattern of land holdings. This is described more fully in a paper by the author[1].

These circumstances have led to the development of three basic patterns of drainage layout which are shown on Figure 2. Figure 2(a) shows the original pattern of drainage prevalent throughout large portions of the country; Figure 2(b) shows a compromise frequently adopted to improve the drainage of a group of estates originally drained in the manner shown in Figure 2(a); Figure 2(c) shows a typical approach adopted in new areas. There are several variations to these three patterns, but they will suffice to illustrate the approach adopted.

RAINFALL INTENSITY

When the first attempts were made to rationalise the method of design statistical information on rainfall intensities was not available. For design purposes therefore, a design run off of 1½ inches in 24 hours was used. In testing the performance of the system it was then assumed that the design run-off was evenly spread over a 24-hour period.

Since that time a considerable amount of information on rainfall intensity has become available, which has revealed, for instance, that 4-day total rainfall of 10 inches is exceeded once every 10 years. This information has also allowed the determination of design drainage coefficients for various crops in a rational manner.

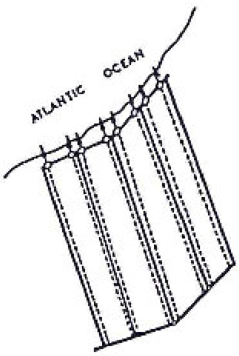


FIG.2(a) – ORIGINAL DRAINAGE LAYOUT FOR GROUP OF ESTATES

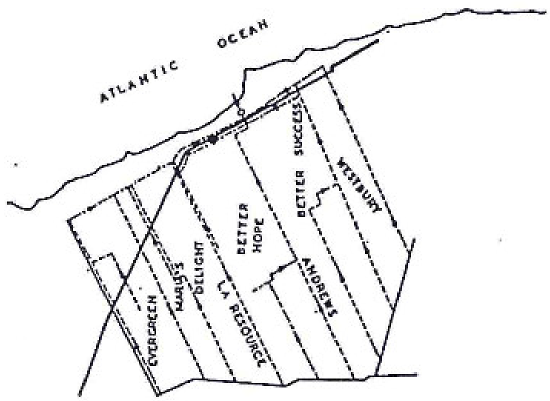


FIG. 2(b) – TYPICAL MODIFIED DRAINAGE ARRANGEMENT FOR DEVELOPED AREA

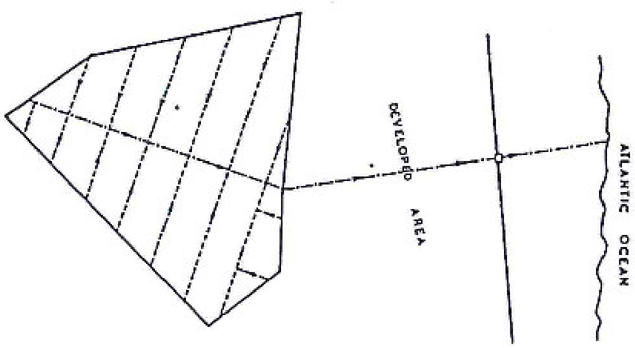


FIG.2(c) – DRAINAGE LAYOUT OF NEW SETTLEMENT AREA

KEY

- Flood or Sea Embankment
- Main Road
- Irrigation Canal
- Subsidiary Drain
- Main or Collector Drain
- Outlet Structure

FIG. 2 TYPICAL DRAINAGE ARRANGEMENTS

Studies of the rainfall behaviour have also revealed that during the rainy season the major portion of heavy rainfall occurs within fairly short periods, and not spread evenly over the entire day. At present this information is of a qualitative nature only, but it is expected that comprehensive information on rainfall intensities will become available shortly from the Hydrometeorological Division of the Guyana Ministry of Works and Hydraulics.

However, the high intensities of rainfall experienced makes it completely uneconomic to design the drains to store excess water while the sluice is shut. Consequently extensive flooding of the lowest land occurs during the period when the sluice is closed, a fact which can be attested to by any visitor to Guyana.

CROP HUSBANDRY

The average land holding is small, and over the past twenty years, there has been a widespread introduction of mechanical methods in rice farming, with a rejection of the methods of transplanting formerly widely used. Consequently, in the weeks immediately following the sowing farmers are not prepared to allow inundation of their fields, as long as it is possible to obtain drainage into the collector drains.

In times of heavy rainfall, therefore, run off from the higher lands in a drainage area gravitates to the lower areas where it ponds up on the fields until such time as the sluice is opened.

THE SOIL CONDITIONS

The soils in the coastal belt are predominantly clays to very great depths, and the heavy showers used in the system analysis occur during the rainy season. Furthermore, during the rainy season humidity is high and evapotranspiration rates are low. Consequently it is assumed that run off accounts for 100% of rainfall as percolation is negligible, the soil moisture content is at a maximum, evapotranspiration is negligible and artificial drainage is provided.

The hydrologic approach adopted therefore makes the following assumptions :—

a) The design shower for the day occurs during the period when the water level in the tidal stream is higher than the water level in the drainage area i.e. when the sluice doors are shut.

b) 100% of the rainfall gravitates to the lowest ground causing flooding to a level that can be calculated by a graph of a type similar to that shown on Figure 3. Such a graph can be prepared from a knowledge of the contours of the drainage area.

Although detailed measurements of the rate of run-off from such drainage areas are not available, the intuitive approach adopted was tested when an analysis of the performances of four drainage systems was carried out by the author[6]. Close agreement was found between measured water levels in the area, and the levels determined by adoption of this hydrologic approach.

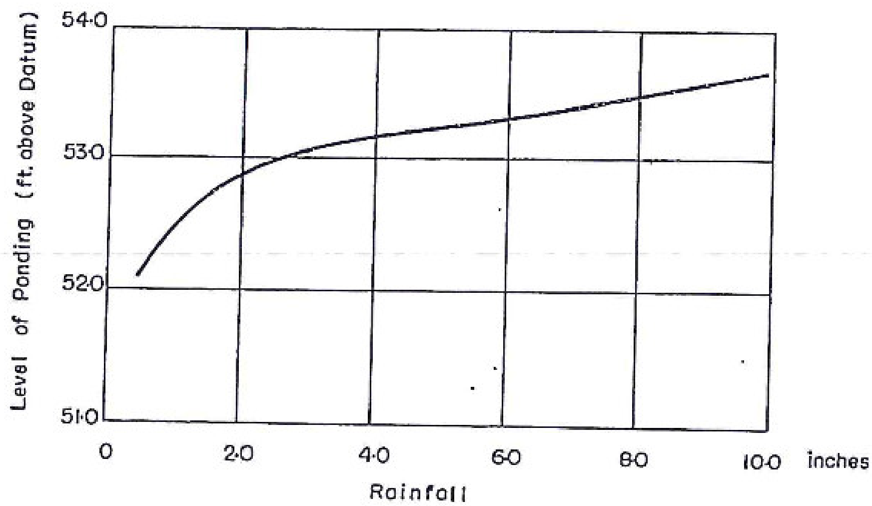


FIG. 3—TYPICAL GRAPH PONDED LEVEL VS. RAINFALL

THE HYDRAULIC APPROACH

SCHEMATIZATION

In its simplest form therefore a simple drainage area, similar to that shown in Figure 2(a), can be schematized into a sluice, a reservoir and a branch channel leading from the storage reservoir to the sluice Figure 4 shows such a schematization, along with the behaviour of the system at the reservoir and at the sluice. Of course the exact behaviour of the water levels at the sluice is dependent on the type of control adopted.

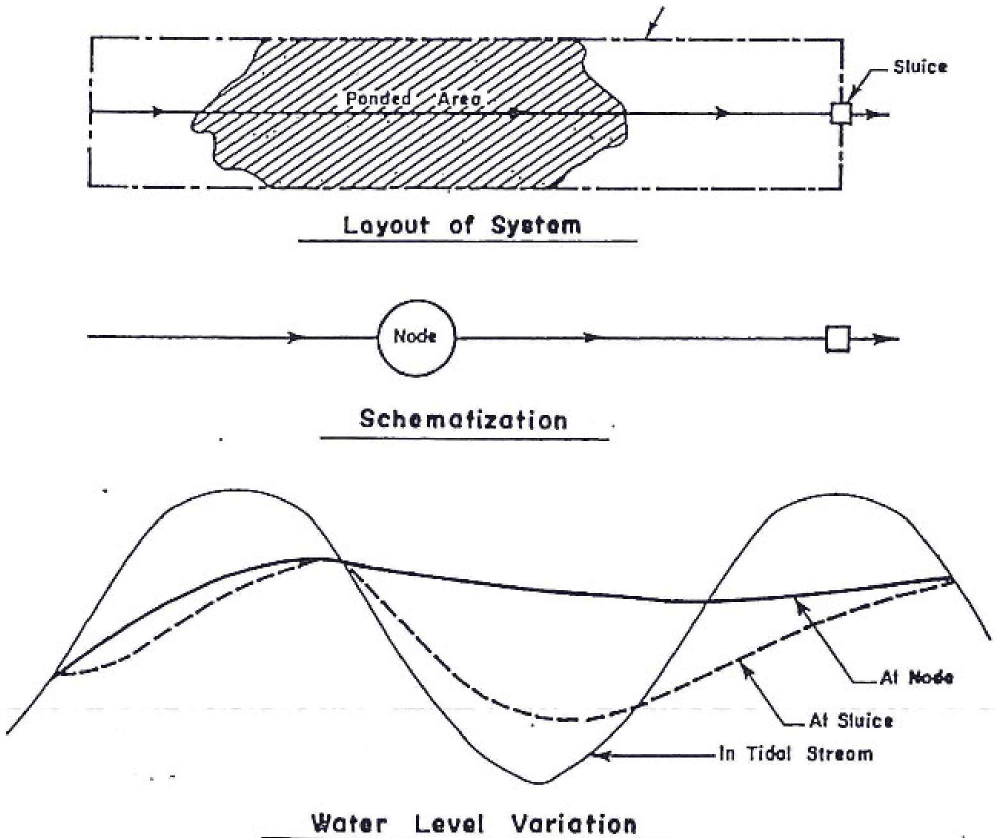
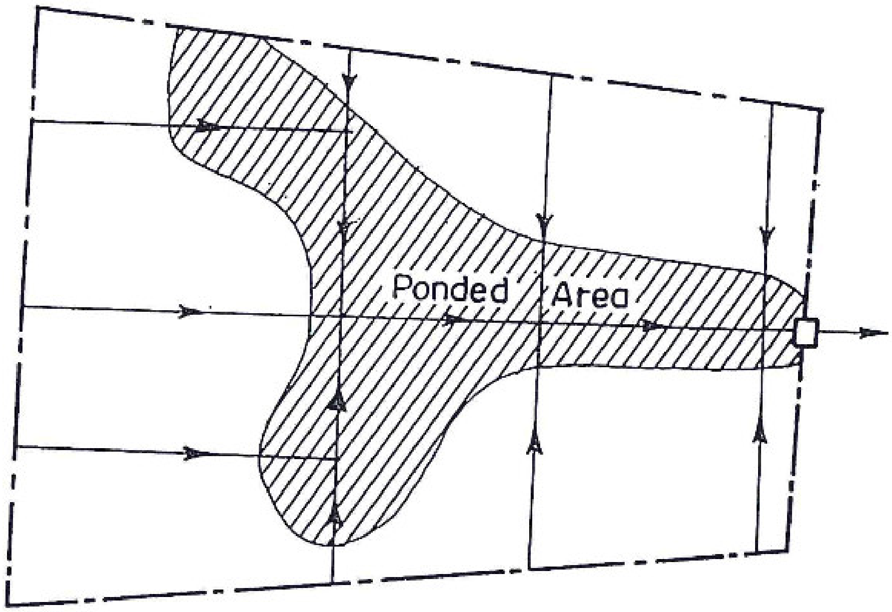


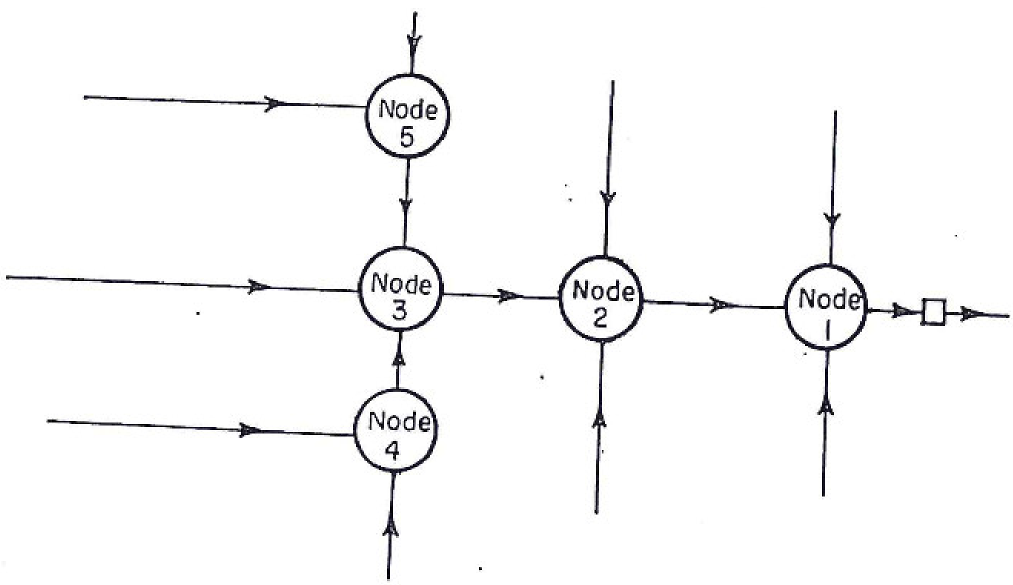
FIG.4 – SCHEMATIZATION OF SIMPLE SYSTEM

In a practical case, with an extensive network of drainage channels the system can be schematized into a network of reservoirs and channels. For purposes of analysis each reservoir can be considered as a node into which one or more branch channels discharge, and from which one channel discharges.

The exact location of each node is dependent on the topography of the drainage area, but in a well-designed system the lowest land would tend to be at the most downstream point of a drainage channel. Storage nodes would then be situated at the junctions of two or more drains. A typical schematization of a large system is shown on Figure 5, which also shows the behaviour of water levels at various nodes.



Layout of System



Schematization

FIG. 5 SCHEMATIZATION OF COMPLEX SYSTEM

Graphs similar to Figure 3 can be prepared for each node, from contoured maps of the drainage area.

The equation of continuity for a node k can be written as

$$- A_k \cdot \frac{dP_k}{dt} + G_k(t) = \Sigma_i \; Q_i(t) \qquad (1)$$

(for K = 1, 2 ....m)

where the summation is over the branches that coincide in node K, and where

A_k = storage area (summation of inundated land areas served by drains leading to node K).

P_k = water level at node K.

t = time

G = inflow in the node

Q = discharge from the node.

Equation of motion for a branch channel

The basic differential equations of unsteady flow in open channels are the well-known momentum and continuity equations.

$$i - \frac{\partial h}{\partial s} = \frac{\partial (v^2)}{\partial s\,(2g)} + \frac{v^2}{C^2 R} + \frac{1}{g}\frac{\partial v}{\partial t}$$

$$\text{and} \quad \frac{\partial F}{\partial t} + \frac{\partial Q}{\partial s} = 0 \qquad (2)$$

where i = slope of channel bed

h = depth of flow

s = distance along the channel

v = mean velocity of channel

C = Chizy coefficient

R = hydraulic radius

F = cross-sectional area of channel

Design practice at present is to allow a maximum permissible velocity in the drains of 3.75 ft/sec based on recommendations by Fortier and Scobey[7]. The velocity head throughout is therefore very low and can be ignored in the analysis. It is also expedient to modify the partial derivatives of v and F with respect to t and s in the following form :

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\frac{(Q)}{(F)} = \frac{1}{F}\frac{\partial Q}{\partial t} - \frac{Q}{F^2}\frac{\partial F}{\partial t}$$

$$\frac{\partial v}{\partial s} = \frac{\partial}{\partial s}\frac{(Q)}{(F)} = \frac{1}{F}\frac{\partial Q}{\partial s} - \frac{Q}{F^2}$$

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial h} \quad \frac{\partial h}{\partial t} = B \quad \frac{\partial h}{\partial t}$$

$$\frac{\partial F}{\partial s} = \frac{\partial F}{\partial h} \quad \frac{\partial h}{\partial s}$$

where B is the upper width of flow.

Furthermore if the water level z is used, with reference ɔ an arbitrarily chosen plane, instead of the depth h then

$$\frac{\partial z}{\partial t} = \frac{\partial h}{\partial t} \text{ and } -\frac{\partial z}{\partial s} = i - \frac{\partial h}{\partial s}$$

quations (2) then become, after some transformation

$$-\frac{\partial z}{\partial s} = \frac{Q^2}{K^2} + \frac{1}{gF}\frac{\partial Q}{\partial t} - \frac{Q}{gF^2} \cdot B \cdot \frac{\partial z}{\partial t}$$

$$\text{and } B\frac{\partial z}{\partial t} + \frac{\partial Q}{\partial s} = 0 \qquad \text{(3)}$$

where $K^2 = F^2 C^2 R$.

the initial stages of opening the sluice gates, and immediately :er closing the sluice gates the friction term in Equation 3 has en found to be very small. If this term and $\frac{Q}{gF^2} \quad B \frac{\partial z}{\partial t}$ is glected Equation 3 reduces to

$$gF\frac{\partial z}{\partial s} + \frac{\partial Q}{\partial t} = 0$$

$$B\frac{\partial z}{\partial t} + \frac{\partial Q}{\partial s} = 0 \qquad \text{(4)}$$

Since z and Q satisfy the second order partial differential equation

$$gF \frac{\partial^2 z}{\partial s^2} - B \frac{\partial^2 z}{\partial t^2} = 0 \qquad (5)$$

Equations (4) represents a wave travelling in the branch with a celerity c given by

$$c = \sqrt{\frac{gF}{B}} \qquad (6)$$

As soon as the wave reaches a particular node Equations (4) are no longer valid. Equation (3) must therefore be applied to the branch between the sluice and the first node, or between two nodes.

Once the storage nodes are contributing to the flow Equation (3) can be applied to the particular branch. In most cases the discharge due to reduction in water level at a node is very much greater than the discharge due to reduction in water level in the drains. Even in cases, where a considerable draw-down has occurred, and the storage in the drains cannot be ignored, it can be assumed that the storage in the drains is concentrated at the node. For each branch, Equations (3) then reduce to the form:

$$\frac{L_i}{gF} \frac{dQ_i}{i\,dt} + \frac{L_i Q_i^2}{K_i^2} = Z_i \qquad (7)$$

where Z_i = difference in water levels at the ends of branch at time t

Q_i = outflow from node at time t

L_i = length of branch

Equations (1) and (7) form a system of differential equations of the first order similar to the system used by Meijer etal [8] for determining flows and water levels for a system of open channel networks.

SOLUTION OF EQUATIONS

A wide variety of finite difference approaches is available for the solution of a system of equations such as those obtained in Equations (1) and (7). However, the stability of the explicit methods is dependent on the choice of very small time units; on the other hand it has been shown (9) that the stability of the implicit methods is independent of the mesh ratio chosen, thus allowing the choice of a large time step in the computations. Despite the greater amount of computing time required for each step when using implicit methods the total time required for computation is less, because of the shorter time step possible.

The following implicit scheme, similar to the one used by Meijer et al (8) for tidal networks has been used for the solution of the system of differential equations

$$-A_k \frac{P_k(t+\Delta t) - P_k(t)}{\Delta t} + G_k\ (t + \tfrac{1}{2}\Delta t) = \tfrac{1}{2}\,\Sigma_i \left[Q_i(t) + Q_i(t+\Delta t)\right]$$

and $$\frac{L_i}{gF_i}\ \frac{Q_i(t+\Delta t) - Q_i(t)}{\Delta t} + \xi_i(t+\Delta t)\,Q_i(t+\Delta t)/Q_i(t+\Delta t)/ = Z_i(t+\Delta t)$$ where $\xi_i = \dfrac{L_i}{K_i^2}$

Starting from the initial values, and taking into account the boundary conditions, this system can be solved at the time instants

$$t = r\,\Delta t\ (r=1, 2\ \ldots\ldots\ldots\ldots n)$$

this system is non-linear in the unknowns, and therefore an iteration process has been developed for its solution. To start the iteration an initial approximation is obtained by extrapolation from previous values of discharges and water levels.

In the vicinity of the sluice the flow is markedly non-uniform; because of the curvature of the streamlines pressure distributions differ from the hydrostatic and velocity distributions are far from uniform. In order to proceed at all, the assumption is made that the difference in water level in the vicinity of a structure can be expressed in terms of a discharge coefficient valid for both steady and unsteady flow.

There are three basic types of sluice used in Guyana viz the broad-crested weir, the culvert and the channel transition. For the first two types of sluice the discharge coefficient used can be taken from the latest published data. However, for the channel transition the discharge coefficients used must be computed in each case by application of the continuity, momentum and energy relationships. In determining discharge coefficients for cases such as these it has been found that in some old drainage systems at low tide rapidly varied flow extended about 200 ft upstream of the sluice. However, in no case tested was the region of rapidly varied flow regime more than a very small percentage of the total length of drainage channels. In any case this condition is highly undesirable and more recent designs have eliminated this possibility.

STARTING METHOD

The method of solution adopted in Equations (8) and (9) is obviously incorrect at time t=o i.e. immediately on opening the sluice gate. Assuming the propagation of a wave of celerity $C = \sqrt{g\frac{F}{B}}$ immediately on opening the of gate, the system of equations represented by Equation (3) was solved for one tide cycle, by an iterative method for a channel 7 miles long between the sluice and the first node.

This solution, for the period before the wave reached the first node, was compared with the solution of Equation (4) by an iterative method, and the difference was found to be negligible.

In both cases the influence of the values obtained for discharges, at the instant when the node began to contribute, were found to vanish very quickly once the first node began contributing. The values obtained for discharge in this initial period were also found to have little influence on the total discharge of the system during a 24 hour period.

As a result of these findings the method adopted for determining the initial discharge and water level to start the iterative process in Equations (8) and (9) has been considerably simplified. The time taken for the wave generated by the opening of the sluice to travel to the first node is determined using Equation (6). At that instant the backwater curve between the sluice and the first node is assumed to be a straight line, and the total discharge during the time period must be equal to the water removed from the drain.

Then $$Q(t_1) = \frac{2x \text{ volume removed from drain storage}}{t_1} \qquad (10)$$

The determination of the volume removed from storage must be determined by a trial and error method as $Q(t_1)$ must equate to the discharge of the sluice.

APPLICATION

To apply the suggested method of computation, the drainage system must first be subdivided into a system of nodes and branches. The density of the network of nodes chosen is dependent on the detail of system performance required. In the initial stages of design it is quite satisfactory to use a maximum of three nodes. Some simple suggested schematizations for typical drainage systems are shown on Figure 6.

The method of computation was applied by Ragwen (10) to the analysis of a simplified system in Guyana. Unfortunately there is a paucity of information on rainfall, discharge and water levels for drainage systems discharging under the conditions assumed in this analysis.

The author has however, made comparisons of water levels obtained by this method with those recorded at Black Bush Polder during the floods of 1962. Rainfall records for this period were available, and water levels were recorded at the sluice and at one other point, but no discharges were measured. It was therefore necessary to schematise the system into a two-node one. Good agreement was found between the measured and calculated water levels at both nodes.

The stability of the method was tested both by Ragwen and the author who found one-hour time steps satisfactory both with respect to stability and accuracy.

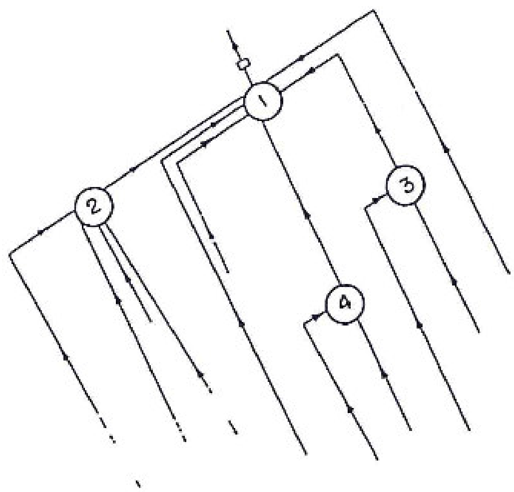


Fig. 6(a) Schematization for Modified System
[Layout as shown on Fig. 2(b)]

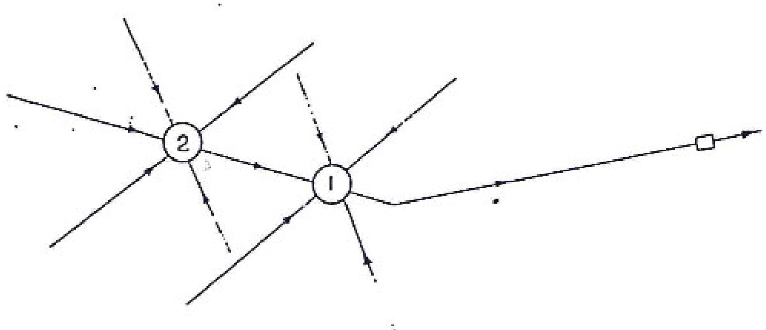


Fig. 6 (b) Schematization for New Settlement Area
[Layout as shown on Fig. 2(c)]

FIG. 6 – SUGGESTED INITIAL SCHEMATIZATIONS FOR COMPLEX SYSTEMS

CONCLUSIONS

The method suggested provides an accurate method for determining the discharge of a drainage system discharging into tidal waters. Although the method has been developed primarily for determining the maximum discharge during the rainy season, with minor modifications to the difference equations developed it can be applied to cases where controlled discharge from drainage fields can be achieved.

Further detailed comparisons of the method with measurements in a well-instrumented drainage system are necessary.

APPENDIX I

REFERENCES

1. Deane , C. A. W.
"Design of Agricultural Drainage in Guyana"
Journal U.W.I. Engineering Society June, 1967.

2. Mazure, J. P.
"Berkening van eeen sluisgang voor een kan"
De Ingenieur November 1933 (In Dutch).

3. Datta, R. R.
"A new aspect of the design of the sluiced drainage channel subjected to diurnal tides".
Irrigation and Power (India) April 1956 pp. 204-216.

4. Combes, G. and Boulan M.
"Efficacite de Clapets ou de Portes a Flats depanchant sur des estuaires soumis a des Marees"
Proc. 10th Congress I. A. H. R.
London 1963 Vol. 1 pp. 1–5 (In French).

5. Scott, P. A., Camacho, R. F. and Sharman, P. A.
"The reclamation and irrigation of the Black Bush Polder in British Guiana"
Conference on Civil Engineering Problems overseas, I. C. E. London 1960. pp. 107–126.

6. Deane, C. A. W.
"Report on Flooding at Black Bush Polder, May-July 1962"
Drainage & Irrigation Dept., British Guiana, September, 1962 (unpublished)

7. Fortier, S. and Scobey, F. C.
"Permissible canal velocities"
Transactions, American Society of Civil Engineers, vol. 89, pp. 940–956, 1926.

8. Meijer Th, Vreugdenhil, C. B. de and Vries, M.
"A method of computation for non-stationary flow in open channel networks."
Proceedings 11th Congress I. A. H. R. Communication 3.28, Lenningrad 1965.

9. Forsythe, G. E. and Wasow, W. R.
"Finite - Difference Methods for Partial Differential Equations." Wiley, 1960.

10. Ragwen, M. R.
"Design of an Agricultural Drainage System for Guyana."
U.W.I. B.Sc. (Eng) Project Report (Unpublished). 1968.

APPENDIX II

NOTATION

The following symbols have been adopted for use in this paper:

A_k = Storage area for node K.

B = Upper width of flow in channel

C = Chezy coefficient

c = Celerity of wave

F = Cross-sectional area of channel

g = Acceleration due to gravity

G_k = Inflow into node

h = Depth of flow in channel

i = Slope of channel bed

K^2 = F^2C^2R.

L_i = Length of channel branch i

P_k = Water level at node K.

Q_k = Discharge from Node K.

R = hydraulic radius of channel,

s = Distance along the channel

t = Time

v = Mean velocity of section of channel

Z = Water level with reference to an arbitrary datum

Z_i = Difference in water levels at ends of channel branch i.

ξi = L_i/k_i2