A NEW DEFINITION OF POWER FACTOR AND ITS IMPLICATIONS FOR POWER SYSTEMS

by

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SUMMARY

This paper indicates the shortcomings of the conventional interpretations of power factor for the assessment of abnormal loads and loading conditions in the electricity supply industry.

A new definition is suggested which encompasses the conventional interpretations for the usual steady state, linear and time invariant loads and which can be applied to abnormal loads and loading conditions.

The paper then considers the basic approaches for power factor correction of abnormal loads and draws some fundamental conclusions. Particular schemes for power factor compensation (correction) are being developed by the authors and will appear in future publications.

1. INTRODUCTION

A number of reference can be made to the literature involving power factor definition and correction as applied to alternating current systems 1–6. There has always been an increasing interest in improving the methods used in the power factor correction of induction motors.7–11. Recently, there has been keen interest in the determination of the maximum capacitance that can be connected across a given induction machine for purposes of power factor correction without causing self exitation. Smith and Sriharan12 have discussed the transients in induction machines with power factor capacitors following disconnection from and reconnection to the supply.

In all of the above investigations the power factor referred to was defined as the ratio of average power (per cycle) to the corresponding volt-amperes as measured by a wattmeter and root mean square voltmeter and ammeter. When the current supplied to a load was sinusoidal and the source was also sinusoidal, then it was seen that this ratio was identical with, Cos ϕ, where ϕ was the phase angle between the supply voltage and the current taken. When the current taken was periodic but non-sinusoidal then the power factor was still calculated by the above ratio. Further, an equivalent sinusoidal wave was postulated for the actual current wave which had a phase shift of, ϕ, and the same power factor.

The above approach has obvious weaknesses. One of them being that the postulated equivalent sinusoidal wave has no relation to the physical system. Also, this approach cannot be applied to non-periodic transient, time varying loads e.g. arc welding, crane lifting. Because of the highly distorted non-periodic nature of the current-voltage characteristic of abnormal loads like these, neither can the power factor be obtained as defined nor can an equivalent sinusoidal be substituted for the original wave form.

11. DEFINITION OF POWER FACTOR

In the normal day to day operation of a power system it is assumed that the load is linear and periodic e.g. heating or lighting loads or induction motor load. This assumption is accepted within some unspecified period of time over which these conditions hold. If an attempt is made to be more precise then it is clear that even the switching on of one of these normal loads presents a non-periodic, non-linear loading condition for an interval of time. The total load of a power system varies slowly with time, giving the familiar daily demand curves with their associated peaks. Over this sort of time interval the load is also non-periodic. However, power system engineers have defined the terms load and diversity factor to give some quantitative meaning to the effects of this 'longer-term' load change.

Load and Power Factors are the two methods in current use for the assessment of load change but no specified time interval is inherent in either definitions. Power factor is used for a "short" time and load factor for a "long" time assessments (with respect to the period of the supply voltage). This vagueness is unsatisfactory and any new definition must overcome this disadvantage. Further, power factor as currently defined cannot handle highly fluctuating non periodic loads.

Consider a general time varying voltage, v(t), supplying some current, i(t), into a passive piece of equipment (Fig. 1).

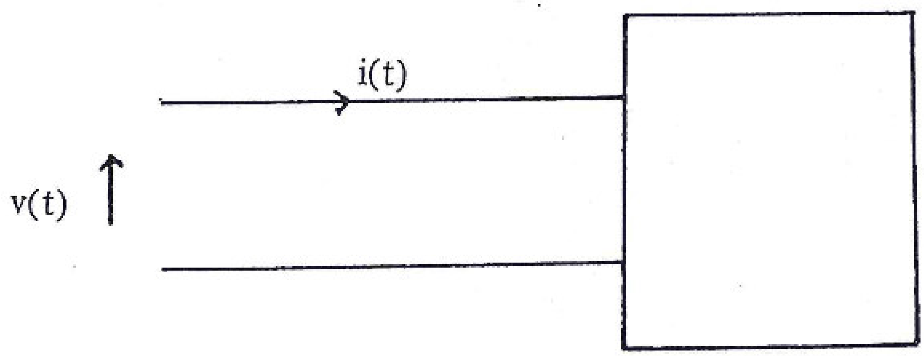

FIGURE 1

The power delivered to the load at time, t, is p(t) where

$$p(t) = v(t)\, i(t) \qquad \text{......(1)}$$

Therefore the energy delivered to the load in the interval

$t_1 \leq t \leq t_2$ is

$$E(t_1,\ t_2) = \int_{t_2}^{t_1} p(t)dt = \int_{t_2}^{t_1} vi\ dt \quad \ldots(2)$$

Define a time varying function, R(t), which is a property of the load such that

$$R(t) \equiv V(t) / i(t) \qquad \ldots\ldots(3)$$

R(t) is called the "Load Function" of the piece of equipment and for physical systems

$$\left|\frac{dR(t)}{dt}\right| < \infty \qquad \ldots\ldots(4)$$

From the theory of functions it is known that

$$\int_{t_1}^{t_2} v^2\ dt \int_{t_1}^{t_2} i^2\ dt \geq \left[\int_{t_1}^{t_2} vi\ dt\right]^2 \qquad \ldots\ldots(5)$$

Therefore, a factor by wich we can rate the energy supplied to the load in the interval $t_1 \leq t \leq t_2$ can be pf(t_1 , t_2) where

$$pf(t_1\ ,\ t_2) = \frac{\int_{t_1}^{t_2} vi\ dt}{\sqrt{\int_{t_1}^{t_2} v^2\ dt \int_{t_1}^{t_2} i^2\ dt}} \qquad \ldots\ldots(6)$$

Define pf(t_1 , t_2) as the "Power Factor" of the energy supplied over the period $t_1 \leq t \leq t_2$ Note that pf (t_1 ,2) is a dimensionless factor.

In words:
'$pf(t_1, t_2)$ is the ratio of the nett energy supplied in the period, $t_1 \leq t \leq t_2$, to the maximum energy which could have been supplied to a maximum efficient load using the same R.M.S. values of current and voltage over the same period.

This definition introduces the term maximum efficient load. If the load is maximum efficient then $pf(t_1, t_2) = 1$ and the equality sign of equation (5) holds. If $R_{max}(t)$ is the load function of this load then;

$$\left[\int_{t_1}^{t_2} v^2/R_{max}(t)\, dt \right]^2 = \int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2 dt \quad \ldots\ldots (7)$$

This occurs only when $R_{max}(t)$ is independent of 't' in the period $t_1 \leq t \leq t_2$ i.e. the load function is time invariant over this interval.

The factor $pf(t_1, t_2)$ does not make any distinction in energy supplied as to whether it is stored or dissipated by the load. Also, the factor gives an indication of the utilization of the power generating capacity and can be applied to any energy system.

Since, $R(t)$, is in general time varying, the concept of load function can be applied to

1) Deterministic, linear and non linear, periodic and non periodic systems,

2) Stochastic systems.

[For stochastic systems the factor $pf(t_2, t_1)$ needs some modification. (See Section 111.5)]

111.1 PERIODIC DETERMINISTIC LOADS

A load, in which $v(t)$ and $i(t)$ are completely known for the interval of time is deterministic in that interval. If $v(t)$ and $i(t)$ repeat themselves after some time, T, then the load is periodic and of period T. In this case

$$pf(0,T) = \frac{\int_0^T vi\,dt}{\sqrt{\int_0^T v^2 dt \int_0^T i^2 dt}} \quad \ldots\ldots\ldots 8$$

and $pf(0,nT) = pf(0\,,T)$ where n is a positive integer. This is so because of the periodic nature of the process.

When $n \longrightarrow \infty$ then

$$pf(0, T) = pf(0, \infty) \quad \ldots\ldots 9$$

Thus, pf(0, T), the factor taken over one cycle for a repetitive operation gives the power factor of the process after an infinite time. This is the justification for using a form of pf(0, T) in the conventional definition for the normal sinusoidal repetitive process.

In the power supply industry, the supply voltage is periodic, of period, T, say. Using p (0, T) as a measure of energy utilization for a non-repetitive load is, of course, unrealistic since this varies from period to period.

111.2 DETERMINISTIC NON-PERIODIC LOADS

In loads of this type the current, i(t), is not repetitive. All that can be attempted is to define the power factor for a particular interval of time, $t_1 \leq t \leq t_2$ as $pf(t_1, t_2)$.

This has the disadvantage that, in general ,

$$pf(t_1, t_3) \neq pf(t_1, t_2) + pf(t_2, t_3) \ldots\ldots\ldots (10)$$

It must be emphasised that most loads met in the day to day operation of a power system are of this type. It now becomes important in calculating a consumer's power factor for a non-periodic load that the period $t_1 \leq t \leq t_2$ be carefully specified.

111.3. LOAD FACTOR

Up to now, the power factor, $pf(t_1, t_2)$ has been tied to the load and mention has been made of the power factor of some load. This idea can be extended easily to measure the efficiencies with which energy is either supplied by a power source or used by a load over the interval. The first usage enables one to associate a power factor with the energy source and the latter allows a factor to be related to the load. The definition of power factor can now be widened to be a measure of utilization or supply of energy.

Consider a power source supplying a 'load' as shown in Fig. 2. This 'load' can be visualised as being made up of, n, sub-loads. If $pf_s(t_2, t_1)$ is the power factor of the source and $pf_L(t_2, t_1)$ that of the complete load, then ignoring losses,

$$pf_s(t_2, t_1) = pf_L(t_2, t_1) \qquad \ldots\ldots(11)$$

If $pf_{Lr}(t_2, t_1)$ is the power factor of the r^{th} sub load then

$$pf_L(t_2, t_1) \leq \sum_{r=1}^{N} P_L f_r(t_2, t_1) \qquad \ldots\ldots(12)$$

By (11) and (12)

$$\frac{pf_s(t_2 - t_1)}{\sum_{r=1}^{n} pf_{Lr}(t_2, t_1)} = K \leq 1 \qquad \ldots.(13)$$

Where K is a positive number.

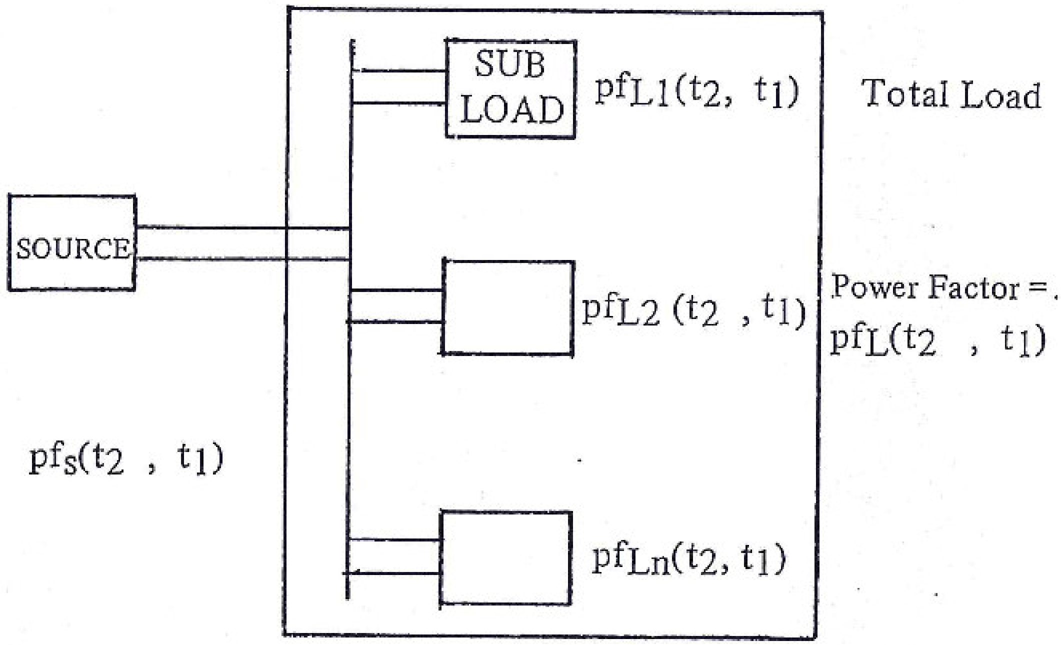

FIGURE 11

III. 4 INTERPRETATION

Diversity factor is an attempt to measure the difference in the demands curves of the various sub-loads wich make the total load. This is done by only comparing the maximum demands so giving a diversity factor of unity when all the loads have their maximum demands at the same time. The factor, K. as defined by equation (13) compares the various demands of the sub loads and is unity only when the sub loads, besides having their maximum demands at the same time, are of the same form.

Load factor as conventionally defined is the ratio of the total energy supplied to the product of maximum average power demand and the interval of interest.

$$\text{Load Factor} = \int_0^T vi \ dt \,/\, \hat{P}_{av} \times T$$

Now $\hat{P}_{av}$ is the maximum value read by the wattmeter over the interval, T. Power System engineers use the load factor so obtained to get an estimate of how efficiently they have been using their generating capacity over the interval. By the above definition, it is possible to obtain a load factor of unity though the load supplied has been of non-unity power factor (conventional definition). According to the load factor, the station has been operating most efficiently but on the basis of conventional power factor the generating plant has not been utilised at its maximum efficiency. This anomaly is even more marked when the load is not constant and the conventional power factor definition does not apply.

If pf(0, T)is used as a measure of station efficiency over the period, T, then this factor is unity only when the load function, R(t), is constant during this time interval. Thus a load which takes a lagging or leading current and is constant in the conventional sense will not give pf(0, T) as unity. This factor, then, is a more precise measure of the efficiency of plant utilization.

III. 5 STOCHASTIC SYSTEMS

For certain processes a parameter of the system may be a stochastic variable, e.g. the on/off time in a manual are welding plant may be function of so many unmeasurable things that it can be conveniently regarded as being stochastic, and the load function, R(t), can only be defined statistically. For stochastic systems such as these estimates of efficiency of energy utilization (power factor etc.) have to be made before the plant is in operation so that decisions can be made as to the purchase of power factor correction devices. Estimates of $pf(t_2, t_1)$ will have to be made on purely statistical information. The definition of power factor as it stands will give $pf(t_2, t_1)$ as a random variable which is of marginal use to the designers. For a process of this type the definition needs some modification. Let v(t) and i(t) be the ensemble averages of a general stochastic process in which v(t) and i(t) are random variables. Thus, we can define

$$pf(t_1', t_2) = \frac{\overline{< v(t)\ i(t) >}}{[<V^2><i^2>]^{1/2}} \qquad \ldots\ldots (14)$$

Where $<x(t)> = \int_{t_1}^{t_2} x(t)\, dt \times \frac{1}{t_2 - t_1}$ the time average of x(t) over

the period $t_1 \leq t \leq t_2$.

For such a process in which v(t), i(t) etc. can be defined statistically for all time then the absolute power factor, $pf(O, \infty)$ is given by (14) where the time averaging is carried out over an infinite time interval. This approach uses statistical techniques which are in use for the design of communication and control systems in which the system itself or its disturbances may be random.

Note, however, that the definition given by equation (14) reduces to that of a deterministic system and can, therefore, be taken as the most general definition of power factor.

IV. MAXIMISATION OF POWER FACTOR

It has been shown that $pf(t_2, t_1) \leq 1$ for any process. It attains its maximum value when R(t) is independent of (t) during the period. For a process in which the load function is varying the power factor, as defined here, cannot be unity.

It is in the interest of power companies to deliver energy over any period of time at as high a power factor as possible. This ensures maximum utilization of plant. Since the power factor of the supplier is equal to the power factor of the total load then the above policy is consistent with the requirement that the individual loads have high power factors.

IV. 2. LINEAR PERIODIC LOADS

The correction of lagging steady state loads by the usual capacitor methods are well known. The only observation that will be made here is that the low power factor of such a load is caused by the load and supply "juggling" some energy between them. This is corrected by giving the load's magnetic field a nearby capacitor with which to transfer energy.

IV. 3. NON LINEAR LOADS

In non linear loads with no storage elements low power factor is caused solely by the harmonic content of the current wave. This is because the load function, R(t), is time varying. Any attempt at compensation must reduce this harmonic content.

IV. 4. THE GENERAL COMPENSATION PROBLEM

Since most equipment is voltage operated the desired compensation is in parallel with it, Fig. 3. However, compensation may be achieved with a series compensator.

The problem can be states as;

Choose a compensator given by $R_1(t)$ such that $pf(t_1, t_2)$

is maximised where
$$pf(t_1, t_2) = \frac{\int_{t_1}^{t_2} vi\, dt}{\left[\int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2\, dt\right]^{1/2}}$$

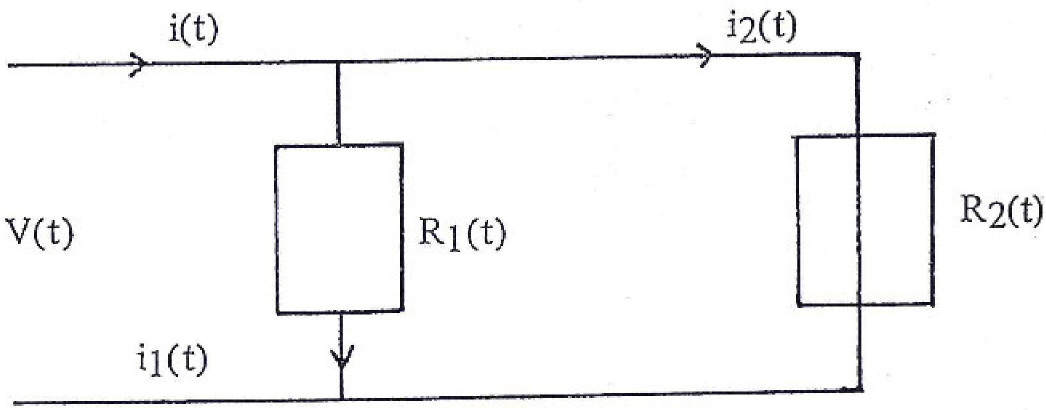

FIGURE III

subject to any physical constraints that may be put on $R_1(t)$. One obvious constraint is that it must exist. The problem as stated may have a trivial answer, for, clearly a short circuit across the load would maximise the power factor. A more practical cost function could be

$$C(t_1, t_2) = n_1(t) \int_{t_1}^{t_2} vi\, dt + n_2(t) \int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2 dt \quad .. (15)$$

This avoids the trivial solution andis equivalent to a tariff method based on the energy taken plus a penalty based on the mean square values of the current nad voltage. This statement of the problem also suits the Calculus of Variations format.

The multipliers, n_1 and n_2 can be made time varying over the interval. This extends the normal power factor correction problem to include those of off-peak loading and seasonal variations of load.

Using the cost function with n_1 and n_2 independent of time the following fundamental statements can be made.

1. In standard lagging (or leading) systems capacitors (or inductors) are ideal compensators.

2. For periodic non-linear systems with no storage elements capacitors and/or inductors cannot be used to increase $pf(t_1, t_2)$.

3. For the compensation of any process the power factor over a period can only be improved by reducing $d\,R(t)/dt$ during the interval. [$R(t)$ being the load function of the resultant load].

V. CONCLUSIONS

The power factor, $pf(t_1, t_2)$ can be defined for any load, deterministic or stochastic. This definition includes the current approaches and when applied to the power source as well as the load envelope the current definition of load factor. As an extension of this factor a cost function can be defined which allows the parallel power factor correction problem to be stated in terms of a Calculus of Variations, Optimisation problem. With time varying multipliers within this cost function the problems of off peak and seasonal loadings can be framed.

VI. FURTHER WORK

This paper only lays the foundation for and outlines the compensation problem. The task which lies ahead is the fabrication of useful compensators for practical systems. The problem to which the authors are aiming is the power factor correction of highly fluctuating non-linear loads as may be presented by a large ship yard or dock yard power demand on a twenty four hour basis where all the information that exists is in the form of statistical descriptions.

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THE COOLING OF GRANULAR SOLIDS IN A MOVING PACKED BED

by

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SUMMARY

Experiments have been carried out on a solids cooler in which a moving packed bed of hot solids was cooled by a stream of air blown crossflow through it. The experiments showed that the Schumann analysis for heat transfer in packed beds of particulate solids was applicable for the 3 mm glass ballotini used in the experiments and that the mechanical design of equipment was satisfactory.

1. INTRODUCTION

For ease of subsequent handling and for safe storage without excessive caking, it is often necessary for a particulate product to be cooled down to a few degrees above ambient temperature. This is especially true for fertilizer material. This can be carried out by various methods some of which are discussed in an article by Schreiber (1). In this article the author compares various types of rotary coolers with a fluidised bed cooler and a special type of vertical counterflow cooler. The author comes to the conclusion that in terms of both convenience and economy the vertical counterflow cooler is preferable. Another possible method which has not attracted attention to date, is to pass the cooling medium, air, crossflow through a moving packed bed of particles. This could have the advantage of space and economy over a rotary cooler and due to its lower pressure drop characteristics, the advantage of economy over a fluidised bed cooler.

The aim of the work described in this paper was to consider the feasibility of a moving packed bed cooler for the cooling of particulate solids and to develop a method for designing full scale units. The work was carried out by building a pilot plant scale cooler, and carrying out an experimental programme to determine the heat transfer characteristics of the system. The specific aim of the work was to determine a design method for the cooling of fertilizer granules, so the material used in the experiments was 3 mm glass ballotini which was of similar size and thermal conductivity to that of fertilizer product material but more convenient to use.

2. THEORETICAL

2.1. Survey of Heat Transfer in Packed Beds

An analysis of heat transfer in a fixed packed bed of solid particles was carried out by Schumann (2) , who arrived at the following relationships:-

$$-\frac{\partial T_g}{\partial y} = T_g - T_s = \frac{\partial T_s}{\partial z} \quad \ldots\ldots\ldots\ldots (1)$$

where:-

$$y = \frac{Hx}{C_g V}$$

$$z = \frac{H}{C_s(1-\varepsilon)} \left[t - \frac{\varepsilon x}{V} \right]$$

with boundary conditions:-

when $y = 0 \quad T_g = T_o$

$z = 0 \quad T_s = 0$

It should be noted that this analysis makes the following important basic assumptions:-

(i) The particles are so small or of such a high thermal conductivity that each one may be assumed to be at a uniform temperature at any given time.

(ii) Compared to the transfer of heat from solid to fluid the transfer of heat by conduction in the fluid itself or between particles is small and may be neglected.

The equations derived by Schumann have been solved by various investigators in different ways and solutions are available in terms of Bessel Functions, Finite Differences, Series Expansions and Error Functions. These solutions have been reviewed by Klinkerberg (3) who suggested that his own solutions in terms of error functions were the easiest to use. The solutions are as follows:-

$$\frac{T_g}{T_o} = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\sqrt{z} - \sqrt{y} + \frac{1}{8\sqrt{z}} + \frac{1}{8\sqrt{y}}\right)\right] \quad (2)$$

$$\frac{T_s}{T_o} = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\sqrt{z} - \sqrt{y} - \frac{1}{8\sqrt{z}} - \frac{1}{8\sqrt{y}}\right)\right] \quad \ldots\ldots (3)$$

He stated that the accuracy of this solution was within 0.006 for y = 2, 0.002 for y = 4 and 0.001 for y = 8 and that the relations should not be used for $y < 2$ and $z < 1$.

For y and z values below this limit a solution by Onsager as cited by Thomas(4) involving error functions but with added terms gives a more accurate result. This solution however, requires more computation.

$$\frac{T_g}{T_o} = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\sqrt{z} - \sqrt{y}\right)\right] + \frac{y^{1/4}}{y^{1/4} + z^{1/4}}\, e^{-y-z} I_o \left(2\sqrt{yz}\right) \quad \cdots (4)$$

$$\frac{T_s}{T_o} = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\sqrt{z} - \sqrt{y}\right)\right] - \frac{z^{1/4}}{y^{1/4} + z^{1/4}}\, e^{-y-z} I_o \left(2\sqrt{yz}\right) \quad \cdots (5)$$

There are thus solutions available which can be used for the accurate application of the Schumann analysis over wide ranges of y and z.

A considerable amount of data on heat transfer coefficients in packed beds is available in the literature, a comprehensive review of which has been compiled by Barker(5). All the reported work shows general agreement with each other. The work considered to be the most useful for the work carried out on the packed bed cooler was that by Denton(6) who obtained the following empirical relation from which the heat transfer coefficient could be obtained:-

$$St = 0.72\, Re^{-0.30} \quad \cdots\cdots\cdots\cdots (6)$$

This relation was obtained from the results of experiments whereby heat was generated in single copper test spheres randomly packed in a bed of glass spheres of similar size to those used in the packed bed cooler experiments.

2.2. Application of Theory to Packed Bed Cooler

The Schumann analysis is derived for unsteady state heat transfer in packed beds. However, this can be applied to the case of cross flow cooling of a moving packed bed if the time of contact of air with the bed is taken as the time the bed takes to traverse the section where heat transfer is taking place. It may be assumed that the time of passage of air through the bed is small compared tothe time taken to traverse the cooling section. This is a reasonable assumption since, in the experiments carried out, the time taken to traverse the cooling section was of the order of minutes, whereas the time of passage of air through the bed was only a fraction of a second.

Now, although the Schumann model had been used to calculate heat transfer coefficients from experimental results (7,8,9) there was no reported work on the conditions under which the analysis would be expected to be valid. Since the particles (3 mm) were relatively large and the thermal conductivity low (0.002 cals $sec^{-1}cm^{-1}{}^{o}C^{-1}$) it was considered that the theory may not be valid because of the possibility of temperature gradients within the particles. It was thus necessary to carry out a series of experiments to determine whether the Schumann analysis held under the conditions envisaged in practice.

The approach used was to use the Denton relation to calculate the heat transfer coefficient for each experimental condition and to compare the experimentally measured outlet air and solids temperatures with those predicted by the given theory. If the experiment results agreed with the theoretically predicted ones then this would indicate that the theory could be used a design method for this type of equipment.

3. EXPERIMENTAL COOLER

3.1. General Description of Experimental Cooler

The apparatus used to investigate the continuous cooling of packed beds of glass beads, shown in Figure 1, consisted of a 6" wide channel, 15" deep, along the bottom of which ran a moving, endless 30 mesh stainless steel belt. The cooler was 9 ft. long, there being a section 2 ft. long (Section 1 of Figure 1) preceding the 2½ ft. long section (Section 2 of Figure 1). The hot glass beads were dropped on to the moving belt at the beginning of Section 1, where the subsequent level of the bed in the cooler was controlled by a gate. Just before reaching the cooling section (Section 2) the temperature of the solids was measured at various levels in the bed. The hot beads were then moved by the belt over the cooling section where the solids were cooled by air, which was blown vertically upwards through the mesh belt. On leaving the cooling section the temperature of the cooled bed was again measured at various levels in the bed. The solids flow rate was measured in the next section (Section 3) after which the solids dropped off the belt, when it went round a roller. The endless belt went round two rollers only, one at each end of the cooler, the belt return coming back through the inlet cooling air.

The belt was driven from 1/15 H.P. motor and its speed was variable in the range 4 to 14 inches/minute. The belt was sealed to the sides of the channel to prevent solids leakage by seating it on suitably angled flat strips of brass fixed to each side of the channel.

The air was blown vertically upwards into the cooling section of the cooler from a duct of cross section 2½ ft. by ½ ft. The ducting supplying air to this section was specially designed to produce as uniform a distribution of cooling air velocities in this section as could be achieved. The mean range of velocities at a given air flow rate measured in this section was + 8% over 45 readings, which was considered to be satisfactory.

3.2. Experimental Measurements

The air temperatures in the experimental runs were measured with mercury in glass thermometers. The cooler inlet air temperature was measured by use of suitable tappings in the duct underneath the bed. The outlet air temperatures were measured by suspending the thermometers directly above the moving bed, at various positions which are shown in Figure 1 .

The solids temperatures were measured with resistance thermometers specially made to suit the system. These thermometers were in the form of a flat strip, the resistance winding which was 5" long and 3/8" wide being covered by and insulated from a brass cover. The total thickness of each thermometer was of the order of 0.040." The thermometers measured the mean solids temperature over the width of the bed at a given level in the bed. Four of these thermometers were calibrated in the range 90^{o} to 150^{o} temperature measurements generally at the hot end of the cooler, and another four in the range 25^{o} to 90^{o} for meeasurements at the cold end of the cooler. Facilities were provided for putting the thermometers in

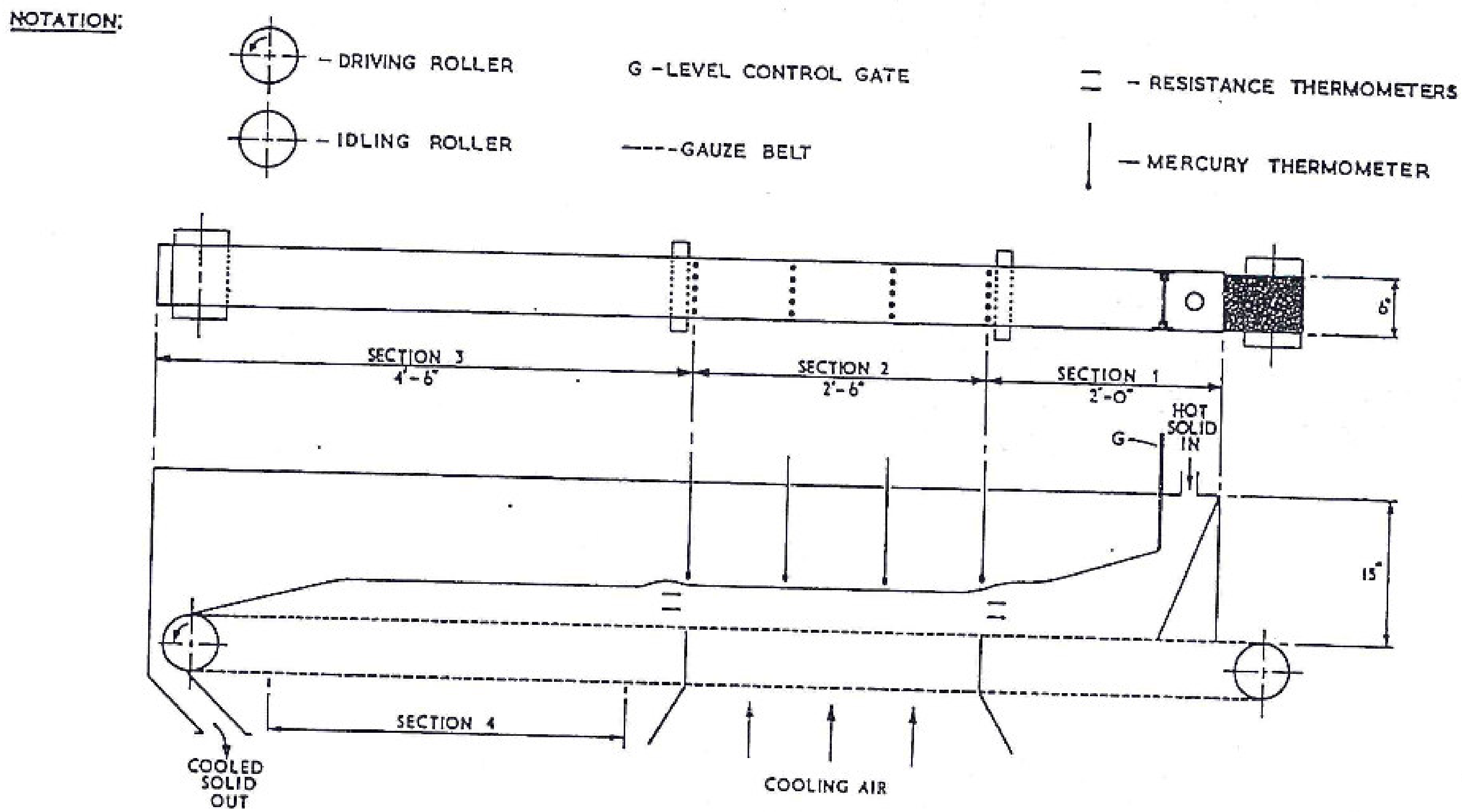

FIG. 1. DIAGRAM OF EXPERIMENTAL BELT COOLER.

the channel at heights of 2 1/8", 3 5/8" 5 1/8" and 6 5/8" above the belt, both immediately preceding and following the cooling section of the cooler, as shown in Figure 1.

The cooler air flow rate was measured by a rectangular section venturi in the duct supplying air to the cooling section. The meter was calibrated by hot wire anemometer traverses in the cooling section before use.

The mass throughput of solid was calculated from measurements of the height and speed of the moving bed in the section following the cooling section. The volumetric solid throughput in the cooling section was also measured, this being used to calculate the bed voidage.

3.3. Experimental Procedure

All the experiments were carried out under steady state conditions. The equipment was first set up to give satisfactory continuous running with a mean solids inlet temperature to the cooling section 120° C. After all the temperatures had settled out there was a 15 minute interval of continuous steady running before the experimental measurements were taken. Two complete sets of readings were taken for each experiment, the mean of the two sets being used in the calculations.

Experiments were carried out over a range of bed heights varying from 2½" to 7", belt speeds from 4" per minute to 14" per minute and superficial air velocities in the cooling section from 2 ft/sec. to 5 ft/sec.

4. EXPERIMENTAL RESULTS

4.1. Heat Transfer

Values of y and z were calculated initially for each set of experimental conditions. In the calculation of these values, the contact time t was taken as the time the bed took to traverse the cooling section. y was found to be in the range 9 to 43 and z in the range 5 to 84. It was thus considered that Klinkenberg's solution of the Schümann analysis could be used since the estimated accuracy would be very much less than 1%. In the calculation of z it was found that the term x/V was always negligible compared to t and so was neglected in each case.

Using the Klinkenberg solution together with the measured inlet air and solids temperature, the theoretically predicted air and solids temperatures leaving the cooling section at the appropriate points where they were measured in practice, were calculated for each experiment in turn.

Typical graphs of solid temperature against time of contact of air and solid are shown in Figures 2 to 7. In these graphs, the continuous lines are the temperature profiles calculated using Equation 3 and the points marked on the graphs are the experimentally measured temperatures. Thus each graph is a direct comparison of theory with practice.

Typical graphs of air temperature above the bed against time of contact of air and solid are given in Figures 8 and 9. The continuous lines are temperature profiles calculated using equation 2 and the points marked on the graphs are the experimentally measured temperatures.

The theoretical curves in each case were not always smooth curves because the bed porosity varied slightly from one experiment to another.

4.2. Operational Behaviour of Cooler

No serious problem were encountered in the mechanical operation of the cooler.

The main problem envisaged was the seal between the belt and the wall of the cooler which had to eliminate solids leakage. The second method tried, that of seating the belt on slightly angled metallic strips, extending ¼ inch into the channel for each side, proved to be entirely satisfactory. The weight of the bed on the seal prevented any particles from passing between the strip and the belt.

The seal between the belt return and the inlet air stream was affected by passing the belt through a slit in a piece of thick rubber sheet. There was no noticeable air leakage with this system. Rubber flaps were used between the end of the air duct and the belt to prevent air leakage under the bed. Only a small amount of leakage was encountered here.

Although the resistance thermometers for solids temperature measurement were only 0.040" thick they did give rise to a resis ance to the flow of solids in the channel. There was a tendency for the level of the bed to rise directly above the thermometer. The effect is depicted in Figure 1. Other than this, the use of the resistance thermometers proved to be satisfactory.

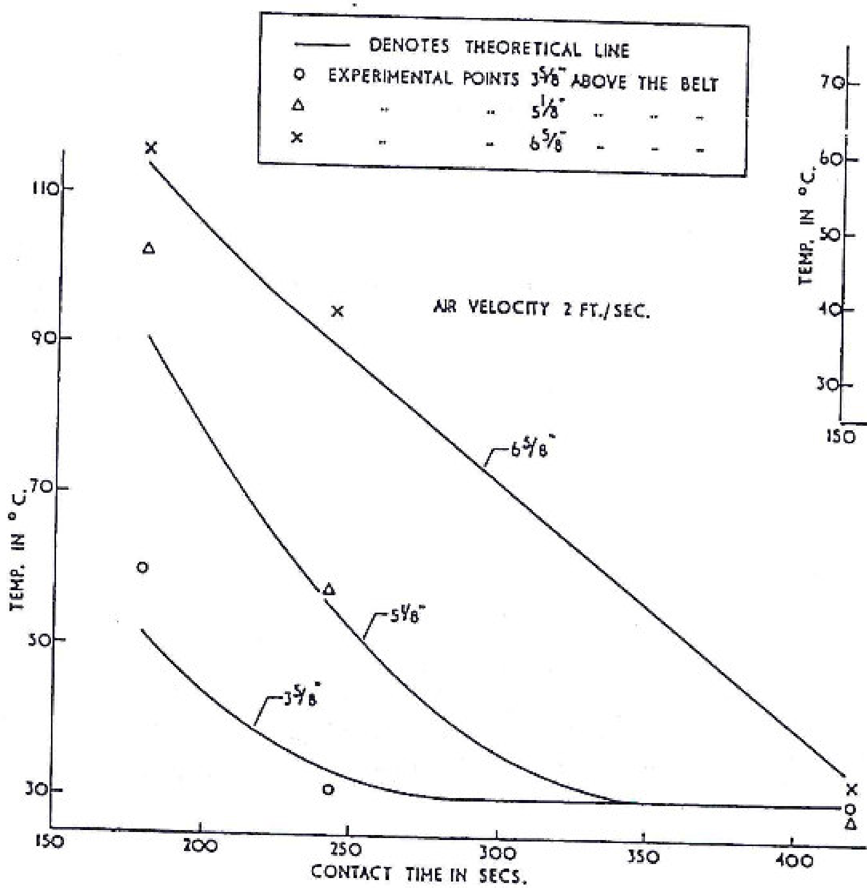

FIG. 2

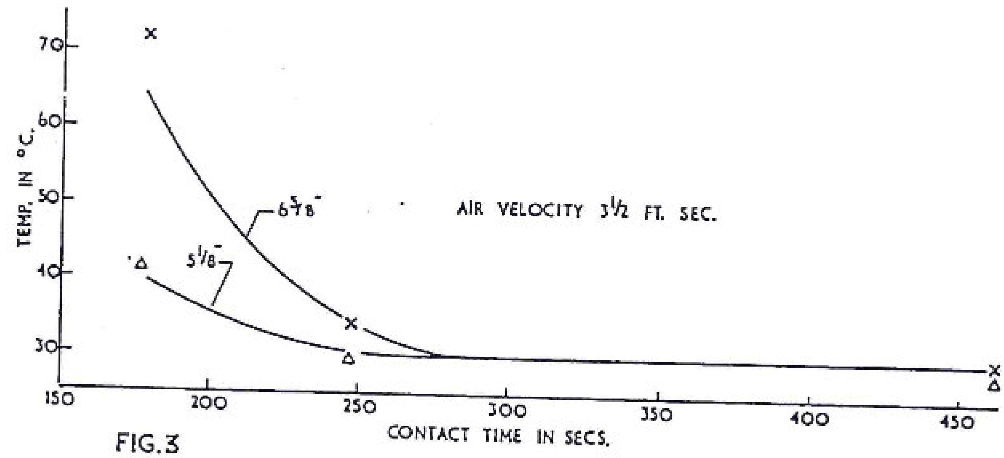

FIG. 3

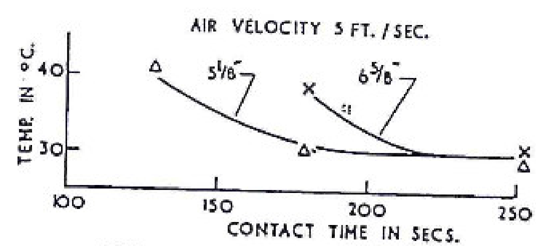

FIG. 4

GRAPHS OF SOLID TEMPERATURES AT EXIT FROM COOLING SECTION V's CONTACT TIME FOR 7" HIGH BED.

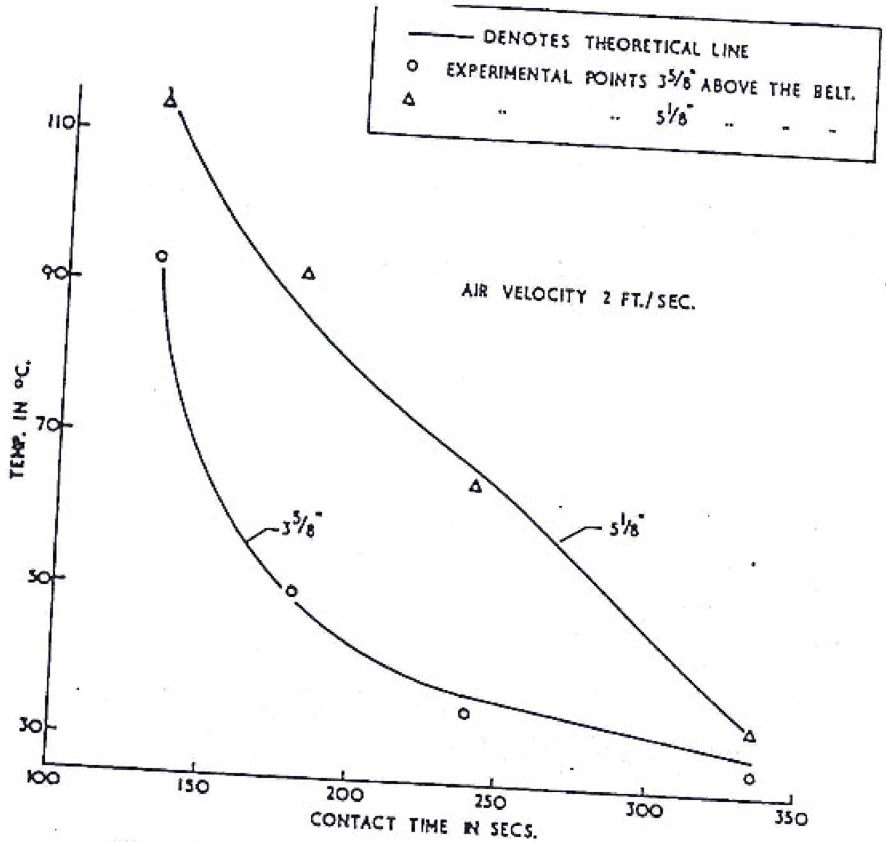

FIG.5

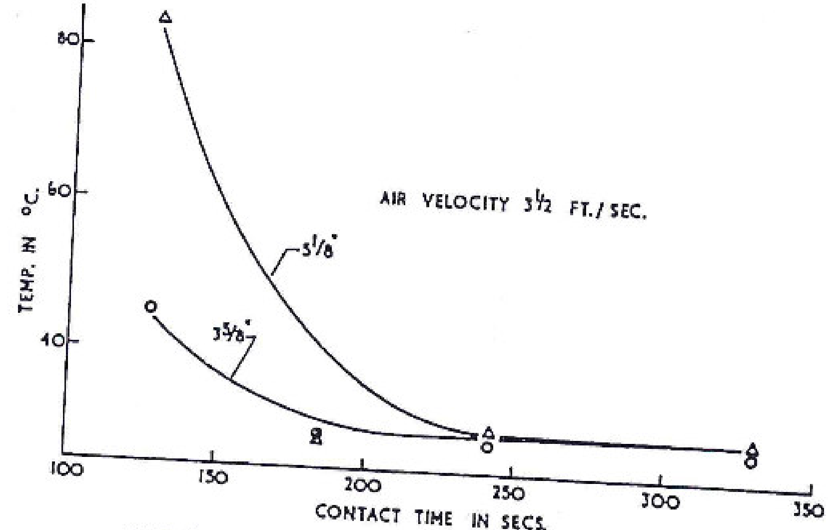

FIG.6.

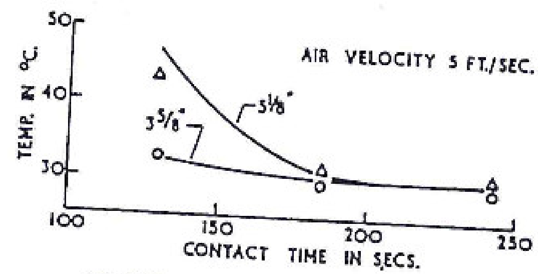

FIG.7

GRAPHS OF SOLID TEMPERATURES AT EXIT FROM COOLING SECTION V's CONTACT TIME FOR 6" HIGH BED.

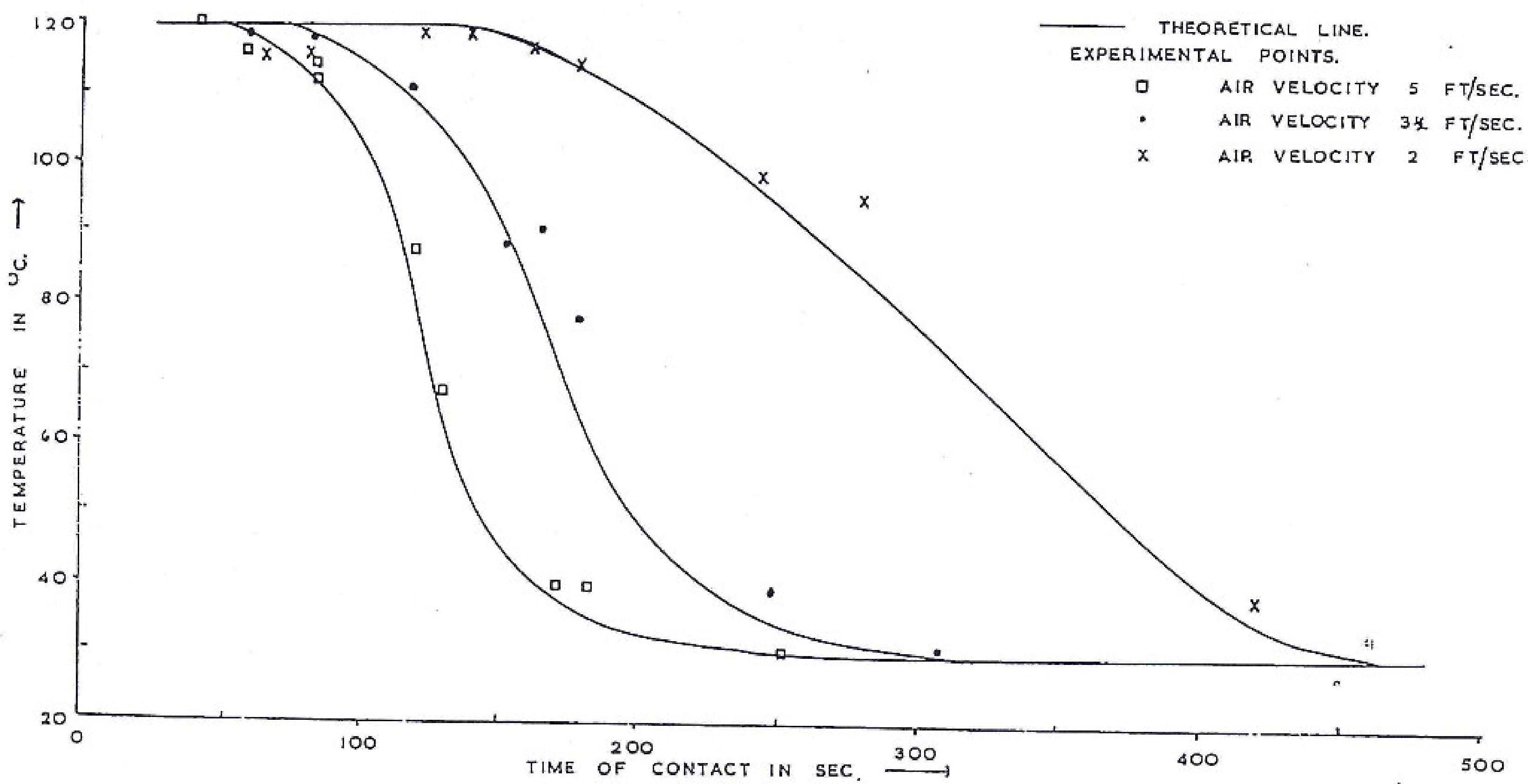

FIG 8 GRAPHS OF AIR TEMPERATURE LEAVING 7" HIGH BED.

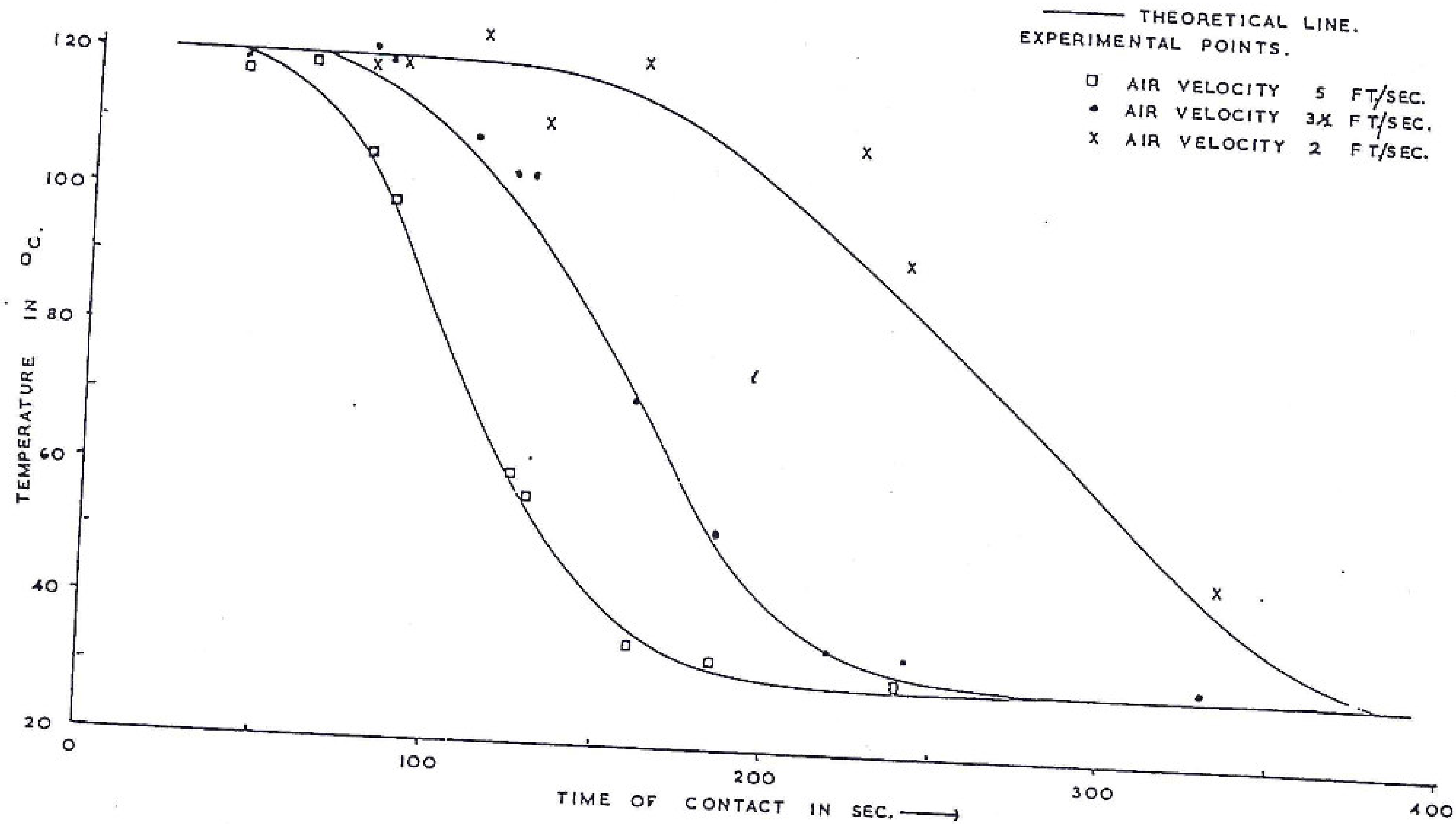

FIG 9 GRAPHS OF AIR TEMPERATURE LEAVING 6" HIGH BED.

5. DISCUSSION

5.1. Heat Transfer Results

The agreement between the experimentally obtained temperatures and the theoretically predicted temperatures was generally within experimental error for both air and solids. There was a tendency for greater discrepancies at the lowest air velocity (2 ft/sec) which could be attributed to the increased difficulty of accurate reading of the micromanometer used to measure the venturi pressure drop at the lower end of the range.

Of the assumptions made in the Schumann analysis it was considered that the weakest was that of assuming that the particles were at a uniform temperature at any given time. If the thermal conductivity of the particles was small enough or the particles large enough to constitute a resistance to heat transfer in this system then it would be expected that the measured solids temperatures would be higher than those predicted. This is not shown up in the results. It would also be expected that this effect would be accentuated at the higher air velocities. However, the agreement between measured and predicted temperatures was found to be very much better at the higher air velocities, probably due to the increased accuracy of the air flow measurement.

The exit solids temperatures were measured about one inch down-stream of the cooling section. If the thermal conductivity were to constitute a resistance to heat transfer then, when the solids were moved by the belt away from the cooling section, the measured temperature of the solid would be expected to rise as the surface temperature came into equilibrium with the core temperature. In runs on the cooler whereby the solids were cooled to a temperature intermediate between the inlet solids temperature and the inlet air temperature, solids temperatures were measured at the same height above the belt at distances 1", 3", 5" and 9" away from the cooling section for a particular mass of solids. There was no significant change in the temperatures measured. This is a further indication that the assumption that the solid particles are at a uniform temperature at a given time is valid.

5.2. Application to Design of Full Scale Equipment

The Schumann analysis has been shown to hold for 3 mm glass ballotini and so can be expected to hold for most particulate solid material up to that size because of its low thermal conductivity. In particular it can be used to design and optimise similar equipment to cool fertiliser granules.

The optimisation can be carried out in a straight-forward manner by inserting the cooling requirements and known physical and thermal quantities into the heat transfer equation and an appropriate equation for the pressure drop across the bed. Combining these gives the running cost i.e. air horse power and also the size of the cooler base area in terms of the variables which require to be optimised i.e. bed depth and air velocity. As the capital cost will be a function of the size of the cooler then the total annual cost can be worked out as a function of these variables also. Adding the running cost, converted to cash, to the annual cost attributable to capital gives the total annual cost as a function of the two variables. a typical calculation indicates that the height of the bed does not optimise, the lower the bed height the lower is the total annual cost. Also, the lower the height of bed, the higher is the optimum air velocity.

It should be noted that this type of equipment can also be used for the continuous drying of particulate solids, if hot air is used. For instance, it could be used for the drying of peas, coffee beans, rice, cocoa, corn or even larger materials.

CONCLUSIONS

The work shows that the Klinkenberg solution of the Schumann relation for heat transfer in packed beds can be applied to particles as large as 3 mm of a material of thermal conductivity as low as 0.002 cals sec^{-1} cm^{-1} $^{o}C^{-1}$. This gives a satisfactory basis for the full scale design of a moving packed bed cooler which has shown itself to be mechanically reliable in pilot plant tests.

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NOMENCLATURE

C_g	– specific heat of air	cals/gm^{o}C
C_s	– specific heat of solid	cals/gm^{o}C
d_p	– particle diameter	cms
h	– heat transfer coefficinet	cals/^{o}C sec cm^2
H	– volumetric heat transfer coefficient	cals/^{o}C sec cm^3
T_s	– outlet solids temperature relative to inlet solids temperature	^{o}C
T_g	– outlet air temperature relative to inlet solids temperature	^{o}C
T_o	– inlet air temperature relative to inlet solids temperature	^{o}C
t	– time of contact of air and solid	secs
V	– fluid flow rate per cross sectional area of bed	cm^3/sec cm^2
x	– height of bed	cms
:	bed voidage	
ρ_g	– density of gas	gms/cm^3
ρ_s	– density of solid	gms/cm^3
μ	viscosity of gas	gm/cm sec

Re $- \dfrac{\rho_g V d_p}{\mu_g}$.

St $- \dfrac{h}{\rho_g \; V C_g}$