

A PRELIMINARY  
INVESTIGATION OF THE  
MANUFACTURE AND  
PERFORMANCE OF A TENOR  
STEEL PAN

by

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## NOTATION

$a$	—	Amplitude
$c$	—	Velocity of Sound in Air
$f$	—	Frequency in Cycles/Sec.
$f_0$	—	Fundamental Natural Frequency
$m$	—	Mass
$t$	—	Time
$T$	—	Periodic Time = $1/f$
$w_a$	—	Density of Air
$w$	—	Radial Frequency = $2\pi f$
$\lambda$	—	Wavelength

## SYNOPSIS

The steel pan is a musical instrument produced from a steel oil drum. It has been developed in Trinidad mainly by the trial and error experimentation of a small number of skilled tuners, but if the development is to continue the efforts of the tuners must be backed up by scientific research. This paper describes preliminary stages of a programme of research initiated in the Faculty of Engineering at the U.W.I. The manufacturing process and the vibrations of the notes of a particular pan have so far been investigated. The pan is shown to consist of flat plate notes flexibly mounted in a stiff elliptical bow. Pitch is directly related to the dimensions of the notes but is also very dependent on the edge restraints of the notes. The partials of the notes are in general non-harmonic, but since the notes appear to vibrate predominantly in their fundamental modes it is possible that the tonal qualities of the pan are influenced to a greater extent by transmission of vibration to other parts of the pan.

### 1. INTRODUCTION

The art of creating music from steel oil drums originated in Trinidad in the early 1940's. From its beginning as a rhythmic accompaniment to Carnival celebrations the music of the steel band has now developed to a stage where it is a serious form of music and one of Trinidad's prime cultural assets. As the music has developed it has become attractive to more and more people outside of Trinidad. The steel band has become a great tourist attraction and bands are able to tour and attract large audiences abroad; many foreigners in fact know Trinidad as the land of steelband and calypso. The steel band has become a national symbol, which is as it should be since it has reached its present position through the efforts and dedication of people involved in the steel band movement in this country,

However, as the music of the steel band reaches a wider audience its commercial potential becomes greater. Already bands are able to support themselves playing in North America and individual tuners working in the U.S.A. find a ready market for their pans. The danger is that the potential may be recognised and exploited elsewhere.

The steel pan has been developed mainly through the isolated experimentation of individual tuners. Their methods have been based on musical knowledge, intuition and trial and error. They have achieved a tremendous amount but if Trinidad is to retain the lead in developing these instruments, then the efforts of the tuners must be backed up by scientific research, and to this end a programme of research has been initiated in the Faculty of Engineering at the U.W.I. The objectives of the three stages of the programme are as follows:-

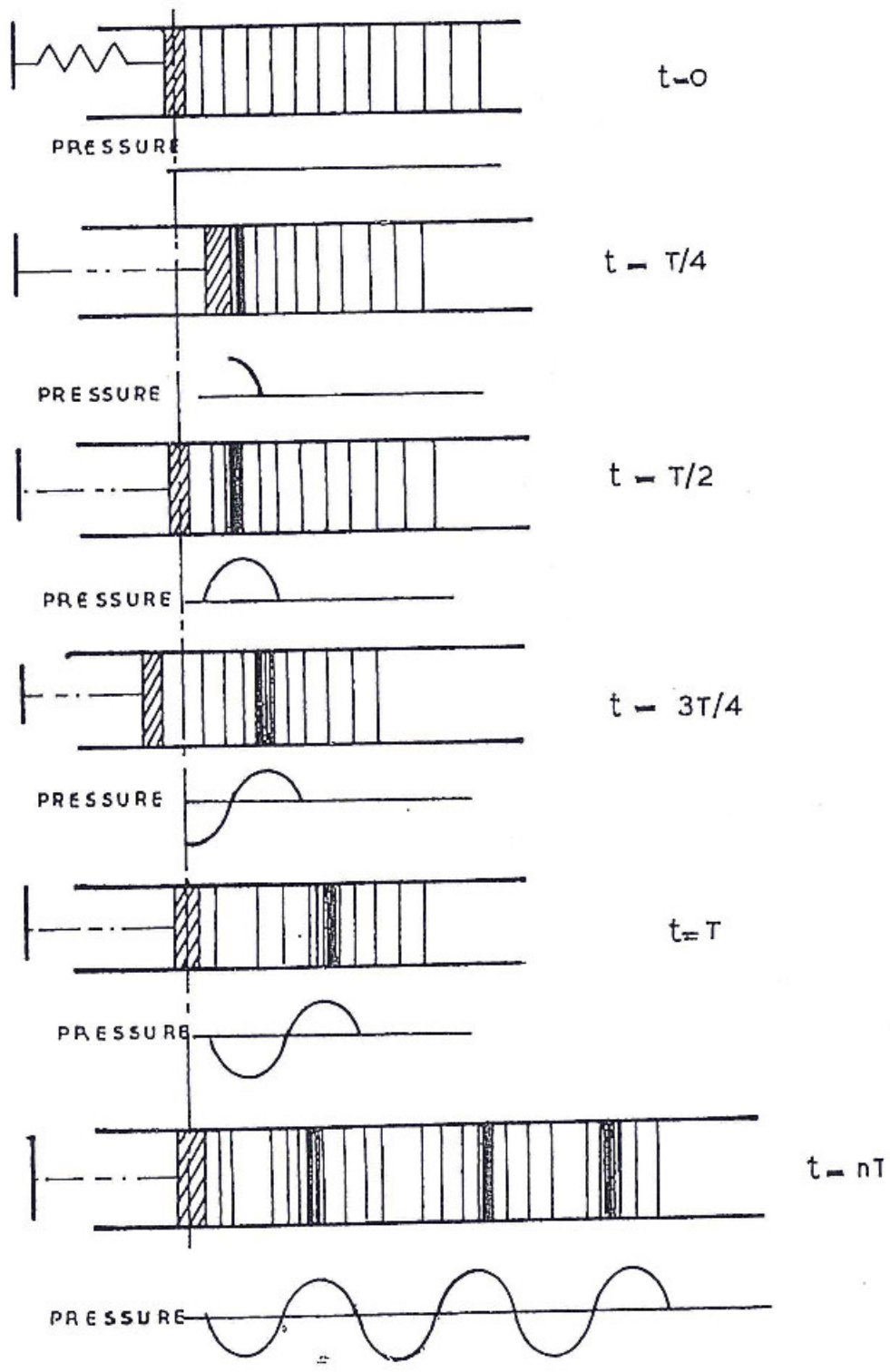
- (i) An investigation of the manufacture, vibration patterns and tonal qualities of a representative range of pans to evaluate and analyse the factors influencing the sounds produced by the pans.
- (ii) A systematic development of the pan based on an appraisal of the information obtained from Stage (1).
- (iii) A study of possible methods of mechanically manufacturing pans to the stage where only the final tuning is required.

This paper describes Stage (i) tests so far carried out on one particular pan. Various types of steel pans are used by bands, each covering part of the musical scale. The tenor pan, on which the melody is normally played, is the most complex since it has the greatest number of notes, and is therefore likely to embody most of the phenomena found in other types of pans. For this reason the programme has started with tests on a conventional tenor pan.

## 2. THE SCIENCE OF MUSIC.

### 2.1. Sound

Sound is created when a disturbance in the form of a pressure alteration or particle movement is set up in an elastic medium. The sound is transmitted by propagation of the disturbance through the medium to the ear which converts the disturbance into an auditory sensation provided that the frequency of the disturbance is within the audible range of the ear of about 20 to 15,000 Hz. Normally the medium is air and any phenomenon which causes a pulsating disturbance in the air



SOUND WAVES GENERATED IN A TUBE  
BY AN OSCILLATING DISC

FIG. I

creates sound. The source of disturbance may be a vibrating system such as the soundboard of a piano or the diaphragm of a loudspeaker, an intermittently throttled air stream such as produced by a siren, the human voice and a trumpet, or a sudden release of air pressure such as an exploding balloon.

A simple illustration of the creation of a sound wave is shown in Figure 1 where the sound generator is a vibrating disc. As the disc vibrates it sets up a cyclic pressure vibration in the adjacent layer of air which is propagated by longitudinal vibrations of the air "particles." If the discs were vibrated in a vacuum then no disturbance would be created and no sound would be generated. The creation and propagation of sound can be compared to the ripples set up when a stone is dropped into a pond, the ripple is propagated outwards from the source but the "particles" of water remain in the same place, the important difference being that whereas in the case of the water the "particles" oscillate transversely to the direction of propagation of the wave, the air particles oscillate parallel to the direction of propagation.

The source of disturbance obviously imparts some of its energy to the sound wave although the fraction imparted is small, most of the energy of a vibrating system for instance is used up in overcoming friction. The energy transmitted by a soundwave per unit perpendicular to the direction of propagation is termed the intensity of the sound wave which may be shown to be directly proportional to the power of the source and inversely proportional to the square of the distance from the source. The level of the resulting auditory sensation is loudness which is closely related to intensity although loudness is also affected to a certain extent by the frequency and timbre of the sound. The power radiated by a vibrating disc producing plane waves may be shown (2) to be given by  $2\pi^2 A \cdot w_a \cdot c.f. \cdot a^2$ , where 'A' is the surface area of the disc, 'f' the frequency and 'a' the amplitude of vibration. In the more general case of waves spreading radially from the source, if one side of the disc is closed off to prevent it generating sound waves then the power radiated assuming an ideal spherical wave is proportional to  $\frac{c.f. \cdot A \cdot a^2}{\lambda^2}$  where  $\lambda$  is the wavelength. If both sides of the disc generate waves, i.e. a double source, one side of the disc produces a compression whilst the other simultaneously

produces a rarefaction with the result that in certain directions there is interference between the waves and less energy is radiated. This may be demonstrated using a tuning fork, along lines inclined at  $45^\circ$  to the axes of the fork there is complete interference between the waves produced by the outer and inner surfaces of the prongs and no sound is heard. If the cross-section of the prongs of the fork is small the disturbance is reduced by circulation of the air, the air on the compression side tending to flow round the prongs to fill up the rarefaction, and the sound output is small. The same problem affects vibrating wires and therefore, in string instruments the wires are flexibly connected to a sound board which is forced into vibration in the same mode as the wire. The sounding board improves the coupling of the vibrating source with the air, thus radiating more power and producing a greatly magnified intensity of sound.

The sound waves considered so far have been simple, pure, sine waves assuming simple harmonic motion of the source. The waves may become very complex if the source is vibrating with a combination of modes (see Section 2.2) and if several sources exist. Sound waves obey the law of super-position provided that the 'particle' displacements are small, and therefore if several sources are creating waves the resultant waveform at any point is obtained by the vector addition of the individual waves. The ear is able to break down the resulting complex waveform into its individual components and therefore we are able to distinguish sounds radiating simultaneously from different sources.

Interference and "beating" are two particular forms of super-position. If two waves of the same frequency and amplitude are moving along the same path they will reinforce each other if they are in phase, or cancel each other if the phase difference is one-half of a wavelength. If the waves are moving in opposite directions as with radiated and reflected waves in a closed tube then at some points, antinodes, the waves reinforce each other, and at others, nodes, they cancel. The antinodes are points of maximum sound intensity and the nodes, points of zero intensity. Interference between radiated and reflected waves may create problems in the acoustics of rooms having walls which are favourable to reflection. In the case of 'beating' the required conditions are that the two waves should have

slightly different frequencies. Thus if the displacements of the waves are described by  $x_1 = a_1 \sin \omega_1 t$  and  $x_2 = a_2 \sin \omega_2 t$ , where  $(\omega_1 - \omega_2)$  is small then vectorial addition gives a resultant wave  $X = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos (\omega_1 - \omega_2)t} \cdot \sin(\omega_1 t + \phi)$  whose amplitude varies between  $(a_1 + a_2)$  and  $(a_1 - a_2)$  with a frequency of  $(\omega_1 - \omega_2)$  i.e. the resultant sound intensity will pulsate with a beat frequency equal to the difference in the frequencies of the super-posed waves.

Above it is shown that the theory of sound may be divided into three main areas of study concerning (a) the vibrating source which radiates the sound wave, (b) the transmission of the sound wave through an elastic medium and (c) the effect of the sound wave on the human ear. In the study of musical instruments we are primarily interested in (a), although we must also be aware of the effect of super-posing sound waves of different frequencies and of the resulting auditory sensations.

## 2.2. Vibrating Systems (4.5)

The simplest form of vibrating system is the single degree of freedom system in which mass and elasticity may be separated, for example the spring-mass system shown in Figure 2. If the mass is given an initial displacement of 'a' and released

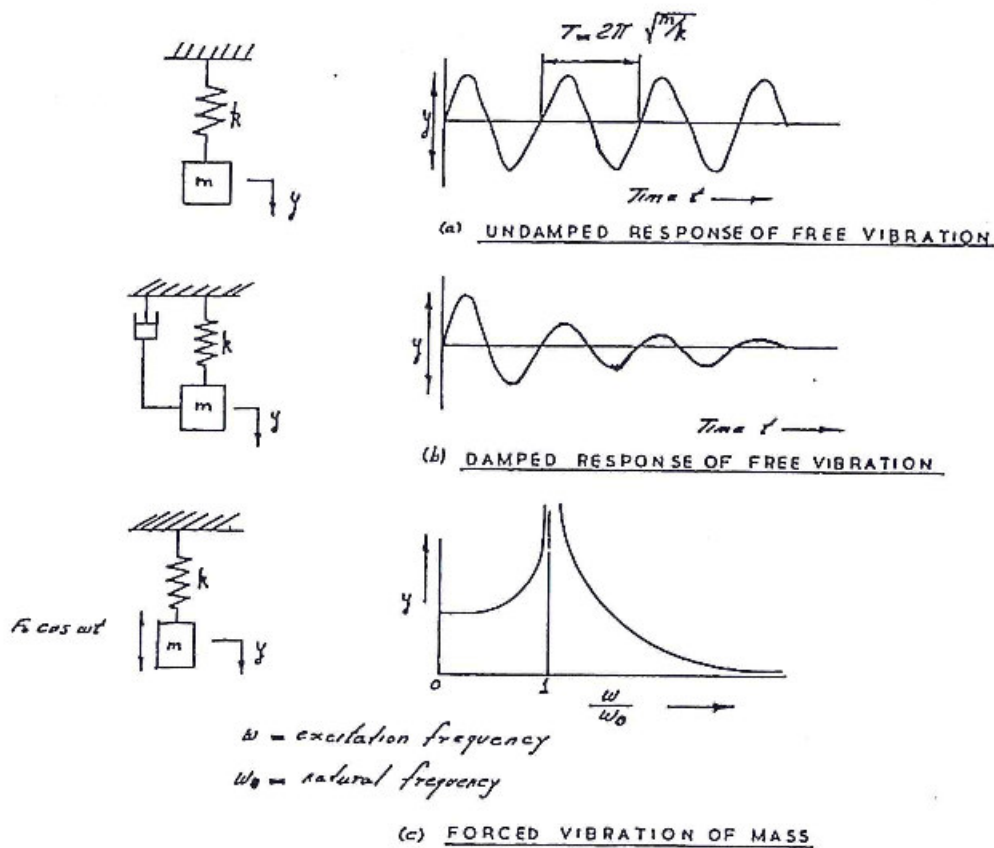
it vibrates at its natural frequency  $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ , with a displacement

form  $x = a \cos t$ . If a viscous damping force is added as in Figure 2 (b) to take into account air resistance and internal friction in the spring, the natural frequency is practically unaffected unless the damping is large, but displacement form

becomes  $x = a e^{-nt} \sqrt{1 - \frac{n^2}{\omega^2}} \cos (\omega t + \phi)$  which represents a

waveform of natural frequency 'w' whose amplitude decreases with time, the term  $e^{-nt}$  specifying the rate at which the vibration dies out. If a simple harmonic force is now applied to the mass as in (c) or the support is given a simple harmonic displacement, there will initially be some transient vibration at the natural frequency of the system which will be damped out as in (b), but in the main the result is a steady vibration at the frequency of the applied force. The response of the system is very dependent on the frequency of the applied force relative to the natural frequency of the system as seen in Figure 2 (c). As



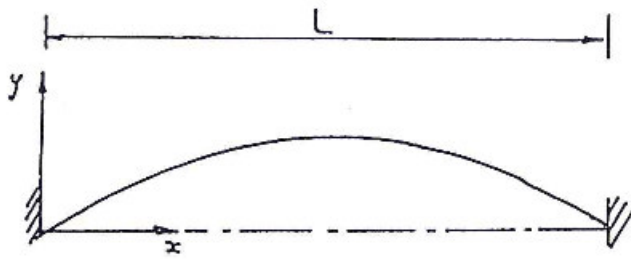


VIBRATION OF SINGLE SPRING-MASS SYSTEM

FIG 2

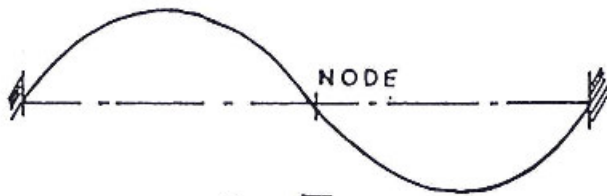
the ratio  $\frac{\omega f}{\omega_0}$  approaches unity the amplitude of vibration of the system increases rapidly until an amplitude peak occurs when the frequencies are equal. This is the condition of resonance which is the cause of most engineering vibration problems but which is used to considerable advantage in musical instruments employing resonators.

The basic phenomena discussed above apply also to the vibration of elastic bodies such as stretched strings, bars and plates which have distributed mass and elasticity, but since an elastic body may be considered to consist of an infinite number of elemental spring-mass systems the analysis is far more complex. Considering the transverse vibrations of a stretched string, Figure 3 in which the tension  $T$  is sufficiently large to be unaffected by displacement of the string, we find that the string can theoretically have an infinite number of natural frequencies



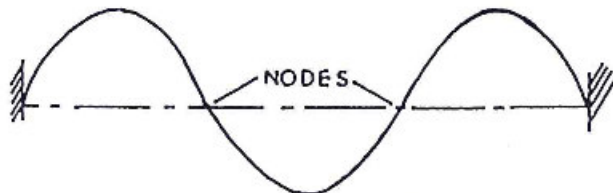
FUNDAMENTAL MODE  $f_0 = \frac{1}{2L} \sqrt{\frac{T}{m}}$  Hz.

$$y = a_0 \sin \frac{\pi x}{L} \cos \omega_0 t$$



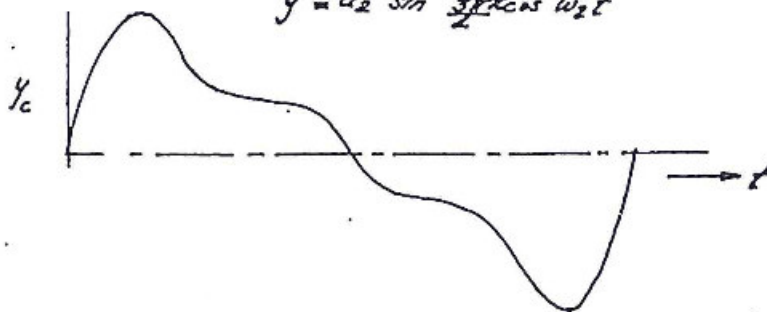
1<sup>st</sup> PARTIAL MODE  $f_1 = \frac{1}{L} \sqrt{\frac{T}{m}}$  Hz.

$$y = a_1 \sin \frac{2\pi x}{L} \cos \omega_1 t$$



2<sup>nd</sup> PARTIAL MODE  $f_2 = \frac{3}{2L} \sqrt{\frac{T}{m}}$  Hz.

$$y = a_2 \sin \frac{3\pi x}{L} \cos \omega_2 t$$



RESULTANT DISPLACEMENT OF CENTRE  
OF STRING FOR FIRST THREE MODES  
WITH  $a_0 = 2a_1 = 3a_2$

VIBRATIONS OF A STRETCHED STRING

FIG.3.

given by  $\omega_n = \frac{n\pi}{L} \sqrt{\frac{T}{m}}$  and each corresponds to a particular mode of vibration  $y = a \sin \frac{n\pi x}{L} \cos \omega_n t + b_n \sin \omega_n t$  where  $n$  is any integer. The lowest or fundamental natural frequency is given by  $n = 1$ , and in this mode the string vibrates in the form of half a sine-wave. The higher natural frequencies being multiples of the fundamental are termed harmonics, the modes of vibration for the second ( $n = 2$ ) and third ( $n = 3$ ) harmonics are shown in Figure 3. If the string is displaced and released the resultant vibration will be complex consisting of the vectorial addition of several modes, the presence and size of modes depending on the method and position of displacement. The summation of the modes for  $n = 1$  to  $n = \infty$  may be represented by a Fourier series as follows:

$$\text{Displacement } y = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} (a_n \cos_n \omega_n t + b_n \sin \omega_n t)$$

If the string is displaced at a point  $P$  by plucking such that when  $t = 0$  the displacement at  $x = d$  is  $y_p = h$  and the initial velocity is zero then  $b_n = 0$  and  $a_n$  may be obtained as follows (6).

$$a_n = \frac{2 h L^2}{\pi^2 n^2 d (L - d)} \sin \frac{n\pi d}{L} \quad \frac{n\pi d}{L} = 0$$

The relative amplitudes of the harmonics are now easily obtained by substituting values of  $n$  into this expression. This shows that, the  $n^{\text{th}}$  harmonic disappears when  $\sin \frac{n\pi d}{L} = 0$  i.e.

when  $\frac{nd}{L}$  is any integer. Since in the  $n^{\text{th}}$  harmonic there are  $(n-1)$  nodal points dividing the string into ' $n$ ' lengths, then if ' $d$ ' is chosen to coincide with any of the nodal points this harmonic will disappear. For example if the string is plucked at

the centre  $d = L_2 \sin \frac{n\pi d}{1} = 0$  for all even values of  $n$ , i.e. all the even harmonics disappear and the resulting vibration consists of only odd harmonics. Therefore, if the string is plucked, struck or bowed at a point,  $P$ , then any harmonics which require point  $P$  as a node will disappear. However, the relative magnitudes of the harmonics will be different from those of the plucked string if the string is bowed as in a violin or struck as in a piano.

The sound radiated by a vibrating string is magnified by coupling the string to a sounding board. The sounding board is set into vibration by the harmonic force supplied by the vibrating string, but in turn the vibration of the wire is modified by the fact that the end of the wire is flexibly mounted. The effect of flexible supports on the frequency of stretched strings has been dealt with by Rayleigh (7). The sounding board should have no predominant resonant frequencies within the frequency range of the instrument otherwise certain frequencies will be accentuated by resonance and the response of the sounding board will be uneven.

The principle difference, therefore, between the vibrations of an elastic body and those of the simple spring-mass system is that the elastic body has many natural frequencies, each of which corresponds to a particular mode of vibration. The free vibrations of an elastic body consist of the fundamental mode with several other modes super-imposed upon it, the actual composition depending on the method of initiating the vibration. If the vibrations are excited by a harmonic force then as the frequency of the force is increased a state of resonance occurs at each of the natural frequencies of the body.

### 2.3. The Nature of Music

The qualities which characterise sound as music are smoothness, pleasantness, regularity and harmoniousness. The important factors are pitch, quality, loudness and duration.

**Pitch.** The pitch of a note is governed by the fundamental frequency of the radiated sound. The ear can differentiate between quite small changes in pitch but only recognises certain differences in pitch as connecting notes musically i.e. regardless of the pitch of the datum note for other notes to be connected to it musically their pitch must bear a definite relationship to the datum pitch. It is found that these relationships consist of

simple ratios of small whole numbers — in order of merit of pleasantness are the octave 2:1, followed by the ratios 3:2, 4:3, 5:4, 6:5, 8:5 and 5:3. Using these ratios a musical scale known as the scale of “just intonation” can be developed. The disadvantage of this ‘ideal’ scale is that many discrete frequencies must be provided to allow the scale to begin on any keynote — for example over thirty notes must be provided on an instrument just to cover one octave of the scale. To overcome this problem the scale of equal temperament was introduced in which the ideal frequency ratios of the scale of just intonation were changed slightly to allow an octave to be divided into twelve equal intervals i.e. the frequency ratio of any two adjacent notes is  $\sqrt[12]{2}$ . Pitch has been standardised by making the frequency of the note A in the treble clef (A4) equal to 440 Hz, and therefore, the frequencies of all other notes can be calculated from the known frequency ratios. Musical instruments must have the pitch of their notes tuned to these frequencies.

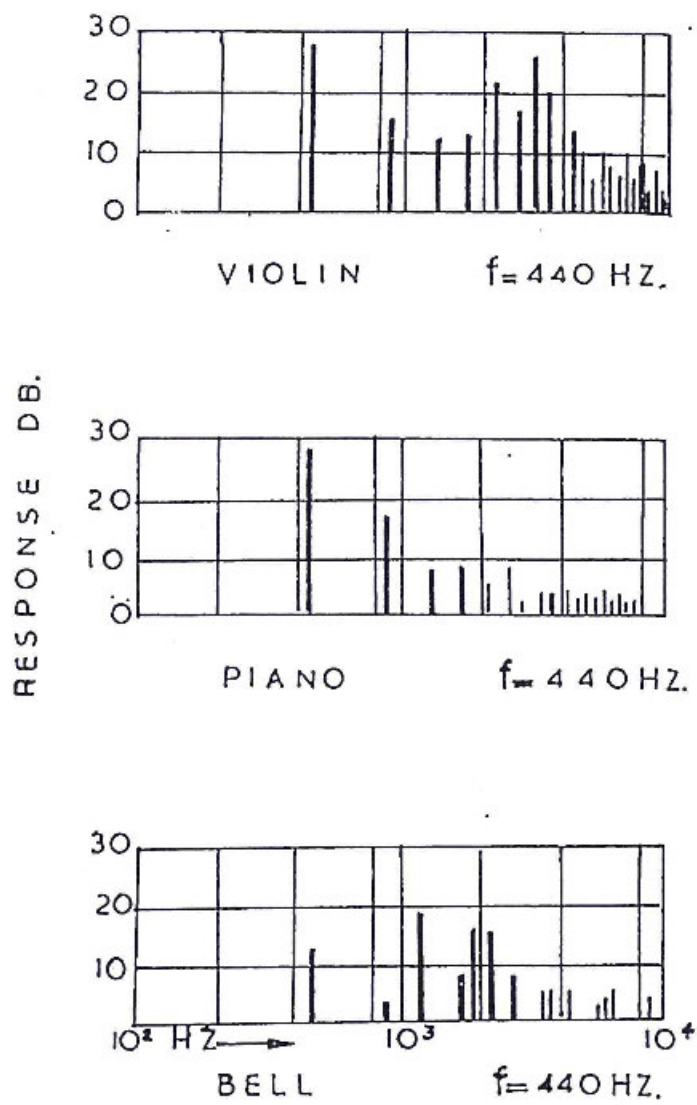
**Timbre or Quality.** Timbre is the most important property of sound, giving the sound an individuality and allowing a distinction to be made between sounds of the same pitch and loudness but produced in different ways i.e. timbre allows a middle C played on a piano to be recognised as different from a middle C played on a violin or trumpet. Timbre is governed solely by the structure of the sound in terms of the presence and relative magnitudes of the fundamental tone and overtones making up the sound wave. For musical notes, the overtones which are produced by the higher natural frequencies of the vibrating source, should be harmonics of the fundamental and should be parts of the common chord of the fundamental. It was shown in the last section that the overtones produced by a vibrating stretched wire are harmonics of the fundamental and this explains its wide usage in musical instruments. In general, vibrating plates and bars produce non-harmonic overtones and therefore they are not often used in musical instruments since non-harmonics introduce roughness or harshness to the sound. An exception is the Xylophone in which the notes are “free-free” beams supported at the positions of the nodes for the fundamental mode of vibration, but since the notes are struck at the centre where the higher overtones require nodes, the overtones are small and the notes tend to be pure.

Returning to the stretched string, the first six harmonics are all clearly part of the common chord of the fundamental, but the seventh, ninth, and higher odd harmonics are not and these tend to add dissonance. It is therefore usual to strike or pluck the strings at  $1/7$  or  $1/9$  of their length from the support so reducing the dissonant harmonics (see Section 2.2).

The characteristics which govern the quality or timbre of a musical note are therefore, (a) the number, distribution and relative intensity of the harmonic partials or overtones, (b) the presence of non-harmonic partials, and (c) the overall intensity (see below). The timbre of a note may be represented by its accoustical spectrum in which the magnitudes of the overtones are plotted on a frequency/response chart. The accoustical spectra of a note of frequency 440 Hz played in turn on a violin, piano and bell are compared in Figure 4 — note the non-harmonic overtones of the bell. The accoustical spectra of several other instruments which are given in Reference 2 show clearly that for a given instrument the accoustical spectrum varies considerably with pitch.

The timbre of a note is also affected by loudness, not only due to the fact that as the instrument radiates more power more partials are created, but also because the ear is known to have the property of adding further overtones to the auditory sensation, the presence and magnitude of which depend on the intensity of the sound.

The duration of a musical note may be split into three regimes — growth, when the response is building up to a maximum; steady-state, the response remaining fairly constant; and decay, when the response dies out due to damping. The accoustical spectrum of the note may change with the regime and therefore the response/time characteristics of the note affect its timbre, with the initial transient period probably having the greatest effect. The growth/decay characteristics of several instruments are discussed in Reference 2, it is noticeable that percussion and plucked and struck string instruments exhibit a rapid growth, zero steady-state and relatively long decay periods.



ACOUSTIC SPECTRUMS OF VARIOUS  
INSTRUMENTS

FIG. 4

### 3. EXPERIMENTAL PROGRAMME

#### 3.1. Introduction.

As explained earlier, the first step in formulating the factors affecting pan performance involves an investigation of the manufacture and behaviour of tenor pans.

Methods of construction of tenor pans vary from tuner to tuner but most involve certain basic stages which are outlined below. Since tuners have generally developed their art by individual experimentation there are bound to be differences in technique, but it is thought that the most significant are likely to be in the shape and arrangement of notes.

##### (i) Sinking

The plain end of the oil drum is hammered inwards to form roughly an ellipsoidal bowl with a maximum depth at the centre of about 6" to 8". The profile may be judged by eye or set by a template.

##### (ii) Marking Out

The boundaries of the note are marked out either by measurement or partly by eye and partly by measurement. The areas of the notes depend on the pitch required, the higher the pitch the smaller the area. In shaping the notes there are principally two modes of design in use – (a) an outer ring of roughly trapezoidal notes formed by radial lines running inwards from the rim of the drum joined at their inner ends by semi-circular arcs and with an arrangement of circular notes of higher pitch over the central portion of the bowl to make a total of approximately thirty notes; (b) three rings of notes formed by the intersection of radial lines and circles, each ring having the twelve notes of an octave.

##### (iii) Grooving

The boundaries of the notes are accentuated by using a flat ended punch to produce continuous grooves of about 1/8" width along the marking-out lines. The force of punching is reckoned to be near the maximum which can be applied without tearing the metal. In some cases double grooves may be used with the intention of providing greater isolation of the notes.



After grooving, the pan is separated from the drum by cutting around the cylindrical section of the drum, leaving a length of about 6" of the cylinder attached to the pan.

#### (iv) Heat Treatment

Normally the pan is heat treated by suspending it over an open fire until some change in the appearance of the metal indicates to the tuner that it has reached the required temperature. This is usually followed by throwing cold water onto the pan to quench the metal.

#### (v) Tuning

During the tuning of the pan the tuner uses his 'ear,' skill and intense concentration to obtain the required pitch and timbre of the pan notes. The process consists basically of tapping the playing surface of the notes with a small hammer to alter their profile – pitch may be raised by tapping around the outside of the note and lowered by tapping the centre of the note. Since the notes are interconnected the process is iterative, converging until the tuner is satisfied with the overall sound of the pan.

The following section describes tests carried out during the manufacture of a tenor pan of type (a) above to investigate the effects of each stage of tuning on the material and structure of the pan and also an experimental investigation of the modes of vibration of individual notes of the pan. Since the investigation of material properties involves the destruction of the pan, four pans were tuned to different stages as follows:-

PAN A – was used to investigate the process of sinking only

PAN B – the sinking and heat treatment operations were carried out and the material properties after heat treatment determined

PAN C – this was fully tuned to investigate the profile and stiffness of the notes

PAN D – was fully tuned and has been used to investigate the modes of vibration of the notes

The four pans were all made from Esso oil drums.

### 3.2. Properties of Oil Drum Material

Standard oil drums are approximately 22½" diameter by 35" long with a capacity of 55 imperial gallons and are manufactured from 18 British Standard Gauge (0.0495" thick) cold rolled sheet of commercial grade mild steel. The manufacturers in Trinidad import sheets cut to shape for the end and cylindrical sections. Owing to the vague material specification it is likely that material properties will vary from one batch of drum to another. Furthermore, since many of the drums from which pans are made are 'used' drums the material properties may have been influenced by the previous history of usage. The material properties of six drums, including the for used in the tests, are compared in Table 1, the properties being determined from specimens cut from the plugged ends of the drums.

Type of Drum	Yield Strength lb/in <sup>2</sup>	Tensile Strength lb/in <sup>2</sup>	%Elongation on 2" Gauge Length
1. Texaco (Trinidad)	30,400	44,000	36.4
2. Texaco (U. S. A.)	49,400	58,400	19.2
3. Esso )	32,800	42,800	32.8
4. " ) Drums used in Project	41,100	50,300	32.0
5. " )	28,600	000	33.8
6. " )	26,800	46,900	32.6

PROPERTIES OF SOME STEEL DRUM MATERIALS      TABLE 1

The results are typical of low grade steel of less than 0.15% content, apart from specimen 2. which has increased strength at the expense of ductility. Drum 2. would appear harder to work than the other drums and would be likely to tear during sinking because of its low ductility. Insufficient work has been completed at this stage to forecast the effect of material properties on the sound produced, although some tuners claim that pans made from the 'harder' drums have a 'richer' sound.

### 3.3. The Sinking Operation

During this operation the material is cold worked by hammering. The process thus involves deformation of the metal, reduction in metal thickness and change in material properties due to strain hardening. It is also likely that residual stresses are present in the sunken pan owing to the variation in the plastic deformation of the metal which results in the permanent deformations of the metal over the bowl being incompatible. However, since the next stage of tuning involves heat treatment these residual stresses are unimportant.

The physical changes involved in the process were measured as follows:-

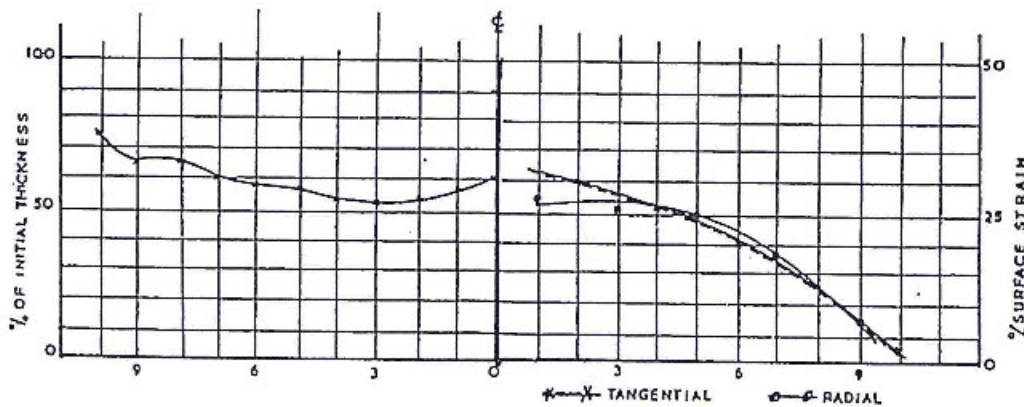
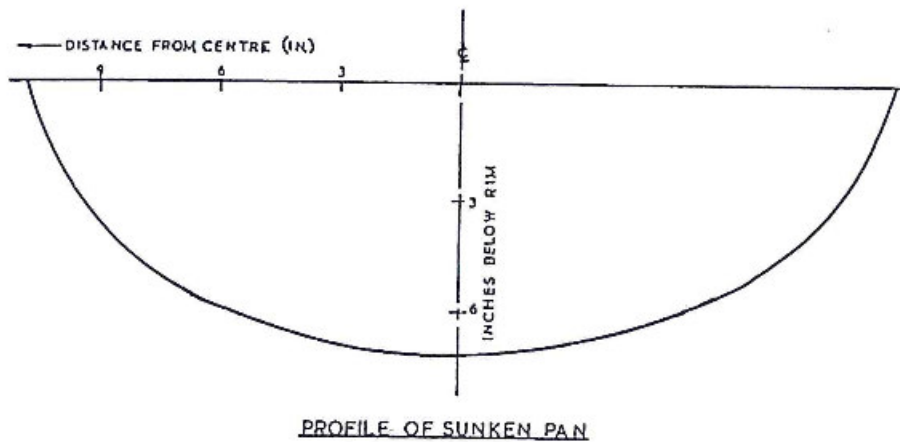
Deformation of the Pan was measured using a grid technique. A grid network consisting of three equally spaced 15 degree radial segments divided by circular arcs at 2" intervals was scribed onto the flat end of the drum. Scale marks 1" apart were marked along the grid lines and the grid photographed. The length of the grid marks on the photograph were measured to an accuracy of 0.1 mm using a vernier travelling microscope, and the distance between the scale marks used to convert the measurements to actual distances on the pan. After sinking, a further set of 1" scale marks were marked along the grid lines using a flexible scale of 1/32" thick brass and the grid again photographed. The photographed grid was measured using the microscope and since the scale marks represent actual 1" lengths on the curved surface of the bowl, the actual lengths of the grid lines on the curved pan surface were easily obtained. The increase in length of the grid lines gave the distribution of deformation over the sunken pan.

The Profile of the sunken pan was obtained by measuring the depth of the pan below a straight edge resting diagonally across the rim of the pan. The depth measurements were made at 1" intervals along the diameter with a vernier depth gauge.

The Variation in Thickness at 1" intervals along a diameter was measured with a micrometer after the pan had been sliced to obtain tensile specimens.

The Effect of Sinking on Material Properties was determined from tensile specimens cut from the sunken pan at radii of 5" and 9¼". The specimens were cut tangentially so that the thickness and deformation of the material over the gauge length was as uniform as possible.

The results showing the deformation, thickness variation and profile are plotted in Figure 5. It is surprising perhaps that



VARIATION IN THICKNESS OF SUNKEN PAN

VARIATION IN SURFACE DEFORMATION OF SUNKEN PAN

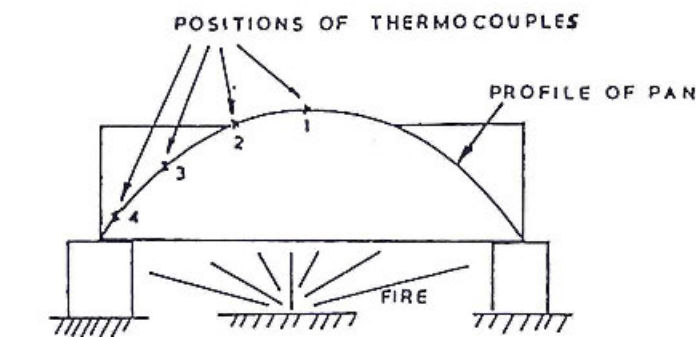
FIG. 5.

the deformation and thickness vary so smoothly considering that the sinking is done by hand, the profile itself being almost elliptical. The changes in material properties, Table 2, are consistent with the effects of cold working, the yield and ultimate strengths being increased and the ductility reduced. The low elongation of the specimen taken from the 5" radius indicates that the material must have been very close to tearing.

### 3.4. Heat Treatment

The sinking operation was repeated for the second pan which was then heat treated by resting it in the inverted position on concrete blocks and burning a worn rubber car tyre underneath the pan. The temperature distribution on the underside surface of the pan during the heat treatment was measured using iron/constantan thermocouples. Thermocouple leads were bent in such a way that their springiness pushed the junctions against the pan to make good contact and the junctions taped to the pan with fibre-glass adhesive tape.

The maximum temperatures in the pan were reached after 10 minutes of heating and are shown in Figure 6. The pan was then removed from the fire and quenched by dowsing with cold water.



MAXIMUM MEASURED TEMPERATURES				
	PT.1	PT.2	PT.3	PT.4
DISTANCE FROM CENTRE (in.)	0	3.5	7.0	10.5
TEMPERATURE, °C	470	500	560	680

TEMPERATURES MEASURED DURING HEAT TREATMENT OF PAN

FIG. 6.

The effects of heat treating the material were found from tests on specimens cut tangentially at radii of 4", 6½" and 10½". The results are given in Table 2 for comparison with the material properties prior to heating. It is seen that the heat treatment returns some ductility to the metal through stress relieving but that this is only really effective over the outer part of the bowl where the temperature reached in the region of 600°C. There is, therefore, a considerable variation in the state of the material across the pan, the central area being hard and almost brittle and the outer area softer and more ductile. In low carbon steels the main importance of rate of cooling concerns residual stresses which may be set up due to differential cooling within the material and therefore, since the thickness of the pan is small the rapid cooling by quenching is unlikely to have any significant effect on the final state of the material.

### 3.5. Tuning

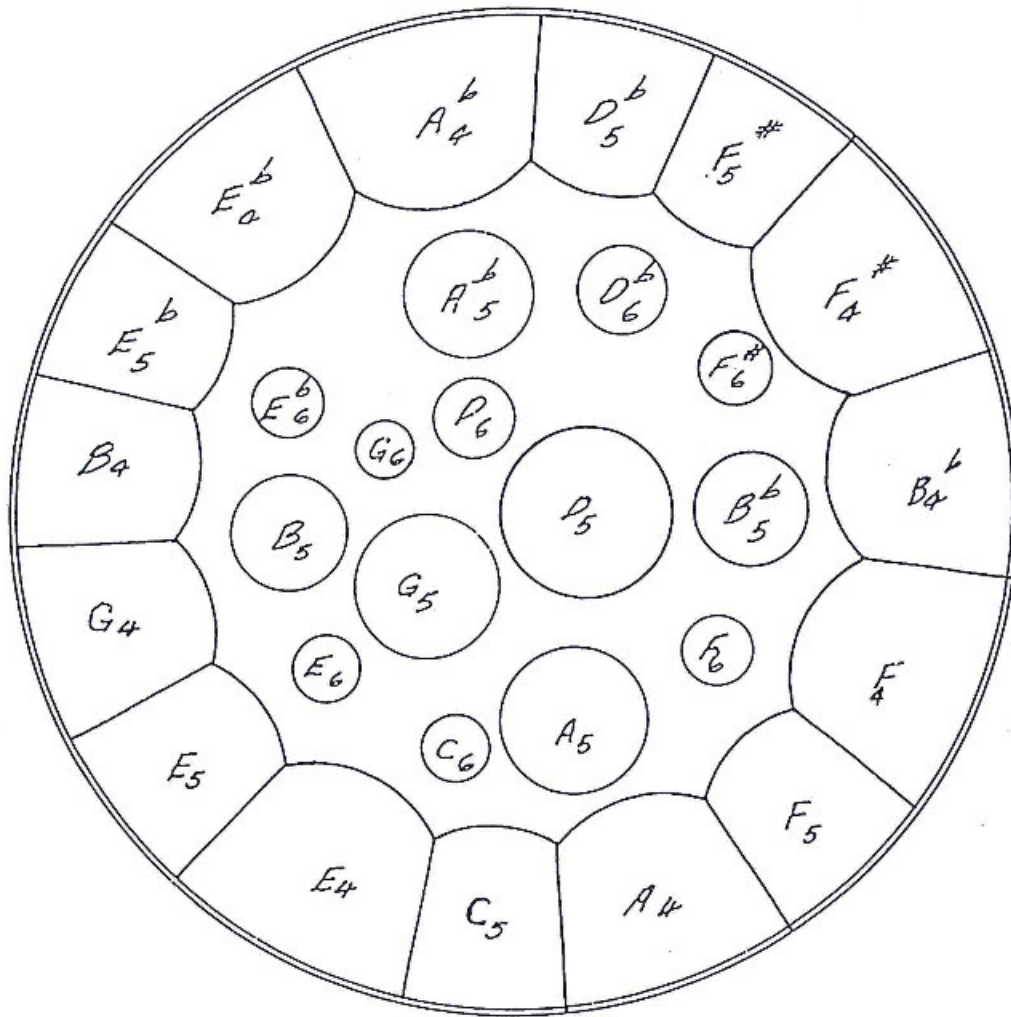
The third pan was brought to the fully tuned state through the processes of sinking, marking out, grooving and heating. The arrangement and shape of the notes of the pan are shown in

State of Specimen	Distance from Drum Centre	Heat Treatment Temp. °C	Yield Strength lb/in <sup>2</sup>	Tensile Strength lb/in <sup>2</sup>	%Elongation on 2" Gauge Length
Original	—	—	32,000	47,000	32.5
Sunken Pan	5"	—	67,000	72,000	2.4
"	9"	—	63,500	66,000	3.6
Heat Treated Pan	4"	505	54,500	59,200	5.5
"	6½"	550	39,500	52,000	14.2
"	10½"	680	45,000	53,200	22.8

COMPARISON OF MATERIAL PROPERTIES AT VARIOUS STAGES OF TUNING

TABLE 2

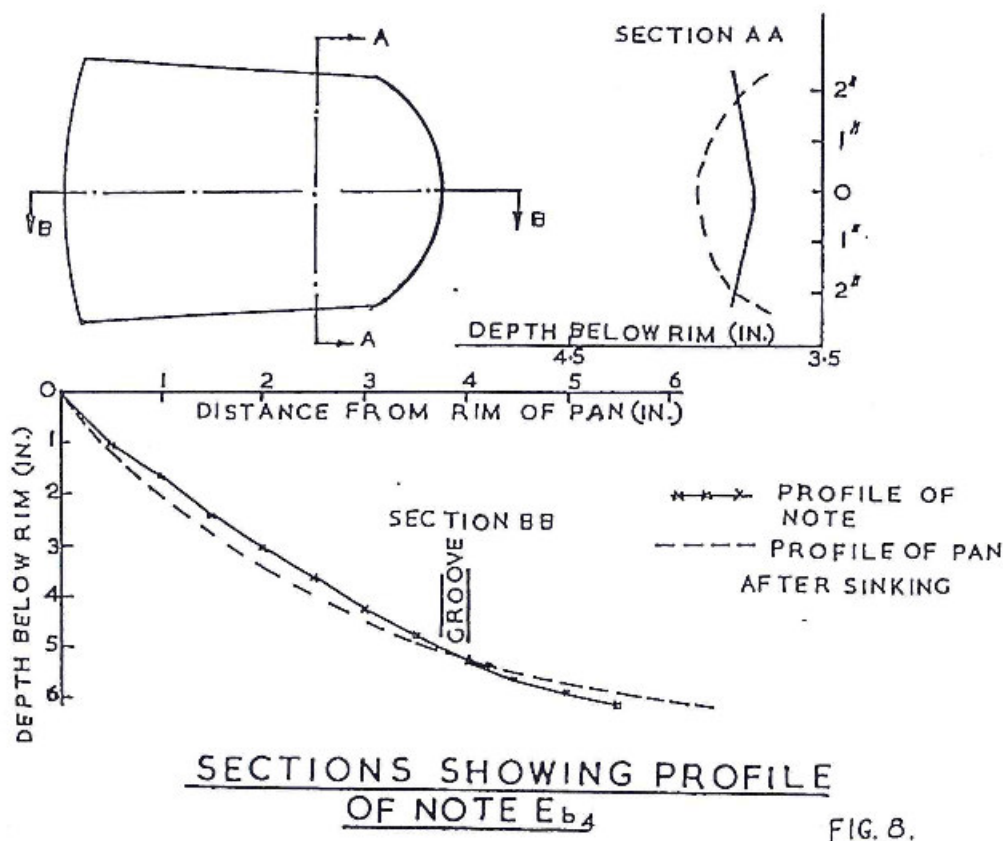
Figure 7. The ratios of the frequencies (see Table 3) of adjacent notes indicate the logical musical arrangement of the notes. In Section 2.3. it was stated that combinations of tones producing pleasant sounds have pitch ratios comprising two small integers, in order of pleasantness the ratios are octave 2:1, fifth 3:2, fourth 4:3, major third 5:4, minor third 6:5, minor sixth 8:5 and major sixth 5:3. All adjacent notes are seen to be connected in this way and therefore, dissonance caused by vibrations being transmitted to neighbouring notes is reduced to a minimum.



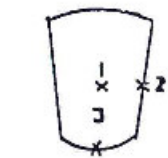
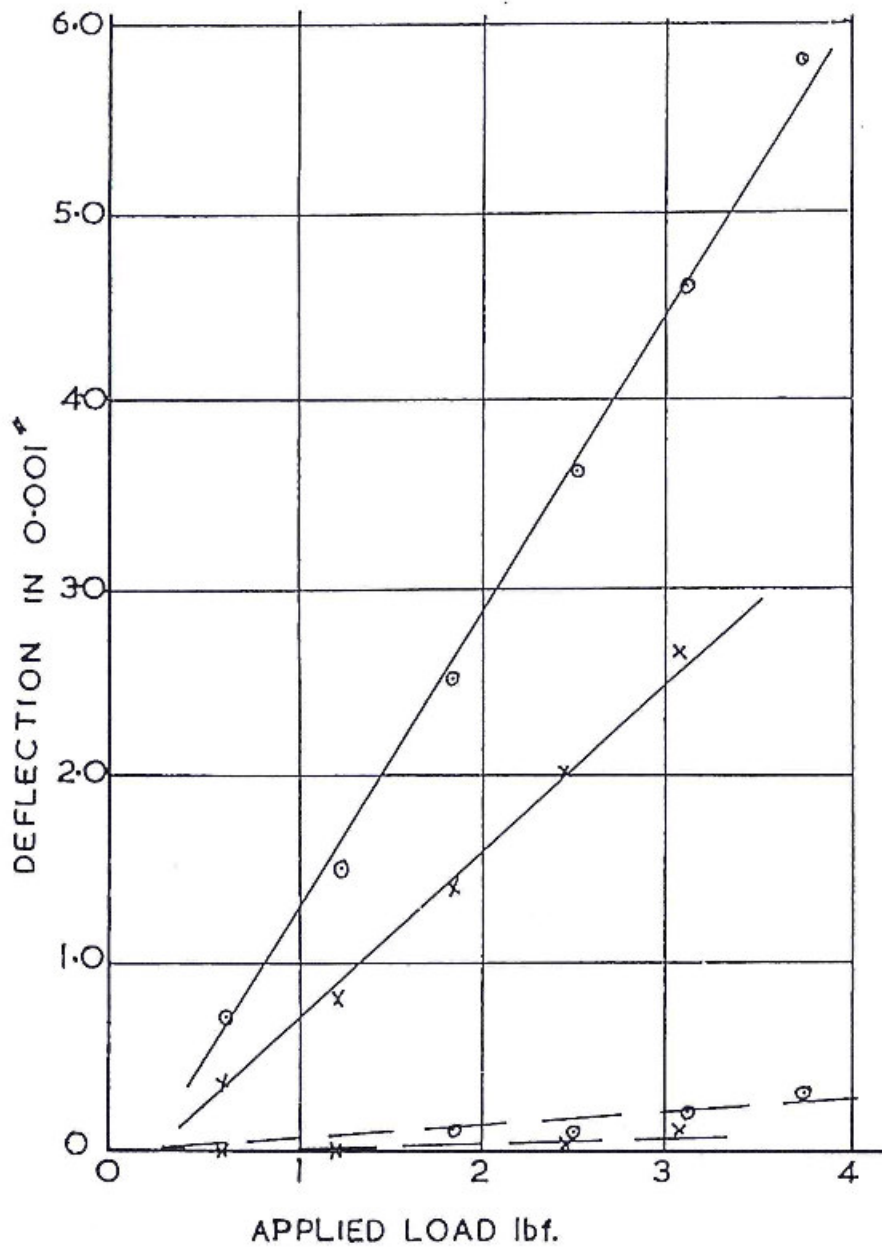
ARRANGEMENT OF NOTES ON THE  
TUNED PAN FIG. 7.

The Profile of the Notes was measured in the same way as for the sunken pan. A typical profile of an outer note is shown in Figure 8, which demonstrates clearly that the effect of grooving and tuning is to depress the boundary of the note and raise up the centre. The note therefore, becomes virtually a flat plate supported at the grooved boundary and if loaded transversely it will tend to bend relative to the boundary rather than act as part of the curved membrane of the bowl. This theory is confirmed by the results shown in Figure 9 which were obtained by applying point loads at the centre of the notes using a lever mechanism and measuring the resulting central and edge deflections. The large deflections of the centres of the notes relative to the grooves indicate that the notes bend as plates rather than stretch as membranes.

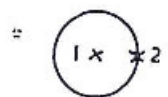
However, the small deflections of the grooves indicate that the plates are flexibly supported. The degree of flexibility will have an important bearing on both the vibrations of the notes and also on the transmission of vibrations through the pan.







○—○—○ POSN. 1.  
 ○—○—○ POSN. 2. NOTE E<sub>4</sub>  
 ZERO POSN. 3.

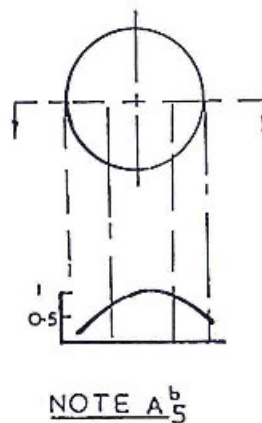
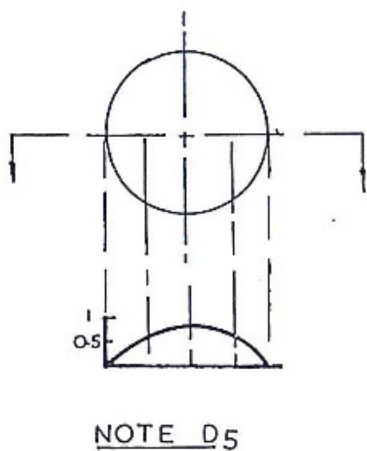
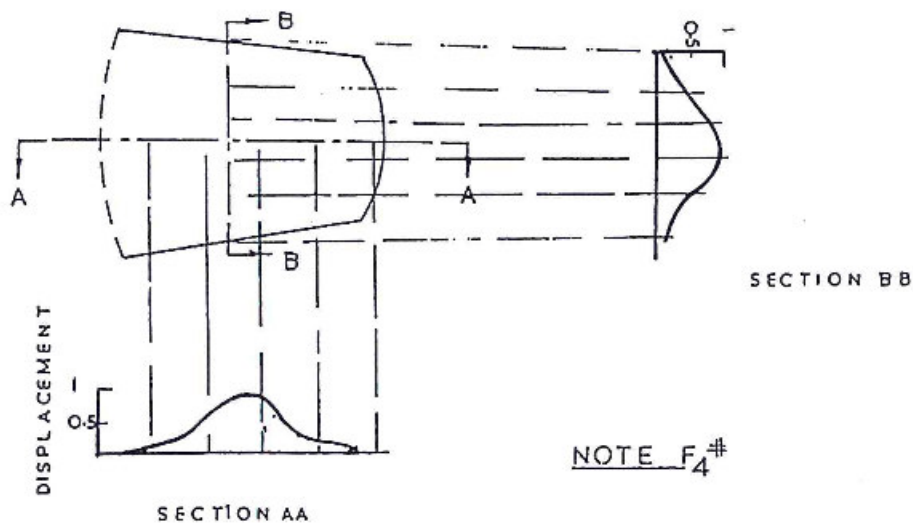


×—×—× POSN. 1  
 ×—×—× POSN. 2 NOTE G<sub>5</sub>

LOAD DISPLACEMENT CHARACTERISTICS  
OF NOTES

FIG. 9

During the final tuning the edge of the note is tapped to raise the pitch and the centre to lower the pitch. Careful inspection of the notes shows a tendency for the metal adjacent to the groove to be flat whilst the remainder is slightly domes. On the outer notes too, the appearance is of an almost circular slightly domes area over the inner region of the note. The final tuning appears therefore, to define the conditions at the edge of the note i.e. the actual location of the edge of the note. This is demonstrated more clearly below by the displacement profiles of the vibrating notes (see Figure 10).



DISPLACEMENT PROFILES FOR VIBRATING NOTES

FIG. 10.

### 3.6. Investigation of Vibration of Pan

The vibrations of the notes were detected using an inductance type displacement transducer having a frequency response range of 0 to 10,000 cps. The sensitivity of the detector was determined by calibration against a vernier gauge and found to be  $160\text{mV/in} \times 10^{-3}$ — thus by displaying the signal on the screen of an oscilloscope a maximum sensitivity of  $0.6 \times 10^{-6}$  in. displacement per cm. of trace movement on the C.R.O. Screen was achieved.

The following tests have so far been completed:-

- (i) Measurement of the fundamental frequencies of all the notes
- (ii) Measurement of the displacement profiles of sample notes
- (iii) Measurement of the frequencies and modes of vibration of the partials of sample notes

(i) Frequency Measurements — The frequencies of the notes were measured by generating Lissajou figures on the screen of the C.R.O., the output signal of the distance detector across the vertical plates of the C.R.O. being coupled with a signal from a variable frequency oscillator across the horizontal plates. Each note was struck repeatedly and the oscillator frequency adjusted until a stationery ellipse on the C.R.O. screen indicated that the frequencies of the two signals were identical.

The measured frequencies of the notes are compared with those of the scale of equal temperament in Table 3. The agreement is good, but the question of which is the more accurate, the tuner's ear or the experimental method, is debatable since it is reckoned that a trained musician can discriminate pitch to an accuracy of about 0.25%. Obviously the most simple and accurate method of tuning is therefore to compare the sound of the note with a pure tone of calibrated pitch such as from a tuning fork.

(ii) Displacement Profiles of Notes — In this case two displacement detector probes were used, one located at the centre of the note and the other at successive positions on the note and grooves. The amplitude of vibration at various

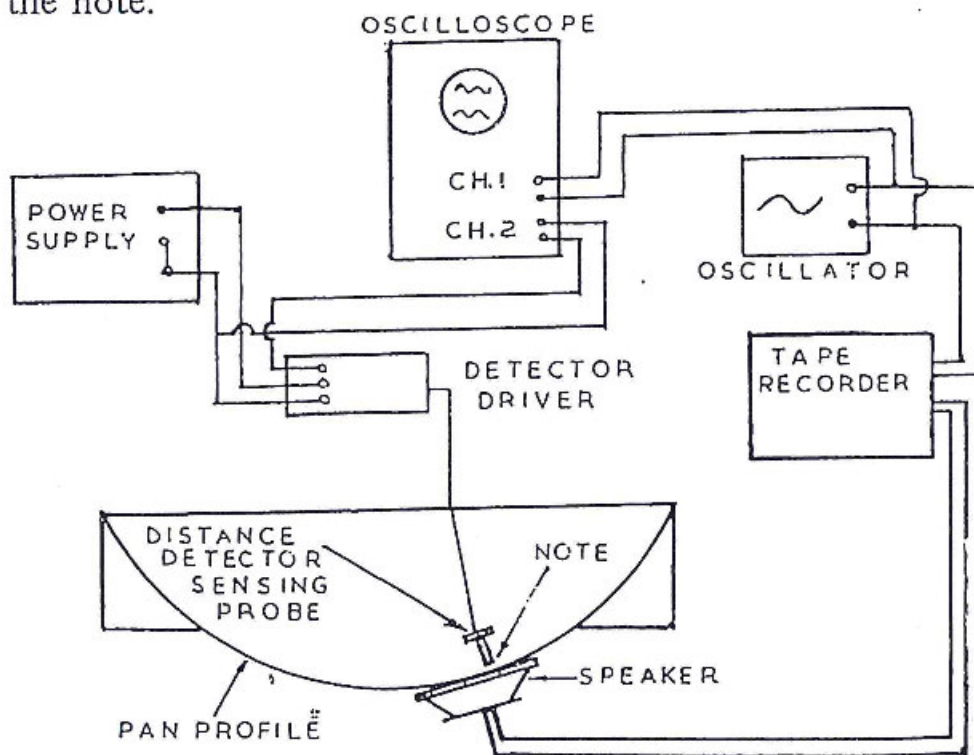
NOTE	E. T.	MEASURED FREQ. (Hz)	NOTE	E. T.	MEASURED FREQ. (Hz)	NOTE	E. T.	MEASURED FREQ. (Hz)
			C <sub>5</sub>	523.25	530	C <sub>6</sub>	1046.5	1060
				554.37	564		1108.7	1109
			D <sub>5</sub>	587.33	594	D <sub>6</sub>	1174.7	1176
E <sup>b</sup> <sub>4</sub>	311.13	314	E <sup>b</sup> <sub>5</sub>	622.25	624	E <sup>b</sup> <sub>6</sub>	1244.7	1270
E <sub>4</sub>	329.63	334	E <sub>5</sub>	659.26	660	E <sub>6</sub>	1318.5	1330
F <sub>4</sub>	349.23	350	F <sub>5</sub>	698.46	715	F <sub>6</sub>	1396.9	1396
# F <sub>4</sub>	369.99	370	# F <sub>5</sub>	739.99	745	# F <sub>6</sub>	1475.0	1480
G <sub>4</sub>	392.00	393	G <sub>5</sub>	783.99		G <sub>6</sub>	1568.0	1568
<sup>b</sup> A <sub>4</sub>	415.31	420	<sup>b</sup> A <sub>5</sub>	830.61	848			
A <sub>4</sub>	440.00	448	A <sub>5</sub>	880.00	880			
<sup>b</sup> B <sub>4</sub>	466.16	452	<sup>b</sup> B <sub>5</sub>	932.33	932			
B <sub>4</sub>	493.88	493	B <sub>5</sub>	987.77	1000			

COMPARISON OF MEASURED FREQUENCIES OF NOTES WITH SCALE OF EQUAL TEMPERAMENT (E. T.) TABLE 3

positions on the note could therefore, be related to the central amplitude independently of the force with which the note was struck and the displacement profile of the note built up.

Examples of the displacement profiles of sample notes are illustrated in Figure 10. The profile of F<sub>4</sub> indicates clearly that the edge condition of the note is modified by the final tuning process and that the vibration appears to be confined mainly to the slightly raised portion of the note. Comparison of the profiles of D<sub>5</sub> and A<sup>b</sup><sub>5</sub> also indicates that the larger circular notes (in this case D<sub>5</sub>) are more rigidly supported at their edge. In each case the notes appear to be vibrating predominantly in their fundamental modes.

(iii) Investigation of Partial of Notes – In order to locate their partials the notes were excited by a small speaker having a variable frequency input, the signal input being displayed on the screen of C.R.O. to enable the frequency to be accurately determined. As shown in Figure 11 the speaker was clamped in such a way that the sound waves emitted impinged directly on the note.



INSTRUMENTATION USED FOR INVESTIGATING  
MODES OF VIBRATION OF NOTES

FIG. 11

The frequency of vibration of the note was measured using the distance detector signal displayed on the C.R.O. screen, and the modes of vibration found by scattering fine sand on the note – to do this the pan had to be suspended in such a way that the note under investigation was in the horizontal plane. As the excitation frequency was gradually increased each partial was detected by a sudden sharp increase in the amplitude of vibration and the mode of vibration indicated by orientation of the sand along the nodes.

The frequencies and modes of vibration of a representative sample of outer notes are shown in Figure 12 (a). The first five partials of the largest note, E<sub>4</sub>, were clearly defined, but for the smaller notes the partials above the second could not be distinguished with any certainty and were further confused by apparent resonance in the adjacent dead metal. The modes of vibration of the first and second partials were similar for each note, and although the smaller notes, C<sub>5</sub> and F<sub>5</sub>, seemed to have a greater variation in their partials at higher frequencies, some common modes existed – for example the crossed nodes mode (No. 5 for E<sub>4</sub>) occurred for C<sub>5</sub> at  $f = 4.41f_0$  and for F<sub>5</sub> at  $f = 4.03f_0$ .

Figure 12 (b) shows the frequencies and modes of vibration of a representative range of circular notes. Because of their high frequencies partials above the third could not be


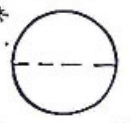
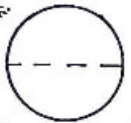




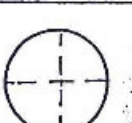



NOTE	FUNDAMENTAL	FREQUENCIES AND MODES OF PARTIALS				
		1	2	3	4	5
E <sub>4</sub>	$f_0 = 326 \text{ Hz.}$	 $f_1 = 2.01f_0$	 $f_2 = 3.1f_0$	 $f_3 = 3.45f_0$	 $f_4 = 4.72f_0$	 $f_5 = 4.92f_0$
C <sub>5</sub>	$f_0 = 523 \text{ Hz.}$	 $f_1 = 1.97f_0$	 $f_2 = 2.71f_0$	HIGHER MODES NOT CLEAR		
F <sub>5</sub>	$f_0 = 710 \text{ Hz.}$	 $f_1 = 1.91f_0$	 $f_2 = 2.56f_0$			
THEORETICAL PARTIALS FOR SQUARE PLATE 		$f_1 = 2.18f_0$	$f_2 = 2.48f_0$	$f_3 = 3.64f_0$	$f_4 = 4.24f_0$	$f_5 = 4.79f_0$

--- SHOWS POSITION OF NODE  
FREQUENCIES AND MODES OF VIBRATION OF PARTIALS OF OUTER NOTES

FIG. 12(a).

distinguished. In each case the first partial having a diagonal mode was very clearly defined but for the smaller notes, A<sub>5</sub> and C<sub>6</sub>, there were two definite conditions of resonance at slightly different frequencies with the node tending to rotate from a tangential to a radial position. Although this behaviour is not fully understood at the moment it is possible that it may be the result of a variation in edge flexibility around the circumference of the note.

Two interesting phenomena were noted during the investigation. The first that all the notes resonated in their fundamental modes when excited by subharmonic forces, the resulting resonance being quite strong especially that at a forcing frequency of  $\frac{1}{2}f_0$ . The circular notes were found to resonate also at frequencies of  $1/5f_0$ ,  $1/4f_0$ ,  $1/3f_0$  and  $1/2f_0$ , the strength of resonance increasing with frequency as expected.

PARTIAL	NOTES			THEORETICAL PARTIALS FOR CIRCULAR PLATES	
	D <sub>5</sub>	A <sub>5</sub>	C <sub>6</sub>	SIMPLY SUPPORTED	CLAMPED EDGE
FUNDAMENTAL	$f_0 = 580H_2$	$f_0 = 877H_2$	$f_0 = 1040H_2$	$f_0$	$f_0$
1	 $f_1 = 2.0f_0$	*  $f_{11} = 1.98f_0$ $f_{12} = 1.99f_0$	*  $f_{11} = 1.94f_0$ $f_{12} = 2.00f_0$	$f_1 = 1.73f_0$	 $f_1 = 2.01f_0$
2	 $f_2 = 3.85f_0$	 $f_2 = 2.1f_0$	 $f_2 = 2.16f_0$	$f_2 = 2.33f_0$	 $f_2 = 3.41f_0$
3	 $f_3 = 4.22f_0$	NOT CLEAR	 $f_3 = 2.68f_0$	$f_3 = 3.91f_0$	 $f_3 = 3.91f_0$

--- SHOWS POSITION OF NODE

\* TWO DEFINITE CONDITIONS OF RESONANCE WITH DIFFERENT INCLINATIONS OF DIAGONAL MODES

FREQUENCY AND MODES OF VIBRATION OF

PARTIALS OF CIRCULAR NOTES

FIG. 12(b).

The second fact noted concerned double resonance conditions of the circular notes similar to those discussed above. Each of the circular notes gave two very definite peaks of resonance when vibrating in the fundamental mode, the pattern of behaviour being similar in each case. As the condition of resonance was approached, the displacement of the note first approached a peak with the displacement signal indicating pure vibration in the fundamental mode – as the frequency was gradually increased a second peak followed almost immediately but in this case the displacement signal indicated that the vibration consisted of the fundamental together with the first partial. The same behaviour was noted when the note was put into resonance by a subharmonic exciting force, for example the note  $A^b_5$  gave double resonances at the following exciting frequencies – 0.2,  $0.202f_0$ ; 0.25,  $0.252f_0$ ; 0.333,  $0.336f_0$ ; 0.50,  $0.505f_0$ ; and 1.00,  $1.01f_0$ . Further tests are required to fully explain this behaviour but it seems possible that it is due to subharmonic excitation of the first partial of the note whose frequency is slightly greater than  $2f_0$  – e.g.  $2.02f_0$  for  $A^b_5$ .

#### 4. DISCUSSION OF THE BEHAVIOUR OF THE PAN

In this section an attempt is made to relate the results of the experimental programme completed so far to the behaviour of the pan as a musical instrument and to indicate the various factors which are likely to affect pan performance.

Starting with the manufacturing process, the sinking operation produces an elliptical bowl which acts as a stiff framework for supporting the notes. The grooving operation subsequently flattens out the material within the groove and the note becomes a flat plate supported at the grooves. The note, therefore, is much more flexible than the bowl and when it is struck it bends and deflects relative to the grooved boundary. However, the notes are not rigidly supported and there is some displacement of the groove when the note is struck, which becomes larger relative to the displacement of the centre of the note as the size of the note decreases. Thus the vibration of the note is transmitted through the flexible supports to the remainder of the pan and the resulting tone is not necessarily due to the note only. If the depth of sinking were reduced the



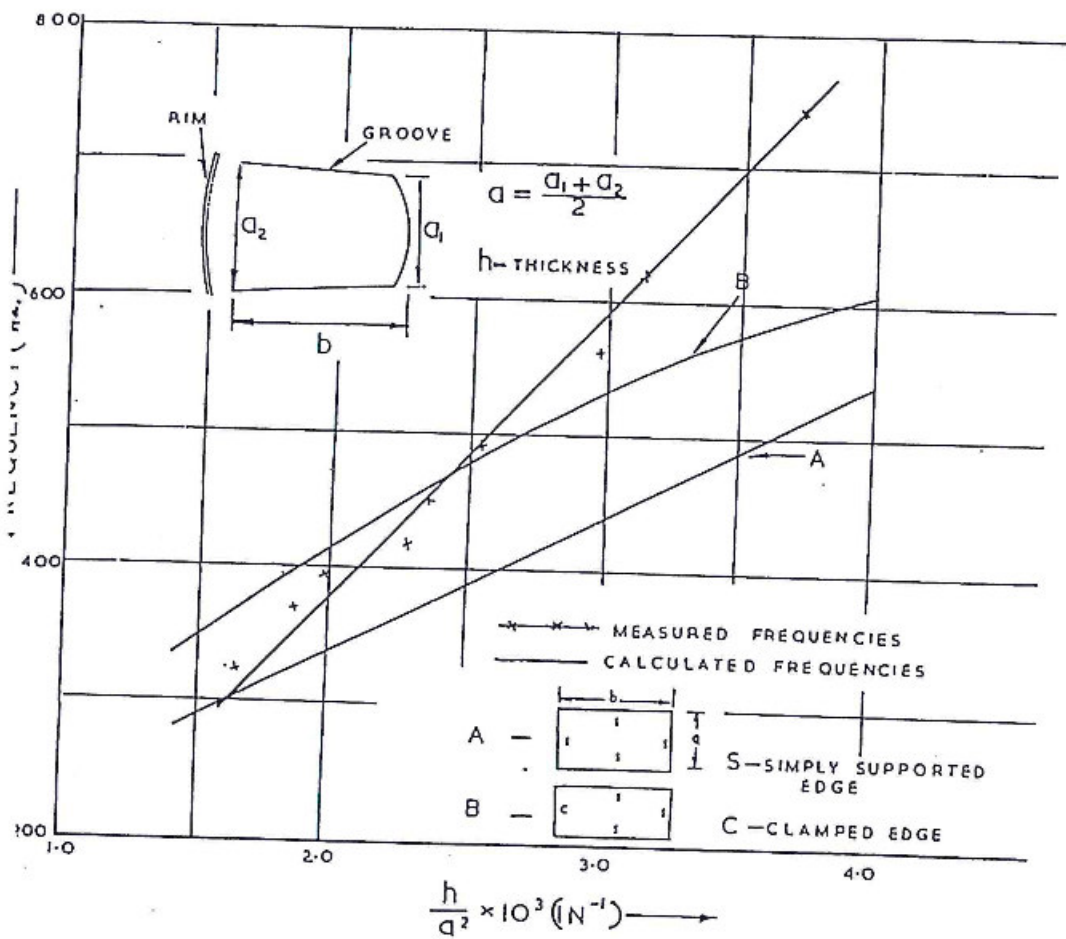
stiffness of the bowl relative to the notes would be decreased resulting in increased vibration of the remainder of the pan and possibly poorer tonal qualities.

The important characteristics of musical notes are pitch, tonal qualities, loudness and duration. The musical performance of the pan may, therefore, be analysed by discussing the factors influencing each of these characteristics.

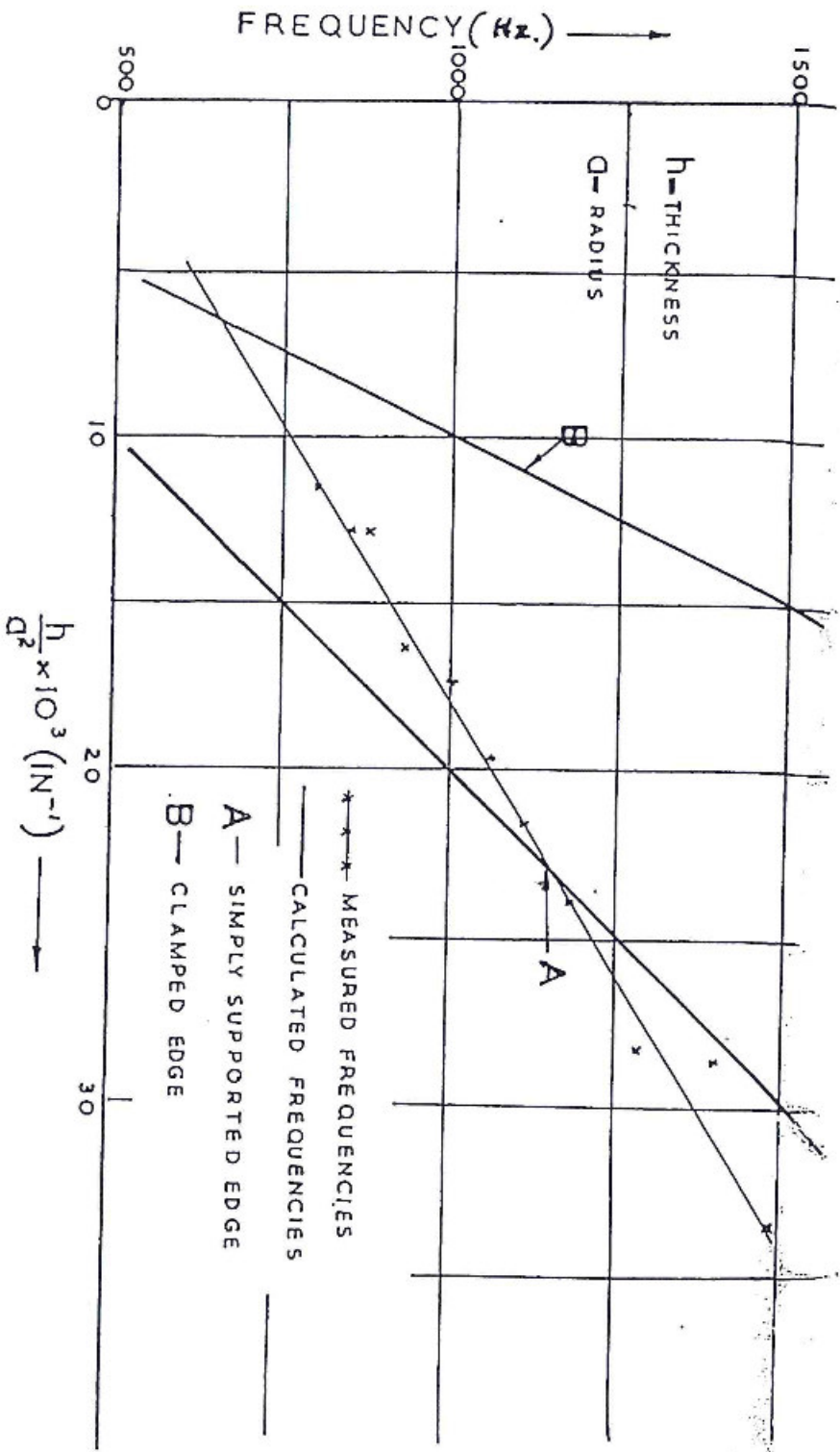
### Pitch

The pitch of a note is governed by its fundamental frequency and, therefore, by the shape, area, thickness, edge conditions (i.e. the restraints exerted on the edge of the note) and possibly to a small extent the profile of the note. The natural frequencies of rigidly supported plates are proportional to the ratio of thickness: area. Graphs plotting the frequencies of the notes against this parameter, Figure 13 (a) and (b), show linear relationships with surprisingly little scatter, but comparison with theoretical relationships for simply supported and built-in plates indicates that the edge conditions of the notes vary with note size. The outer notes were approximated by rectangular plates for this analysis and Figure 13 (a) shows that the frequencies of the larger notes agree reasonably well with those calculated assuming the plate to be clamped at the rim edge and simply supported at the grooves. As the size of the notes decrease the note frequencies increase above those calculated suggesting that a greater restraint exists on the edges of the notes. Evidently the opposite is true for circular notes, Figure 13 (b), since the comparison of measured and calculated frequencies suggests that as the note size increases the edge restraint increases. In fact the frequencies of the smallest notes are less than those calculated for simply supported plates owing to the increased lateral flexibility of the groove relative to the note, i.e. the note may be considered to be supported at its edge by springs and as the ratio of the spring stiffness to note stiffness decreases the natural frequency of the plate decreases.

The effect of the final tuning process on the frequencies of the notes is not apparent from the above comparisons and requires further investigation. However, it may be of significance with regard to the slightly domed circular regions of the outer notes mentioned in Section 3.5. that if a circle is



COMPARISON OF MEASURED AND CALCULATED  
 FREQUENCIES OF OUTER NOTES FIG. 13(d).



COMPARISON OF MEASURED AND CALCULATED  
 FREQUENCIES OF CIRCULAR NOTES. FIG. 13(b).

drawn to fit inside the boundaries of the notes and the radii used to plot the frequencies of the outer notes on Figure 13 (b) the points are almost coincidental with Line A.

### Tonal Qualities

The tonal qualities of musical notes are governed by the presence and relative magnitudes of harmonic overtones produced by the vibrating source. The tonal quality of a pan note will also be affected to a considerable extent by sound waves radiated by vibrations of other parts of the pan.

The signals recorded from the displacement pick-ups and the vibration patterns indicate that the notes vibrate predominantly in their fundamental modes. The absence of significant partials is probably a result of hitting the note near its centre where both the first and second partials of all the notes require nodes and therefore, these partials are largely destroyed. This in fact probably improved the tonal qualities of the notes since the partials are non-harmonic and would produce dissonance. The first partial occurs at very nearly twice the fundamental frequency for all the notes but the frequencies of the higher partials are apparently dependent mainly on the shapes and edge conditions of the notes.

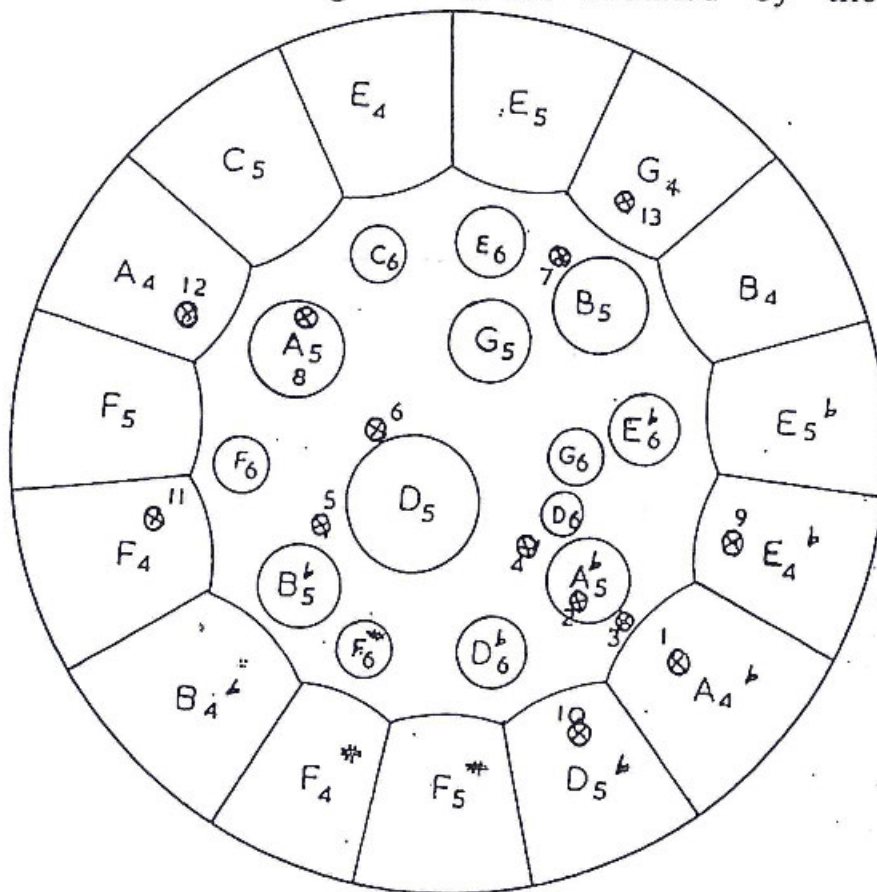
The resonance of a note by subharmonic excitation is clearly noticeable where notes an octave apart are placed together, striking of the lower note producing a distinct ring of the upper note which can be eliminated by damping the note with a finger. Again a significant first partial of the lower note whose frequency is almost but not quite  $2f_0$  could give rise to dissonance through beating with the higher note.

Some preliminary tests have been carried out to investigate the response of the remainder of the pan when individual notes are struck. The results of one such test are shown in Figure 14. No definite conclusions can yet be drawn but it appears that other notes may either respond to the excitation frequency or to the initial impulse of striking the note in which case they vibrate at their own natural frequencies. In the latter case the effects on the tone produced will depend on the ratio of the frequency to the notes sounded. If the ratio consists of two small integers as mentioned previously for adjacent notes a pleasant tone will be produced but for other ratios dissonance will occur.

Other factors influencing the tonal qualities of the pan include the type of hammer used and the force with which the note is struck. To fully understand the impact of all these factors on the sounds produced by the pan it will be necessary to carry out an intensive programme of spectrum analysis to break down the tones into their individual components. The results of this programme together with the results already obtained will enable the sources of the individual components to be traced and hence the influence of the various factors on the tonal qualities of the pan to be analysed.

### Loudness and Duration

Loudness is the auditory sensation associated with the intensity of a sound wave. The intensity of the sound wave radiated by a vibrating plate is dependent on the plate area, amplitude and frequency of vibration and the coupling of the plate with the air. It was shown in Figure 14 that the whole of the pan vibrates to some extent and therefore it is possible that the surface of the pan may act to a certain degree as a soundboard reinforcing the sound radiated by the note.



POINT	NOTE	FREQ. (HZ)	MEASURED VIBRATION		COMMENTS
			FREQUENCY (HZ)	RELATIVE AMPLITUDE	
1	A <sub>4</sub> <sup>♯</sup>	420	420	1.0	
2	A <sub>5</sub> <sup>♭</sup>	840	420	0.066	Fundamental of A <sub>5</sub> <sup>♭</sup> at 840 Hz is superimposed on 420 Hz oscillation opposite phase to A <sub>4</sub> <sup>♯</sup>
3	Dead Metal	-	420	0.132	Dead Metal Vibrates at frequency of note
4, 5, 6, 7	"		420	Very Small	Vibrates at frequency of first partial of A <sub>4</sub> <sup>♯</sup>
8	A <sub>5</sub>	880	840	0.033	
9	E <sub>4</sub> <sup>♭</sup>	314	314	0.13	Vibrates at own frequency
10	D <sub>5</sub> <sup>♭</sup>	564	420	0.40	Vibrates at frequency of A <sub>4</sub> <sup>♯</sup>
11	F <sub>4</sub>	350	350	Very Small	Vibrates at own frequency
12	A <sub>4</sub>	448	420	0.03	Vibrates at frequency of A <sub>4</sub> <sup>♯</sup>
13	G <sub>4</sub>	393	393	0.12	Vibrates at own frequency

VIBRATION PATTERN OF PAN WHEN

NOTE A<sub>4</sub><sup>♯</sup> IS STRUCK

FIG. 14.

However, the effect is obviously very complex, being closely coupled with the tonal qualities of the notes, and requires further investigation.

The duration of a note of a percussion instrument consists of a transient build-up period followed by a decay period. The rate of decay of pan notes is quite high because damping is increased by transmission of the vibrations of the note to other parts of the pan and even to the support stand. Although some of the energy of vibration is converted into sound energy the major damping effect results from hysteresis losses through internal friction within the material. The latter is known to be affected by the crystal structure of the metal and by the presence of stresses in the metal and is therefore, likely to be influenced by the cold working and subsequent heat treatment of the metal during the manufacturing process. However, the actual degree of influence cannot at present be assessed.

## 5. CONCLUSIONS

For the pan under investigation the notes vibrate in the fashion of flat plates flexibly supported around their grooved boundaries by the much stiffer framework provided by the sinking process. As such their pitch is governed mainly by shape, area, thickness and the edge restraints imposed by the surrounding material. The frequencies of both the outer trapezoidal notes and the inner circular notes are linearly proportional to the ratio of thickness: Area with surprisingly small scatter.

The notes vibrate predominantly in their fundamental modes possibly because the position of striking the notes tends to destroy the first and second partial modes. The partials in any case are generally non-harmonic and would add dissonance to the tones. It is likely that the tonal qualities of the notes are more dependent on the sounds radiated by vibrations of other parts of the pan than on the partials of the notes themselves.

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7. Lord Rayleigh "Theory of Sound" Dover Publications Inc.  
Vol. 1 p. 203