THE OPTIMISATION OF POWER SYSTEMS LOADS FOR A LEAST COST PERFORMANCE CRITERION

PART I – THEORETICAL BASIS

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ABSTRACT

This paper provides a critical review of the conventional definition and interpretation of 'power-factor' and identifies the limitations of such a definition when applied to abnormal loads.

A new definition is suggested which encompasses the conventional interpretations for the usual steady state, linear and time invariant as well as for abnormal loads and loading conditions.

The paper then considers the basic approaches for power factor correction and draws some fundamental conclusions. Particular schemes for load compensation (correction) are developed.

1.0. INTRODUCTION

A number of references can be made to the literature involving power factor definition and correction as applied to alternating current system RUSSEL, C.J., ELLIS, T., DRAKE, C.W., HALBERG, M.N., RUDRA, J.J., TERRY, H.W. There has always been an increasing interest in improving the methods used in the power factor correction of induction motors, AZACETA, E., TRUEBA, A., KUCRERA, M.J., BELOQUIST, W.C., BOYCE, W.K., WARNER, R.G., KNOWLTON, D.E., DESIENO, C.F. and BEAUDOIN, B.J. Recently, there has been keen interest in the determination of the maximum capacitance that can be connected across a given induction machine for the purposes of power factor correction without causing self-excitation. SMITH, I.R. and SRIHARAN have discussed the transients in induction machines with power factor capacitors following disconnection from and reconnection to the supply.

In all the above investigations the power factor referred to was defined as the ratio of average power (per cycle) to the corresponding volt-amperes as measured by a wattmeter and root mean square voltmeter and ammeter. When the current supplied to the load was sinusoidal and the source was also sinusoidal, then it was seen that this ratio was identical with, Cos ϕ where ϕ was the phase angle between supply voltage and current taken. When the current taken was periodic but non-sinsusoidal then the power factor was still calculated by the above ratio. Further, an equivalent sinusoidal wave was postulated for the actual current wave which had a phase shift of, ϕ ,and the same power factor.

The above approach has obvious weaknesses. One of them being that the postulated equivalent sinusoidal wave bears no relation to the physical system. Also, this approach cannot be supplied to non-periodic transient, time varying loads, e.g. arc welding, crane lifting. Because of the highly distorted non-periodic nature of the current-voltage characteristic of abnormal loads like these, neither can the power factor be obtained as defined nor can an equivalent sinusoidal be substituted for the original wave form.

The design of tariff-structures by public utility companies and rate fixing authorities has utilised the power factor concept. In very general terms the basic principle of rate fixing hinges on payment for energy consumed plus an appropriate sharing of fixed charges. The latter factor produces the greatest difficulty because of the wide variety of usage patterns that exist among consumers as well as the social and economic aspects of the electric utility industry.

The consumer on the other hand, having accepted the rate and tariff structure, attempts to optimise his costs through various techniques which are recognised by the supply authority and which are usually anticipated in the design of the tariff structure. The classical situations of power factor correction, off peak rates, interruptible loads etc. are well known and accepted techniques for achieving optimisation of costs are well established.

Again, such tariff structures and techniques for optimisation normally assume a sinusoidal, periodic and linear loading. For example the classic power factor definition and compensation techniques based on power factor improvement assume a sinusoidal, periodic load, although some recognition has been given to the fact that non-sinusoidal loads do exist and require a different approach GEORGE EWING, BEDFORD AND HOFT. The problem is aggravated when such a load (non linear, non periodic – e.g. an automatic welding unit) exists in isolation as is common in small developing countries where single product industries are common, and the utility companies find on the one hand that computation of costs utilising existing metering equipment and tariff structure leads to serious anomalies. On the other hand the consumer finds that the traditional methods of correcting power factor reducing maximum KVA demand cannot produce any meaningful results.

This paper presents some results of work in this area which started with a critique of the classical power factor definition and then develops a new and more generalised approach to the problem.

2.0. DEFINITION OF POWER FACTOR

In the normal day to day operation of a power system it is assumed that the load is linear and periodic. This assumption is accepted within some unspecified period of time over which these conditions hold. If an attempt is made to be more precise then it is clear that even the switching on of one of these normal loads presents a non-periodic, non-linear loading condition for an interval. The total load of a power system varies slowly with time, giving the familiar daily demand curves with their associated peaks. Over this time interval the load is also non-periodic. However, power system engineers have defined the terms load and diversity factors to give some quantitative meaning to the effects of this "longer-term" load change.

Load-Factor and Power-Factor are the two methods in current use for the assessment of load change but no specified time interval is inherent in either definition. Power factor is used for a "short time" and load factor for a "long time" assessments (with respect to the period of the supply voltage). This vagueness is unsatisfactory and any precise definition must overcome this disadvantage Further, power factor as currently defined cannot handle highly fluctuating non-periodic loads.

Consider a general time varying voltage $v(t)$ supplying some current $i(t)$ into a passive piece of equipment. The power delivered to load at time (t) is

$$p(t) = v(t)\, i(t) \quad \ldots\ldots (1)$$

and the energy to the load in the interval $t_2 \leq t_1$ is

$$E(t_1, t_2) = \int_{t_2}^{t_1} p(t)\,dt = \int_{t_2}^{t_1} v(t)\, i(t)\,dt \quad \ldots\ldots (2)$$

Define a time varying function, $R(t)$, which is a property of the load such that

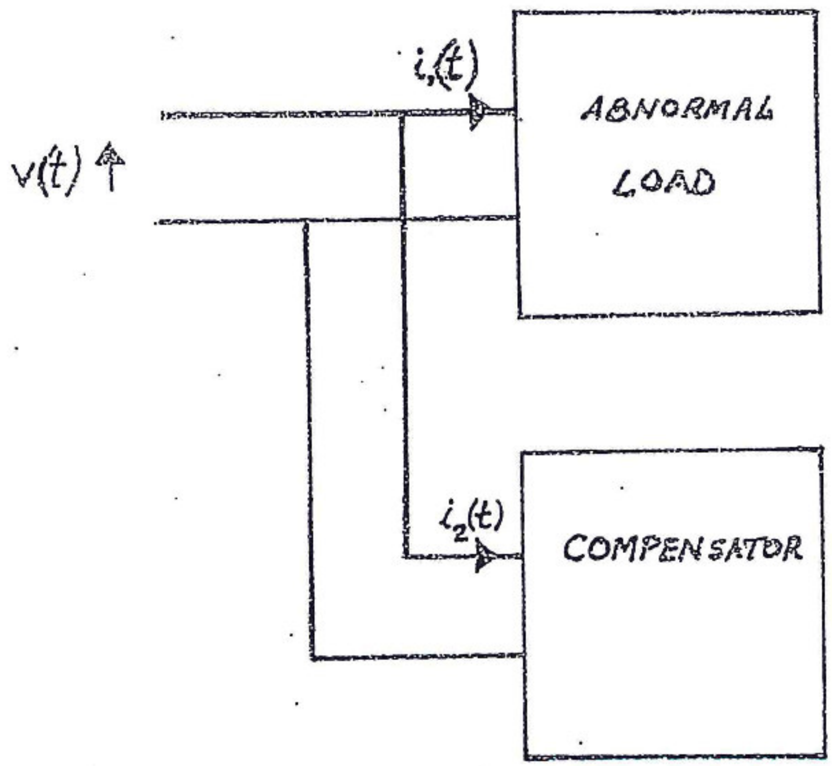
$$R(t) = v(t)/i(t) \quad \ldots\ldots (3)$$

$R(t)$ is called the "Load Impendance Function" and for physical systems

$$\left|\frac{dR(t)}{dt}\right| < \infty \quad \ldots\ldots (4)$$

From function theory

$$\int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2 dt \geq \left(\int_{t_1}^{t_2} vi\, dt\right)^2 \quad \ldots\ldots (5)$$



$$R_2(t) = \frac{v(t)}{i_2(t)} = \frac{1}{G_2(t)}.$$

FIG. 1

Therefore, a non-dimensional factor by which we can rate the energy supplied to
the load in the interval, $t_1 \leq t \leq t_2$ can be $pf(t_1,t_2)$ where

$$pf(t_1,t_2) = \frac{\int_{t_1}^{t_2} vi \, dt}{\left[\int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2 dt\right]^{\frac{1}{2}}} \quad \ldots\ldots(6)$$

Define $pf(t_1,t_2)$ as the "Power Factor" of the energy supplied over the period and in words:

> "$pf(t_1,t_2)$ is the ratio of the nett energy supplied in the period $t_1 \leq t \leq t_2$ to the maximum energy which could have been supplied to a maximum efficient load using the same R.M.S. values of current and voltage over the same period".

This definition introduces the term "maximum efficient load". If the load is "maximum efficient" then $pf(t_1,t_2) = 1$ and the equality sign of (5) applies. If $R_{max}(t)$ is the load impedance function of this load then

$$\left[\int_{t_1}^{t_2} v^2/R_{max}(t) \, dt\right]^2 = \int_{t_1}^{t_2} v^2 dt \int_{t_1}^{t_2} i^2 dt \quad \ldots\ldots(7)$$

This occurs only when $R_{max}(t)$ is independent of (t) in the period $t_1 \leq t \leq t_2$; i.e. the load function is time invariant over this period. The factor $pf(t_1,t_2)$ does not make any distinction in energy supplied whether it is stored or dissipated by the load. Also, the factor gives an indication of the utilisation of the power generating capacity and can be applied to any energy system.

For periodic deterministic loads of period, T, pf(0,T) is given by

$$pf(0,T) = \int_0^T vi \, dt / \left[\int_0^T v^2 dt \int_0^T i^2 dt\right]^{\frac{1}{2}} \quad \ldots\ldots(8)$$

and $pf(0,T) = pf(0,nT)$ where n is a positive integer. when $n \to \infty$,
$pf(0,T) = pf(0,\infty)$ ......(9)
Thus pf(o, T), the factor taken over one cycle for a repetitive operation gives the power factor of the process after an infinite time. This is the justification for using pf (0, T) as a measure of energy utilisation for non-repetitive loads (where T is the voltage period).

3.0. LOAD FACTOR

Consider a power source supplying a number of sub-loads. If $pf_s(t_1,t_2)$ is the power factor of the source and $pf_L(t_1,t_2)$ is that of the total load then

$$pf_s(t_1,t_2) = pf_L(t_1,t_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

assuming no transmission losses.

If $pf_{Lr}(t_1,t_2)$ is the power factor of the r^{th} sub-load then

$$pf_L(t_1, t_2) \leq \sum_{n=1}^{n} pf_{Lr}(t_1, t_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where n is the number of sub-loads.

Hence
$$\frac{pf_s(t_1,t_2)}{\sum_{n=1}^{n} pf_{Lr}(t_1,t_2)} \equiv K \leq 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where K is a positive number, which can be used to compare the demand patterns of individual loads to that of the total load.

3.1. INTERPRETATION

Diversity factor is an attempt to measure the difference in demand curves of the various sub-loads which make the total load. This is done by only comparing the maximum demands over the intervals of interest. The factor, K, as defined by (12) compares the various demands of the sub-loads and is unity only when the sub-loads, besides having their maximum demands at the same time are of identical forms. Hence, K, is a more accurate measure of "diversity".

Load factor as conventionally defined is the ratio of the total energy supplied to the product of the maximum average power demand over the interval of interest and the period of interest.

$$\text{i.e. Load Factor} = \int_0^{T_i} vi \; dt / P_{av} \times T_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Now P_{av} is the maximum value read by the wattmeter over the interval T_i.

The power factor as defined by $pf(0,T_i)$ over the interval is a more efficient measure of the efficiency of equipment usage over the interval than both diversity factor and Load Factor since it is possible for either to be unity without total efficient usage of equipment.

4.0. MAXIMISATION OF POWER FACTOR

It has been shown that $pf(t_1,t_2) \leq 1$ for any process. It is in the interest of power companies to deliver energy over any period of time

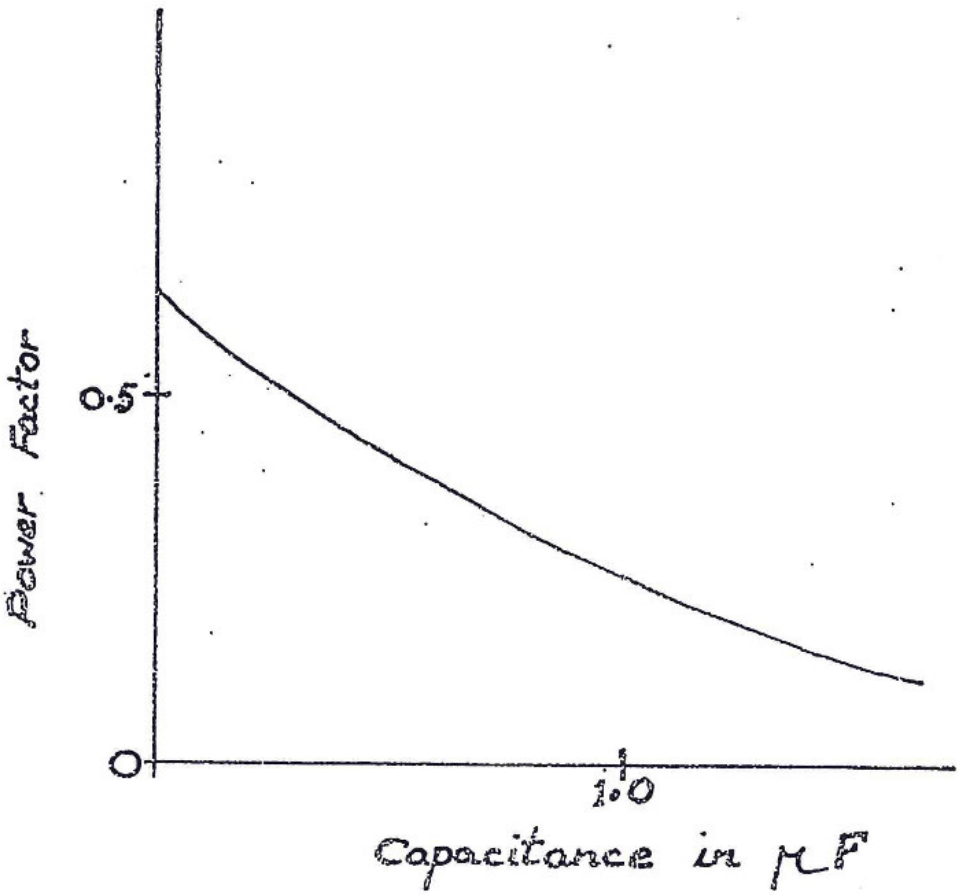


Fig. 2

t as high a power factor of the supplier is equal to the power factor of he total load then the above policy is consistent with the requirement hat the individual loads have high power factors. It is a physical fact hat certain pieces of equipment have fixed v-i characteristics. So, any ower factor improvement must be carried out by attaching external ompensating devices.

.1. THE GENERAL PARALLEL COMPENSATION PROBLEM

Since most equipment is voltage operated the desired ompensation is in parallel with it. Hence the problem can be stated:

Choose a compensator given by $R_2(t)$ such that $pf(t_1,t_2)$ is aximised, where $pf(t_1,t_2)$ is the combined power factor of the ompensator and load, Fig. (1), subject to any physical constraints that ay be put on $R_2(t)$. It is the purpose of this section to develop a chnique for load compensation which will lead to a computer rogramme. It is obvious that the problem as stated can have trivial swers (e.g. a short circuit across the load will maximise the power ctor) unless other constraints (educated guesses) are put on $R_2(t)$ e.g. e compensator should consume as little energy itself over the period f interest. Since the form of the compensator may be stipulated before nd e.g. resistive, capacitive, inductive or a combination of these three, gue constraints on the energy consumed will not be satisfactory.

Since the consumer will compensate so as to optimise his costs it ould appear that a convenient cost function for the optimisation oblem should be based on tariff structures.

2. COST FUNCTION

Consider a consumer whose supply voltage is $v(t)$ and current ken at any instant t, is $i(t)$, Fig. 1. The energy taken over time, T, is ven by E_s where

$$E_s = \int_0^T v(t) \cdot i(t) \, dt \quad \ldots (14)$$

t η_1 be unit average price to the consumer of energy supplied. erefore the consumer's cost for energy over some period T is $\eta_1 E_s$. fine a quantity E_m where

$$E_m = \int_0^T v^2(t)dt \int_0^T i^2(t)dt \quad \ldots (15)$$

before $E_s \leq E_m$

E_m is the maximum amount of energy that can be supplied with the ne R.M.S. values of voltage and current over the interval of interest.

An effective cost function could be Q, cost/interval where

$$Q = \eta_1 E_s + \eta_2 (E_m - E_s) \quad \ldots\ldots (16)$$

where the first term gives the cost of energy for the interval and the second relates to the excess equipment being tied up so as to supply the particular demand pattern (η_2 is the cost assigned to this 'mis-use' of equipment).

The cost, Q, over the interval, T, is an operating or variable cost and to this must be added such fixed charges as are applicable. By equ. (14), (15) and (16) the cost function, Q can be defined as:

$$Q = (\eta_1 - \eta_2) \int_0^T vi\ dt + \eta_2 \left[\int_0^T v^2 dt \int_0^T i^2 dt\right]^{\frac{1}{2}} \quad \ldots\ldots (17)$$

If the supply voltage is independent of loading this equation becomes

$$Q = (\eta_1 - \eta_2) \int_0^T vi\ dt + \eta_2 k \left[\int_0^T i^2 dt\right]^{\frac{1}{2}} \quad \ldots\ldots (18)$$

where

$$k^2 = \int_0^T v^2 dt \quad \ldots\ldots (19)$$

Let C be a cost function such that $Q/\eta_2 = \bar{C}$. Since η_2 is positive $\bar{C}$ is given by equ. (20) is also an equivalent cost function.

$$\bar{C} = \bar{\eta} \int_0^T vi\ dt + k\left[\int_0^T i^2 dt\right]^{\frac{1}{2}} \quad \ldots\ldots (20)$$

where

$$\bar{\eta} = \frac{\eta_1 - \eta_2}{\eta_2} \quad \ldots\ldots (21)$$

Another cost function which attempts to do the same as $\bar{C}$ is C where

$$C = \eta \int_0^T vi\ dt + k \int_0^T i^2 dt \quad \ldots\ldots (22)$$

In this case the load current is weighted as its mean square instead of its root mean square. Since η is under the control of the authority the cost function given by equ. (22) can be equally effective in forcing the consumer to correct his demand. The advantage of using, C, is that conventional optimisation techniques are more easily applied to it.

It should be noted that the aim of the authority is that its equipment should be used as efficiently as possible. Many tariff structures attempt to do this and they succeed to varying degrees. Cost functions (20) and (22) can be equally effective in obtaining efficient usage.

4.3. SOLUTION OF GENERAL PARALLEL COMPENSATION PROBLEM

With reference to Fig. 1 the problem of parallel compensation can be stated as follows:

> "Choose a compensator, whose load function is $R_2(t)$, which can be put in parallel with the normal load such that the overall cost function is minimised".

The cost function for this case becomes

$$C = \int_0^T [n\, v(i_1+i_2) + k(i_1+i_2)^2]dt \quad \text{.....(23)}$$

Since i_1 and v are known the cost function can be simplified to

$$C_1 = \int_0^T [n i_2 v + k(i_1+i_2)^2]\, dt \quad \text{.....(24)}$$

Since

$$G_2(t) = \quad = i_2/v \text{ then} \quad \text{.....(25)}$$

$$C_1 = \int_0^T [nG_2v^2 + k(i_1 + G_2v)^2]dt \quad \text{.....(26)}$$

The problem now is to choose $G_2(t)$ such that C_1 is minimised. By Pontryagin's Maximum Principle, ATHANS and FALB, the Hamiltonian, H, is given by

$$H = -[nG_2v^2 + k(i_1 + G_2v)^2] \quad \text{.....(27)}$$

and H must be maximised for each value of $i_1(t)$ and $v(t)$ by choosing $G_2(t)$ subject to the constraints imposed. $i_2(t)$ is not completely free in that, in certain cases we may require a non-linear resistance compensator. In this case $G_2(t) \geq 0$. In other cases $G_2(t)$ may be related to other system variables or physical constraints. The complete compensation problem can now be stated as;

> "Choose $G_2(t)$ which maximises the Hamiltonian, H, subject to the constraints on $G_2(t)$;"

4.3.1. CONVENTIONAL POWER FACTOR CORRECTION

It is of interest to apply the technique to the classical problem of power factor correction of inductive loads. In this case the optimal solution is obtained for a completely free $G_2(t)$. This is done in detail in Appendix I and the optimum $G_2(t) = \text{Sin}\ \frac{\text{Cos}\ \omega t}{\text{Sin}\ \omega t}$, where α is the power factor angle and Sin is the input voltage. The load admittance function $G_2(t)$ is that of a capacitor as expected. Note that in the solution no physical experience of the particular problem at hand interpretation is a bit more difficult.

In this case $G_2 \leq 0$ for $\sin \omega t \geq 0$ and $G_2 \geq 0$ for $\sin \omega t \leq 0$, i.e. we require a compensator which stores and consumes energy in the time segments $\sin \omega t \geq 0$; some sort of rechargeable a.c. battery. A simpler compensator which is wholly resistive can be obtained by choosing $G_2(t)$ from the set $G_2(t) \geq 0$ in which case the solution is as given if $-\eta/2k \geq I/V$

and $G_2 = 0$ when $\sin \omega t \geq 0$ . . . .(33)
when $-\eta/2k < I/V$
$G_2 = -\eta/2$ when $\sin \omega t \leq 0$
This, of course, does not give unity power factor.

This problem is a very simple one but it clearly demonstrates the reason behind the use of a precise Cost Function in determining compensators. In this problem it is assumed that the energy used by the compensator is of no use. It is possible to use heating loads as compensators but the format of this problem is different. Problems of this type are receiving further attention.

5.0. CONCLUSIONS

The power factor, $pf(t_1,t_2)$ can be defined for any deterministic load over the period $t_1 < t < t_2$. This definition includes the traditional approach and when applied to the power source as well as the load envelops the current definition of load factor.

It has been shown that although $pf(t_1,t_2)$ is adequate for rating the performance of an energy system it has certain deficiencies when used as a Cost Function in the analytical design of load compensators from the economic point of view. Because of these limitations a cost function which may be taken as the basis of a tariff structure was developed which allows the compensating problem (and others) to be stated in a precise mathematical form in which the type of compensator to be used (if known) can be included by constraining certain system variables to lie in particular sub-sets.

With time varying multipliers within this cost function (i.e. by marking η a function of time) the compensation problem can be framed so as to include off peak and seasonal loadings. The cost function allows the problem of load efficiency correction to be handled by standard optimisation techniques which includes the calculus of variations. Minimisation of this cost function will reduce operating costs irrespective of tariff structure provided that the tariff structure penalises inefficient users (i.e. low power factor users). The method is very general and among its possible applications is to the demand pattern of regulated converters where an optimum regulating pattern is required.

is built in (i.e. it is not assumed that the optimum design will require a compensator which only supplies reactive energy). This is the essence of any automated optimisation scheme even though the solution time in so doing may be longer.

4.3.2. EXAMPLE

Consider the simple non-linear example of a load presented by a half wave rectifier defined by

$$v(t) = V \sin \omega t$$
$$i(t) = I \sin \omega t \text{ when } \sin \omega t > 0$$
$$= 0 \text{ when } \sin \omega t < 0 \quad \ldots\ldots (28)$$

It is required to compensate the system by choosing parallel compensation. It has been, demonstrated practically that any static capacitor connected in parallel with this load will further degrade its performance (performance here being measured by its power factor). See. Fig. (2).

If $G_2(t)$ is left free then as before $G_2(t)$ must be chosen such that H is maximised i.e. $\frac{\partial H}{\partial G_2} = 0$

By equ. (27)

$$\frac{\partial H}{\partial G_2} = -[\eta v^2 + 2kv(i_2 + G_2 v)] \quad \ldots\ldots (29)$$

$$\text{i.e. } G_2 = -\frac{\eta}{2k} - \frac{i_2}{v} \text{ when } v \neq 0 \quad \ldots\ldots (30)$$

When $v = 0$, G_2 can take any practically convenient value.

Hence $G_2 = -\eta/2k - I/V$ when $\sin \omega t > 0$

$= -\eta/2k$ when $\sin \omega t < 0$ ……(31)

$= -\eta/2k$ when $\sin \omega t = 0$ for convenience.

When the load is compensated the total load admittance function is $G_2(t) + I/V$ for $\sin \omega t > 0$ and $G_2(t)$ for $\sin \omega t < 0$ ……(32)

i.e. $-\eta/2k$ for all values of 't'. Hence the compensated load is maximum efficient and the power factor is unity. This is as we would expect that for power factor maximisation a unique value of $G_2(t)$ cannot be chosen unless it is required to do so using minimum energy. Then $G_2 = I/V$ when $\sin \omega t < 0$ and $G_2 = 0$ when Sin 0 would be the solution. But the cost of compensating energy may outweigh the cost due to "mis use" of equipment, hence it would still be uneconomical. [Note this solution would be correct economically if $-\eta/2k = V/I$].

The optimum solution can be physically interpreted very easily when $-\eta/2k > I/V$ i.e. $G_2 \geq 0$. When $G_2 \leq 0$ for any segment then

A.1. APPENDIX

It is of interest to apply the technique outlined in the paper to the classical problem of correction of inductive loads. In this case let $v(t) = \mathrm{Sin}\ \omega t$ and $i_1(t) = \mathrm{Sin}\ (\omega t - \alpha)$. The problem is to choose $i_2(t)$ such that

$$C = \int_0^T \eta(i_1+i_2)v + k(i_1+i_2)^2\, dt \qquad \text{A.1.}$$

is minimised. Since the compensator must be passive

$$\int_0^T v i_2\ dt \geq 0 \qquad \text{A.2.}$$

Define a time function $g(t)$ such that

$$\int_0^T (v i_2 - g) = 0 \qquad \text{A.3}$$

where $g(t) \geq 0$

A.3 is then equivalent to A.2

Using the Lagrange multiplier technique the augmented cost function becomes

$$\int_0^T [\eta(i_1+i_2)v + (v i_2 - g)\lambda + k(i_1+i_2)^2]dt$$

where λ is the Lagrange multiplier. By the Pontryagin Maximum Principle we must choose i_2 and g such that the Hamiltonian, H, is maximum for all values of i_1 and v.

where

$$-H = \lambda(v i_2 - g) + \eta(i_1+i_2)v + k(i_1+i_2)^2 \qquad \text{A.4}$$

Let $i_2 = \bar{i}_2$ when $\frac{\partial H}{\partial i_2} = 0 \quad \therefore \quad \bar{i}_2 = \frac{(\eta+\lambda)v}{k} - i_1$

H is maximised with respect to 'g' when $g = 0$ if λ is < 0 and $g = \infty$ when $\lambda > 0$

$$\bar{i}_2 = -\frac{(\eta+\lambda)}{k}\,\mathrm{Sin}\ \omega t - \mathrm{Sin}(\omega t - \alpha)$$

$$= -\mathrm{Sin}\ \omega t\left(\frac{\eta+\lambda}{k} + \cos\alpha\right) + \mathrm{Cos}\ \omega t\ .\ \mathrm{Sin}\ \alpha$$

Using the constraint A.3

$$\int_0^T v\bar{i}_2\ dt = -\int_0^T \mathrm{Sin}^2\ \omega t\left(\mathrm{Cos}\ \alpha + \frac{\eta+\lambda}{k}\right)dt \geq 0 \qquad \text{A.5}$$

obviously $g \neq \infty$ since this would make the augmented cost function go to infinity. Therefore $\lambda < 0$ and

$$\int_0^T v\,\bar{i}_2\ dt = 0$$

$$\int_0^T \mathrm{Sin}\ \omega t\ .\ \left(\mathrm{Cos}\ \alpha + \frac{\eta+\lambda}{k}\right)dt = 0 \qquad \text{A.6}$$

then $\mathrm{Cos}\ \alpha + \frac{\eta+\lambda}{k} = 0 \qquad \text{A.7}$

Therefore the optimum compensator must take a current i_2 such that $i_2 = \sin\alpha \quad \cos\omega t$ which is the condition for capacitive compensation. This novel technique gives the same answer for the classical problem as the classical approach.

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