GENERATION OF SUBHARMONICS BY STOCHASTIC LOADS

by

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SUMMARY

Under particular conditions of discontinuous and repetative varying current flow, subharmonic currents should exist. These harmonics can be significant and affect the performance of the system. In this study an attempt was made to isolate the subharmonics under the above conditions and compare their magnitudes with calculated values.

1. INTRODUCTION

In electrical engineering, solid state devices are becoming increasingly popular for the control of speed, torque and output power of both a.c. and d.c. machinery. One of the main reasons is the quick response of these devices to changes of current and voltage that allow them to be used in applications where other forms of control would be inadequate. Solid state inverter devices are typical of such rapid control.

These devices - principally thyristors and thyristor modules - have found in the electrical industry for the regulation of large power flow. One such use is the reduction of voltage fluctuations and the improvement of stabilizing the generator loadings, of a relatively weak electrical system connected to a large varying load. The major problem is the methods of current regulation applied to these devices which make them sensitive to the generation of superharmonics and subharmonics.

A search of the literature on harmonics indicates extensive investigation in the detection and elimination of superharmonics, but very little has been don in identifying the causes, detection and elimination of subharmonics.

This paper considers the conditions giving rise to the occurrence of subharmonics in power systems. It then describes a mathematical model and the simu lated equivalent circuit using solid state equipment that may be used for the detection of generated superharmonics. A case study is herein included to demon strate the validity of the approach and to show that under actual operating conditions experienced by most electrical processes, subharmonics will be produced.

2. THEORY

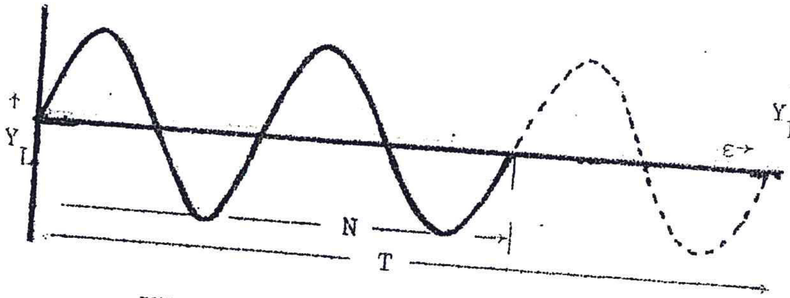
The control of solid state devices is accomplished by

(a) Integral cycle control, or
(b) Phase angle control.

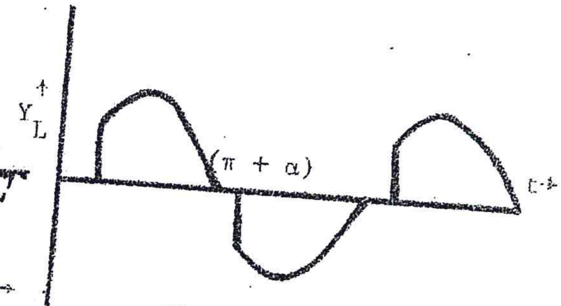
Integral cycle control is made possible by conduction of a complete number of current sinusoids followed by an extinction interval equivalent to a further number of complete sinusoids[1]. A particular example is given in Figure 1 of conduction for two cycles and inactive operation for one cycle.

The other method known as phase angle control consists of switching the solid state devices at identical points of their respective anode voltage cycles The load voltage has the[2] symmetrical form of a sinusoid with portions of the applied voltage missing. An example of phase angle control can be seen in Figure 2.

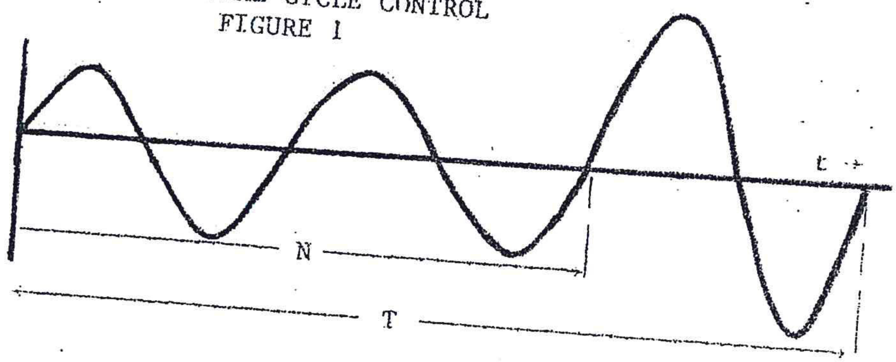
Basically both forms of control are commonly used to alter the voltage and currents of a.c. machinery. In so doing objectionable higher harmonics 3, 5, 7, etc. are generated. In phase angle controlled circuits these harmonics assume significant proportions and must be reduced to tolerable levels by the insertion of appropriate filters in the circuit.



INTEGRAL CYCLE CONTROL
FIGURE 1

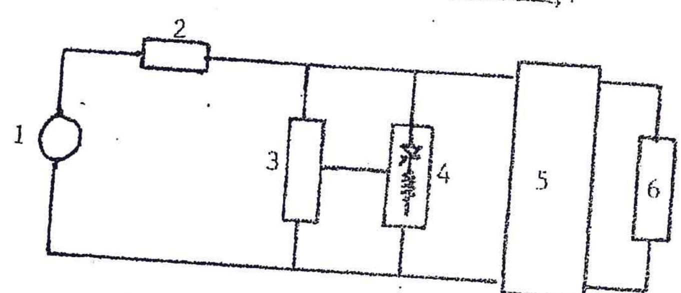


PHASE ANGLE CONTROL
FIGURE 2



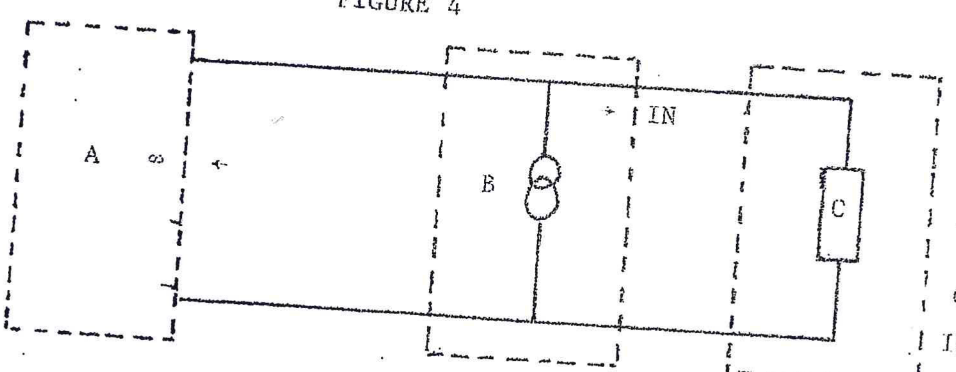
CONTINUOUS REPETATIVE
VARYING VOLTAGE

FIGURE 3



1 SUPPLY
2 BAND STOP FILTER
3 SWITCHING CIRCUITS
4 THYRISTOR LOAD
5 FILTER
6 TERMINATION

MODELLED SYSTEM USED
FIGURE 4



A INFINITE IMPEDANCE
B SOURCE OF SUBHARMONIC
C FILTER LOAD
IN SUBHARMONIC CURRENT FLOW

EQUIVALENT OF MODELLED SYSTEM AT SUBHARMONIC
FREQUENCIES

FIGURE 5

A closer look at the Fourier analysis for both waveforms indicates the presence of superharmonics and subharmonics. The synthesised current function can be described as

$$i(\theta) = \hat{V}/R \; f\,(\sin\theta) \quad \text{(for a resistive load)}$$

But $\quad i(\theta) = a_o + a_n \sin n\theta + b_n \cos n\theta + \ldots\ldots$

a_o, a_n, b_n are coefficients of the series $n = 1,2,3 \ldots\ldots$

For phase angle controlled loads

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\pi} \frac{\hat{V}}{R} \sin\theta \sin n\theta \, d\theta$$

$$= \frac{\hat{V}}{R\pi}\left[\frac{\sin (n+1)\alpha}{(n+1)} - \frac{\sin (n-1)\alpha}{(n-1)}\right]$$

$n = 1,2,3 \ldots\ldots$

$$a_o = 0$$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\pi} \frac{\hat{V}}{R} \sin\theta \cos n\theta \, d\theta$$

$$= \frac{\hat{V}}{R\pi}\left[\frac{\cos (n-1)\pi - \cos (n-1)\alpha}{(n-1)} - \frac{\cos (n+1)\pi - \cos (}{(n+1)}\right.$$

$n = 1,2,3 \ldots\ldots$

for $\quad \theta = wt$ the lowest frequency components will be at supply frequency and all others will be superharmonics. Hence a supply that applies a phase angle controlled voltage waveform will contain no subharmonics.

For integral cycle controlled loads

$$a_n = \frac{1}{T\pi} \int_0^{2\pi T} \frac{\hat{V}}{R} \sin \quad \sin \frac{n\theta}{T} \, d\theta$$

$$= \frac{\hat{V}}{R\pi} \frac{T}{(n^2 - \pi^2)} \quad \sin 2 N \frac{n\pi}{T}$$

$$n = 1,2,3 \ldots\ldots$$

$$a_o = 0$$

$$b_n = \frac{\hat{V}}{RT\pi} \int_0^{2\pi T} \sin \theta \cos \frac{n\theta}{T} \, d\theta$$

$$= \frac{\hat{V}}{R\pi} \frac{T}{(T^2 - n^2)} \quad (1 - \cos 2 N \frac{n\pi}{T})$$

$$n = 1,2,3 \ldots\ldots$$

for $\theta = wt$ the lowest frequency component will be lower than supply frequency and as N/T approaches 1 the harmonic content of the spectrum will increase from almost d.c. to infinity.

The case of a repetative varying current can be analysed as a continuous current and a periodically varying discontinuous current which can be synthesised as before

$$i(\theta) = \frac{\hat{V}_1}{R} f(\sin \theta) \qquad \theta < N$$

$$= \hat{V}_2/R \quad f(\sin \theta) \qquad T \geq \theta > N$$

Considering the case

$$i(\theta) = \frac{\hat{V}_1}{R} \sin \theta \qquad \theta < N$$

$$i(\theta) = \frac{\hat{V}_1}{R} \sin \theta + \frac{\hat{V}_x}{R} \sin \theta \qquad T > \theta \geq N$$

The synthesised Fourier expression becomes

$$i(\theta) = \frac{\hat{V}_1}{R} \sin \theta + a'_o + a'_n \sin \frac{n\theta}{T} + b'_n \cos \frac{n\theta}{T}$$

$$n = 1,2,3 \;.....$$

$$a_n = \frac{\hat{V}_x}{R} \frac{1}{(n^2 - T^2)} \sin 2\pi \frac{Nn}{T}$$

$$b_n = \frac{\hat{V}_x}{R} \frac{T}{(T^2 - n^2)} \left(1 - \cos 2n \frac{N\pi}{T}\right)$$

$$n = 1,2,3 \;.....$$

An example of this is included in Figure 3.

Hence a phase controlled load in which the phase angle is continually vary: as in the case of a static inverter (a.c. to a.c.) will produce subharmonics.

In electrical power systems the source impedance is much smaller than the load impedances. The result is at subharmonic frequencies a flow of subharmonic current generated by non-linear circuit elements back to the supply[7]. Any filte system used, would have to be of lower impedance than the source, or the filter would not be a current sink at subharmonic frequencies.

3. SYSTEM MODELLED

Using a thyristor/load arrangement controlled by appropriate circuits seer Figure 4, it was possible to generate discontinuous or repetative varying currer of any type. The combination could be considered as a source of subharmonics w:

its impedance being proportional to its load. The detection circuit consisted of a high quality filter system[4] (constant K) in parallel with the supply. This acted as a current sink at subharmonic frequencies and offered infinite impedance otherwise. The filter was made using realizable components and terminated by its iterative impedance. This filter offered the advantage of good sensitivity and selectivity.

In order to prevent the flow of subharmonic current back to the supply a series band stop filter tuned for resonance at subharmonic frequency was placed in the supply line. The system can be seen in Figures 4 and 5.

4. RESULTS

A typical example of the generated waveforms corresponding to two cycles on, two off, discontinuous current flow, and examples of the results obtained can be seen from Figures 7, 8, 9, 10. In case (1), Figures 7 and 8 results were obtained using the band stop filter tuned at subharmonic frequency. The results corresponded to within 10% of the calculated value. The band stop filter was removed and detection taken for case (2)(Figures 8 and 9).

In Figures 9 and 10, N = on period, T = total period. The removal of the filter reduced the amplitude of the detected subharmonics considerably. It can be said that significant improvement was affected by using better quality filter systems to isolate the subharmonics and better reactive elements in the filter systems. Hence the accuracy of detection was a function of available quality filters, their sensitivity and selectivity.

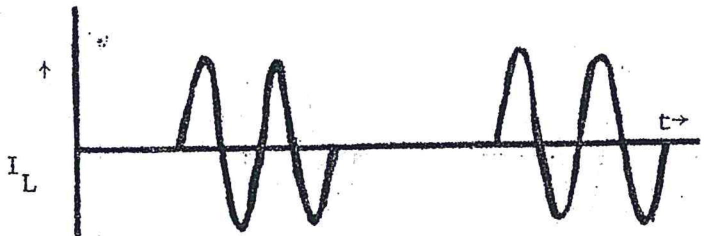
5. CASE STUDY

There are many processes in electrical systems that utilize discontinuous and repetative varying current flow. The most common types are thyristor controlled loads, welding loads, etc.

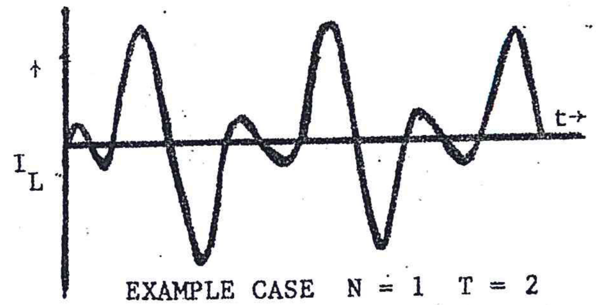
The welding process is generally believed to take a steady current under operating conditions. This type of load was investigated for continuous and intermittent operation. An oscillograph recording of Figure 11 of the arc current shows that for continuous operation the average value of current is quite constant but the instantaneous values change rapidly to give subharmonics that extended from 5 Hz to 60 Hz. The arc current therefore is quite random and depends on several variables such as electrode spacing, arc voltage, metal characteristics, etc.[5]. The non-linear behaviour of the welding transformer in saturation aggravated the random current pulses.

Figure 12 shows the results obtained when the arc current was controlled by thyristors randomly fixed. The level of detected subharmonics was also from 5 Hz to 60 Hz but was more consistant in amplitude and the P.U. values in this case showed a marked increase. This proves that discontinuous current circulation produces objectionable subharmonics.

FIGURE 6



EXAMPLE CASE N = 2 T = 4
2 CYCLES ON 2 CYCLES OFF



EXAMPLE CASE N = 1 T = 2
1 CYCLE ON 1 CYCLE OFF +
CONTINUOUS CURRENT

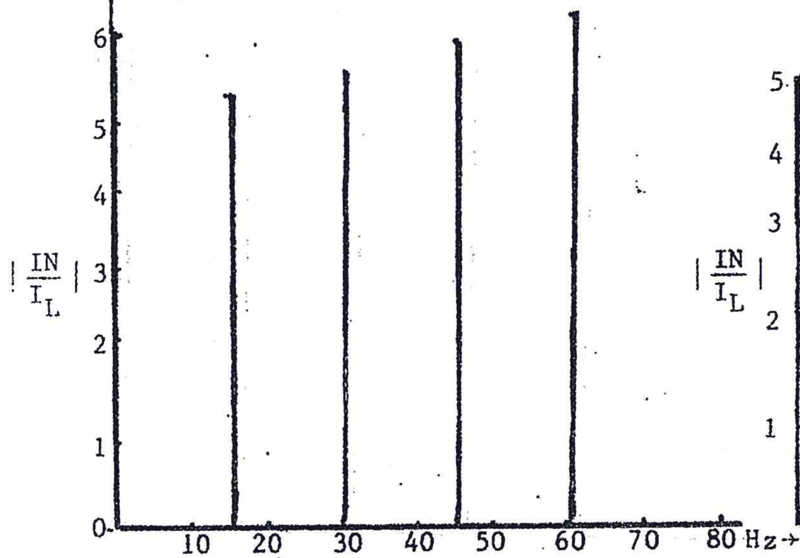


FIGURE 7

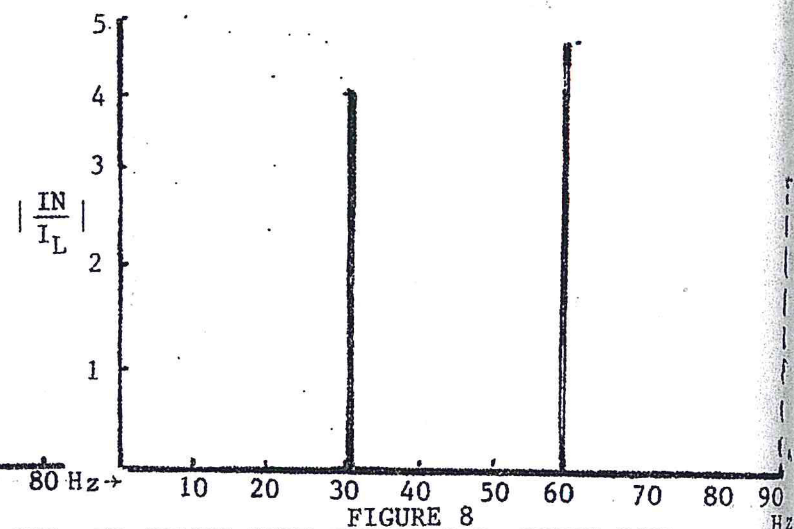


FIGURE 8

DETECTED SUBHARMONICS WITH FILTER USED IN SERIES WITH SUPPLY P.U. VALUE USED

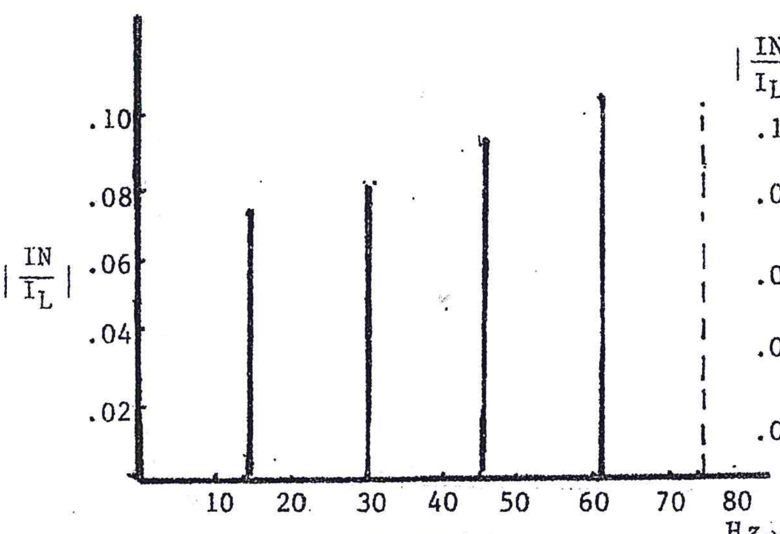


FIGURE 9

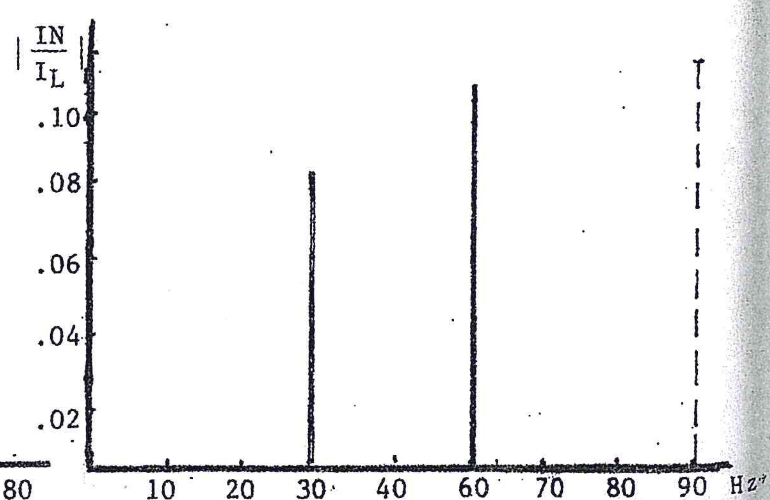


FIGURE 10

DETECTED SUBHARMONIC WITHOUT FILTER IN SERIES WITH SUPPLY PER UNIT VALUES

NOTE DIFFERENT P.U. VALUES USED

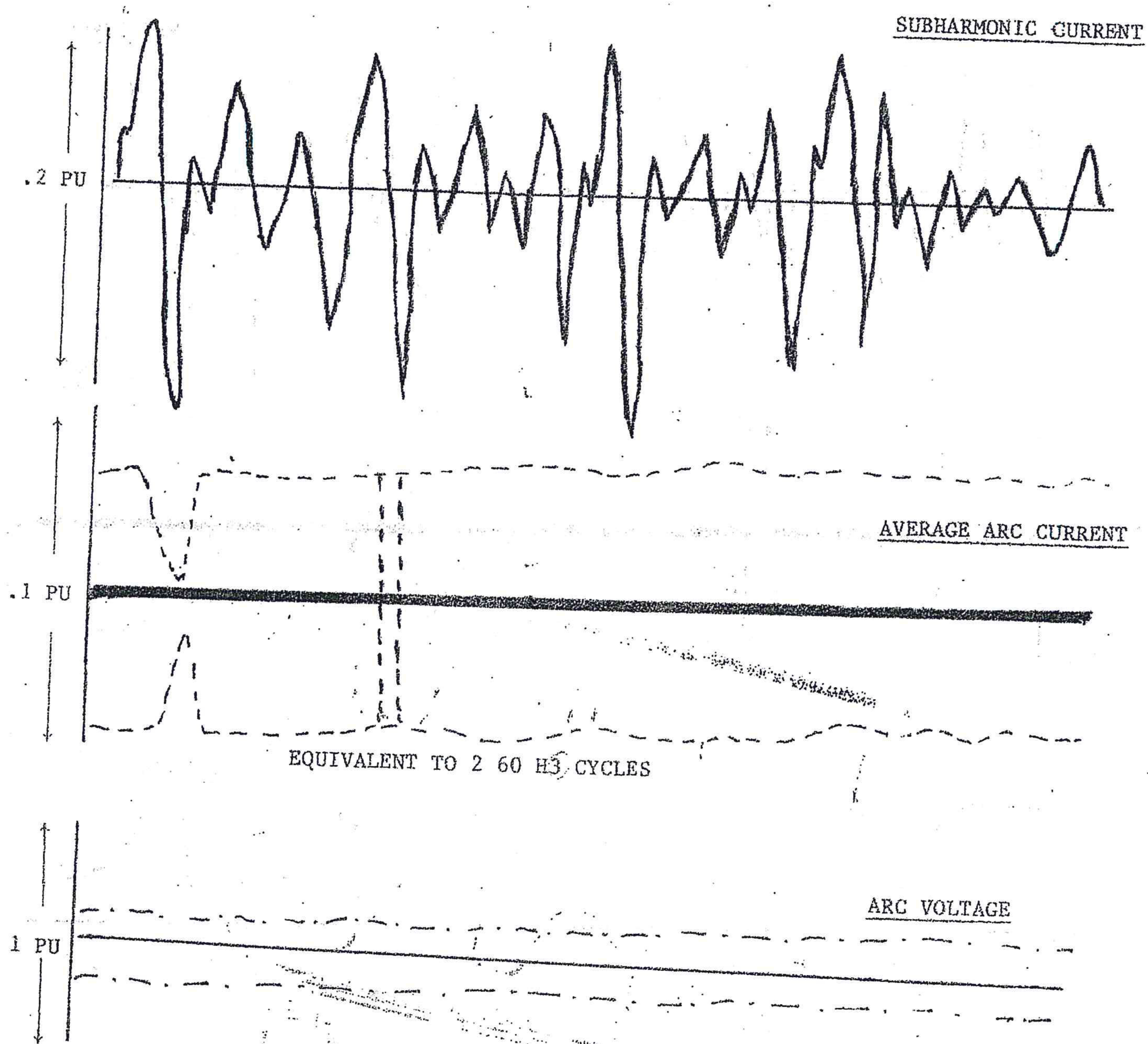

SUBHARMONIC CURRENT
.2 PU
AVERAGE ARC CURRENT
.1 PU
EQUIVALENT TO 2 60 H3 CYCLES
ARC VOLTAGE
1 PU

FIGURE 11

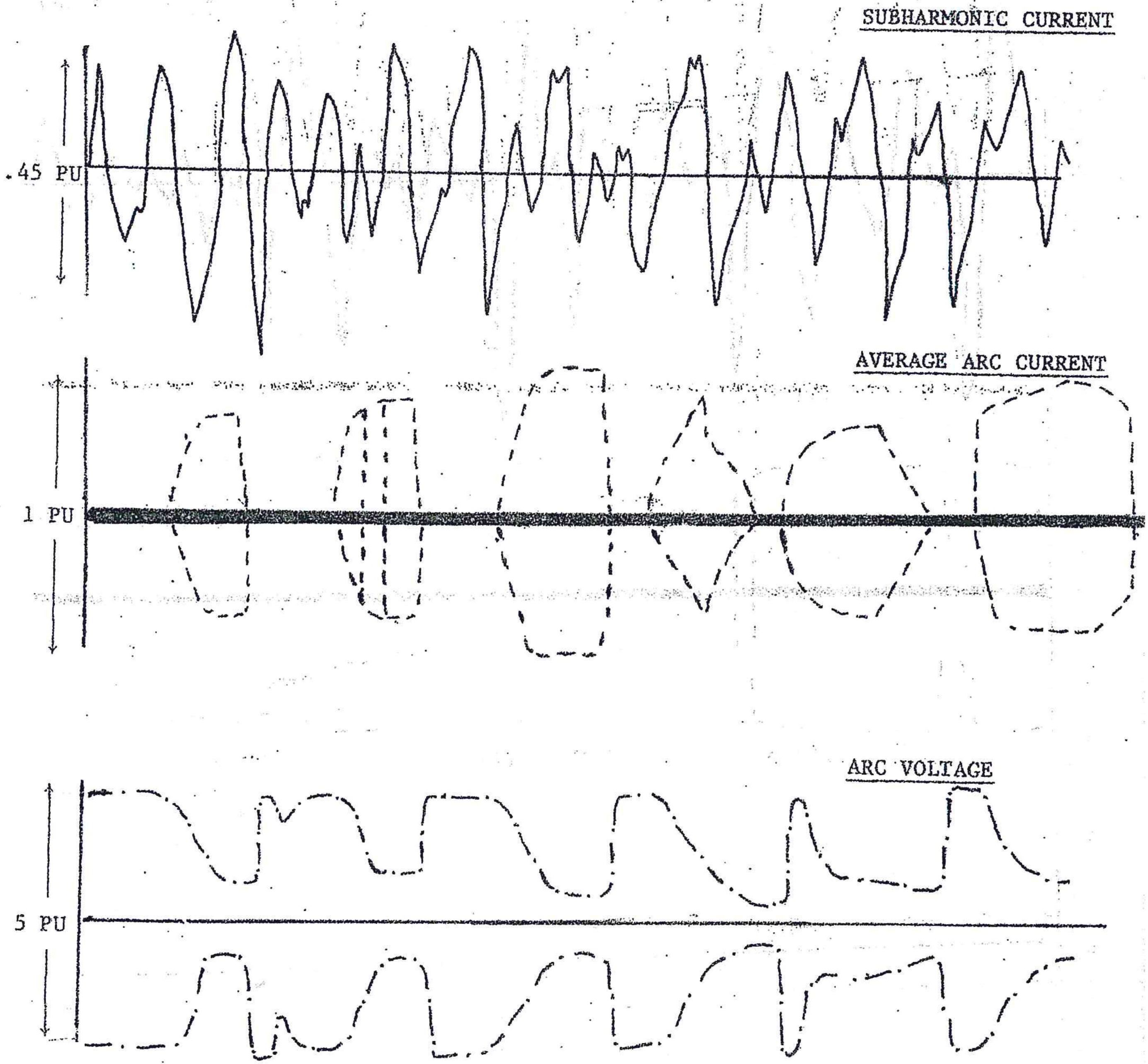

SUBHARMONIC CURRENT
.45 PU
AVERAGE ARC CURRENT
1 PU
ARC VOLTAGE
5 PU

FIGURE 12

6. CONCLUSIONS

The discontinuous current and repetative varying current flows produced by integral cycle and phase angle controlled thyristor currents produce subharmonics that are significant. Because the source impedance in power system circuits is much lower than load impedances, subharmonic currents will not flow through the system, but through the source. This leads to

(a) reduction of effective power transfer

(b) saturation of magnetic devices.

Attempts to isolate the subharmonics using the circuit described lead to

(a) ferroresonance[6]

(b) magnetic saturation

(c) circuit oscillation

at subharmonic frequencies. The circuit described is good for experimental investigation. It is not proposed as an alternative for the elimination of subharmonics. Research in this field should evolve systems capable of eliminating them, since standard filter systems used in power systems are incapable of dealing effectively with subharmonic currents.

NOMENCLATURE

$\hat{V}$ - maximum voltage

$\hat{V}_1$ - maximum amplitude of continuous varying voltage

$\hat{V}_2$ - maximum amplotude of continuous component of varying continuous voltage

$\hat{V}_x$ - maximum amplotude of discontinuous component of varying continuous voltage

R - resistance

θ - angular displacement

α - delay angle before conduction starts with phase angle delayed finity

T - period

N - on time in cycles

M - off time in cycles

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