USING THE ULTRASONIC GONIOMETER TO GENERATE ULTRASONIC SURFACE WAVES FOR MATERIALS EVALUATION (CRITICAL ANGLE REFLECTOMETRY)

by

W. Weston-Bartholomew
Department of Electrical Engineering
University of the West Indies

SUMMARY

This paper discusses one of the uses of the Ultrasonic Goniometer. The goniometer applies the theory of critical angle reflection from a liquid-solid interface. The temperature of the liquid has to be controlled. A theoretical analysis was done to determine the limit control of the temperature.

Further, when measurements of critical angle are done under this controlled condition, the temperature coefficient of elasticity for the surface wave on materials can be calculated.

1. INTRODUCTION

There are many ways of generating ultrasonic surface waves[1]. This paper will discuss the method of using "critical angle reflectometry".

The theory of critical angle reflectometry is as follows:

Water is used as a coupling medium in which dilatational waves are propagated. These waves become incident on the surface of the solid (see fig.1). As in optics Snell's law can be used i.e. $\sin \theta_1 = \left(\frac{C_1}{C_2}\right) \sin \theta_2$. C_1 and C_2 are velocities in the liquid and solid respectively. When the angle of incidence is increased the dilatational wave in the solid becomes critical then the shear wave in the solid becomes critical. Immediately beyond this criticality almost all the energy in the incident dilatational wave is converted into a surface wave which propagates along the surface of the material (see fig. 2). It is known that this perturbation is confined to one wavelength thick. At about 1.2 wavelengths the amplitude of the perturbation is negligible[2,3]. The amplitude of the wave dies away, in both x and z directions.

2. THE ULTRASONIC GONIOMETER (CORNER REFLECTION TECHNIQUE)[4]

The ultrasonic goniometer is the instrument used to generate these surface waves. The main advantages of the goniometer is that specimens of various shapes can be interrogated since there is no physical contact between transducer and specimens. The main disadvantage is that the velocity of the wave in the coupling media varies with temperature. This means that the temperature of the coupling media has to be controlled.

The author considered this in detail because the sensitivity of the system is dependent upon the velocity of the dilatational wave.

3. TEMPERATURE CONSIDERATIONS

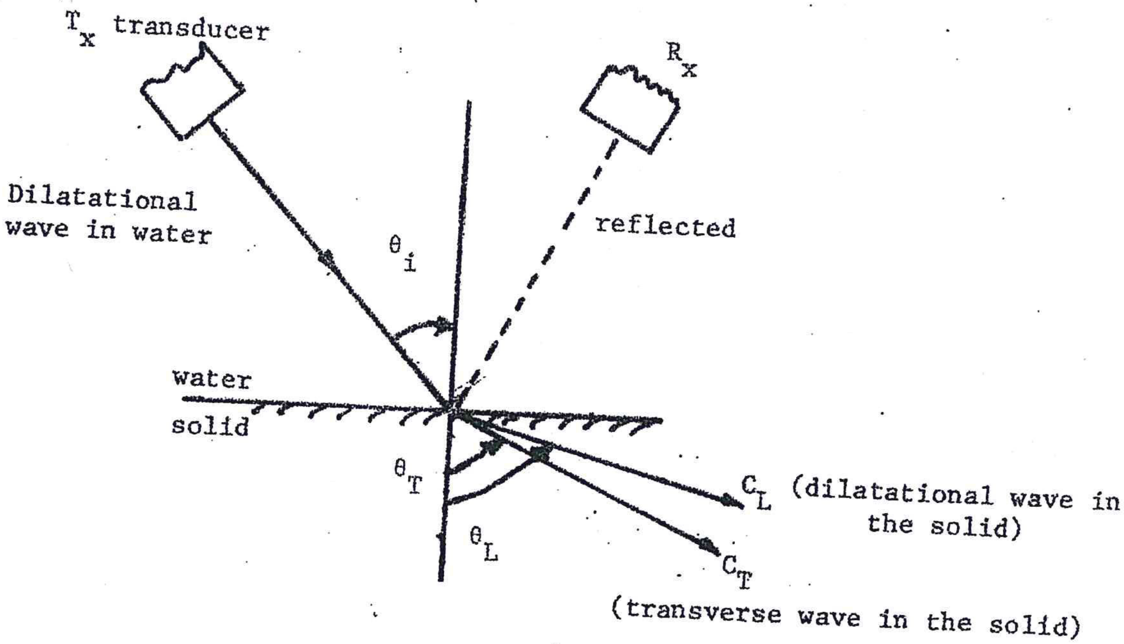
The velocity change is due to the change in density and bulk modulus of the fluid.

The relation between density (ρ) bulk modulus (κ) and velocity (C_1) is

$$C_1 = \sqrt{\left(\frac{\kappa}{\rho}\right)}$$

Hence the change in velocity due to temperature becomes

$$\frac{1}{C_1}\frac{dC_1}{dT} = \frac{1}{2\kappa}\frac{d\kappa}{dT} - \frac{1}{2\rho}\frac{d\rho}{dT}$$



A Liquid-Solid Interface

Figure 1

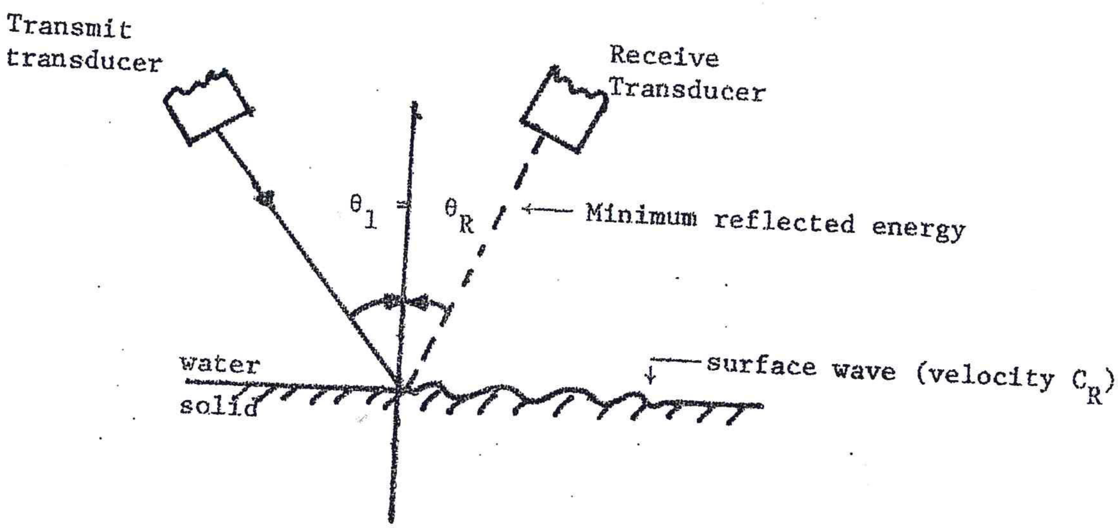


Figure 2 (Showing critical or Rayleigh angle θ_R)

When the coupling medium is water the density change can be neglected and the velocity change is due mainly to bulk modulus changes.[6]

As can be seen from figure 2 at the critical angle $(\theta_2 = \frac{\pi}{2})$

$\sin \theta_1 = \frac{C_1}{C_R}$ where C_R is the velocity of the surface wave on the solid.

Now $$d\theta = \frac{1}{C_R \cos \theta} \left[dC_1 - \frac{C_1}{C_R} dC_R \right]$$

And $$\frac{d\theta}{\theta} = \frac{C_1}{\theta\, C_R \,\mathrm{Cos}\, \theta} \left[\frac{dC_1}{C_1} - \frac{dC_R}{C_R} \right] \qquad (1)$$

After dividing through by $C_1\theta$

Therefore we can write

$$\frac{1}{\theta} \frac{d\theta}{dT} = \frac{C_1}{\theta\, C_R \,\mathrm{Cos}\, \theta} \left[\frac{1}{C_1} \frac{dC_1}{dT} - \frac{1}{C_R} \frac{dC_R}{dT} \right]$$

The surface wave velocity C_R can be written in terms of μ, the shear modulus, ρ, the density and η_R where

$$\eta_R = \frac{0.87 + 1.12\, \nu}{1 + \nu} \qquad \text{(See Bergnann}^5\text{)} \qquad (2)$$

and ν is Poisson's ratio.

$$C_R = \eta_R \sqrt{\frac{\mu}{\rho}}$$

differentiating and multiplying through by $\frac{1}{dT}$ we get

$$\frac{1}{C_R} \frac{dC_R}{dT} = \frac{1}{2} \left[\frac{1}{\mu} \frac{d\mu}{dT} - \frac{1}{\rho} \frac{d\rho}{dT} \right] + \frac{1}{\eta_R} \frac{d\eta_R}{dT} \qquad (3)$$

Substituting eqn. (3) into eqn. (1) we get

$$\frac{1}{\theta} \frac{d\theta}{dT} = \frac{C_1}{\theta\, C_R \,\mathrm{Cos}\, \theta} \left[\frac{1}{C_1} \frac{dC_1}{dT} - \frac{1}{2\mu} \frac{d\mu}{dT} + \frac{1}{2\rho} \frac{d\rho}{dT} - \frac{1}{\eta_R} \frac{d\eta_R}{dT} \right] \qquad (4)$$

Now considering eqn. (2)

$$d\eta_R = \frac{d\nu}{4\,(1+\nu)^2} \qquad \text{and}$$

$$\frac{d\eta_R}{\eta_R} = \frac{d\nu}{4(1+\nu)\,(0.87+1.12\nu)} \tag{5}$$

Substituting equation (5) into equation (4) the equation relating change in ical angle with change in temperature can be written as follows:

$$\frac{1}{\theta}\,\frac{d\theta}{dT} = \frac{C_1}{\theta\, C_R \operatorname{Cos}\theta}\left[\frac{1}{C_1}\,\frac{dC_1}{dT} - \frac{1}{2\mu}\,\frac{d\mu}{dT} + \frac{1}{2\rho}\,\frac{d\rho}{dT} - \frac{1}{4\,(1+\nu)\,(0.87+1.12\nu)}\,\frac{d\nu}{dT}\right] \tag{6}$$

In some cases where the density and Poisson's ratio of the specimen do not over the temperature range considered we can write:

$$\frac{1}{\theta}\,\frac{d\theta}{dT} = \frac{C_1}{\theta\, C_R \operatorname{Cos}\theta}\left[\frac{1}{C_1}\,\frac{dC_1}{dT} - \frac{1}{2\mu}\,\frac{d\mu}{dT}\right] \tag{7}$$

Both $\frac{1}{\mu}\frac{d\theta}{dT}$ and $\frac{1}{C_1}\frac{dC_1}{dT}$ are given in Bradfield[6] for most materials at 20^0C. that θ on the right hand side of equation (7) is in radians. It was found in order to obtain accurate and repeatable results the temperature of the (water) has to be controlled to $\pm 0.01^0$C

If the elastic moduli of an isotropic solid are considered

i.e. $C_{11} = \lambda + 2\mu$ and $C_{44} = \mu$ we can express $\frac{d\eta_R}{\eta_R}$ in

equation (5) as follows:

$$\frac{d\eta_R}{\eta_R} = \frac{C_{44}\left[\frac{1}{C_{11}}\,dC_{11} - \frac{1}{C_{44}}\,dC_{44}\right]}{2C_{11}\left[8.58 - 23.38\left(\frac{C_{44}}{C_{11}}\right) + 15.92\left(\frac{C_{44}}{C_{11}}\right)^2\right]} \tag{8}$$

where $\nu = \dfrac{C_{11} - 2\,C_{44}}{2\,C_{11} - 2\,C_{44}}$ See reference (7)

The author re-arranged equation (1) in order to get an expression for the temperature coefficient of elasticity for the surface wave in any material, i.e.

$$\frac{1}{C_R}\frac{dC_R}{dT} = \frac{-\theta C_R \cos\theta}{C_1}\left[\frac{1}{\theta}\frac{d\theta}{dT}\right] + \frac{1}{C_1}\frac{dC_1}{dT} \qquad (9)$$

Substituting $C_R = C_1/\mathrm{Sin}\ \theta$ into eqn. (9) we get

$$\frac{1}{C_R}\frac{dC_R}{dT} = -\frac{\theta}{\tan\theta}\left[\frac{1}{\theta}\frac{d\theta}{dT}\right] + \frac{1}{C_1}\frac{dC_1}{dT} \qquad (10)$$

Note that θ outside the bracket is in radians.

Therefore $$\frac{1}{C_R}\frac{dC_R}{dT} = \left[\frac{\pi}{180\ \tan\theta}\right]\frac{d\theta}{dT} + \frac{1}{C_1}\frac{dC_1}{dT} \qquad (11)$$

Substituting $C_R = \sqrt{\dfrac{G_{sR}}{\rho}}$ (where G_{sR} is the elastic modulus of the surface wave) into equation (11) we get

$$\frac{1}{C_R}\frac{dC_R}{dT} = \frac{1}{2G_{sR}}\frac{dG_{sR}}{dT} - \frac{1}{2\rho}\frac{d\rho}{dT}$$

Substituting for $\dfrac{1}{C_R}\dfrac{dC_R}{dT}$ from equation (11) we have

$$\frac{1}{2G_{sR}}\frac{dG_{sR}}{dT} = -\left[\frac{\Pi}{180\ \tan\theta}\right]\frac{d\theta}{dT} + \frac{1}{C_1}\frac{dC_1}{dT} - \frac{1}{2\rho}\frac{d\rho}{dT}$$

Therefore

$$\frac{1}{G_{sR}}\frac{dG_{sR}}{dT} = -2\left[\frac{\Pi}{180\ \tan\theta}\right] + \frac{1}{C_1}\frac{dC_1}{dT} - \frac{1}{\rho}\frac{d\rho}{dT}$$

or $\frac{1}{G_{sR}} \frac{dG_{sR}}{dT} \simeq \frac{2}{C_R} \frac{dC_R}{dT}$ when the temperature coefficients

of linear expansion and density are neglected.[6] The author used accurately measured values of critical angles versus temperature, to form Table 1. This table gives the temperature coefficient of elasticity for the surface wave for some materials. The temperature was controlled to $\pm$ 0.01°C.

4. CONCLUSIONS

A corner reflector type goniometer was used. The temperature of the fluid (water) was controlled to $\pm$ 0.01°C. The ultrasonic exciting frequency was 5MHz and pulsed continuous waves were used.

The paper shows that if the temperature of the water controlled, and careful measurements of critical angle done, useful results can be obtained.

Using equation (11) and carefully measured values of θ_R the temperature coefficient of elasticity for the surface wave can be obtained for any material.

REFERENCES

(1) de Klerk J., "Ultrasonic Transducers", Ultrasonics, Jan. 1971, p. 35.

(2) Weston-Bartholomew, W., Ph.D. Thesis, University of Warwick, Coventry, England.

(3) Viktorov, I.A. "Rayleigh and Lamb Waves", Plenum Press, New York (1967).

(4) Rollins, F.R. "Ultrasonic Examination of Liquid-Solid Boundaries Using a Right Angle Reflector Technique", Journal of the Acoustical Society of America, 1968, 44, 431.

(5) Bergmann, L. "Ultraschall und Seine Anwendung and Technik", Edwards, Ann Arbor, Michigan (1964).

(6) Bradfield, G.E. "Notes on Applied Science No. 30", National Physical Laboratory, U.K. (1964).

(7) Goobermann, G.L. "Ultrasonic Theory and Applications", English University Press, London (1968).

Material	Measured		From Bradfield[6]		Calculated	
	θ_R (degrees) (Measured)	$d\theta_R$ (degrees) (Measured)	C_1 (ms^{-1})	dC_1 (ms^{-1})	$\frac{1}{C_R}\frac{dC_R}{dT}$ ($^oC^{-1}$)	$\frac{1}{C_{sR}}\frac{dG_{sR}}{dT} \times 100$ ($^oC^{-1}$)
Titanium	30.7	0.09	1482.66	3.02	-0.000678	-0.1356
Mild Steel	30.5	0.08	"	"	-0.000398	-0.0796
Stainless Steel	31.5	0.1	"	"	-0.00878	-0.1756
Copper	45.24	0.11	"	"	+0.000125	+0.0250
Brass	47.88	0.12	"	"	+0.000134	+0.0268
Aluminium	30.8	0.12	"	"	-0.001541	-0.382

TABLE 1. Table of calculated values of Temperature Coefficient of Elasticity for the Surface Wave in some Materials, from measured values