SHORTER COMMUNICATION

HYDRODYNAMIC LUBRICATION IN HYDROSTATIC EXTRUSION OF A WORK-HARDENING MATERIAL

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SUMMARY

An analysis for hydrodynamic lubrication in hydrostatic extrusion of a work-hardening material is presented. A refined stress-strain relationship, which matches closely the stress-strain characteristics of the material, is used.

1. INTRODUCTION

The presence of high pressure fluid in hydrostatic extrusion could result in hydrodynamic lubrication between the die and the deforming metal. The possibility of hydrodynamic lubrication becomes strong when the speed of extrusion and the viscosity of the fluid used are large. There are various theories of hydrodynamic lubrication for rigid-plastic and work-hardening materials (1,2,3,4). In the analysis presented in this paper the stress-strain relationship of the material is approximated by an expression which closely fits the experimental data.

2. NOMENCLATURE

D_1, D, D_2	Diameters of the billet, deforming metal and the product respectively
B	$e^{\gamma p}$
G	$\gamma\sigma$
H	A constant in $\sigma = \sigma_o + (H + \beta\varepsilon - \sigma_o)(1 - e^{-n\varepsilon})$
h_1, h, h_2, h_3	Film thicknesses at the inlet, in the deforming zone, at the die throat and at the exit to the die land respectively
p_1, p, p_2, p_3	Fluid pressures at the inlet to the deforming zone, in the deforming zone, at the die throat and back pressure respectively

p_A	Fluid pressure in augmented extrusion
p_{FD}	Frictional component of extrusion pressure
Q	Lubricant flow rate/2 Π sin α
R	Extrusion ratio
U_1, U_2, U_3	Billet, deforming zone (surface) and product velocities respectively
v	Velocity of lubricant at (x, y)
α	Die semi-angle
β	A constant in the stress-strain relationship
ε	Effective strain
η_o η	Viscosity of lubricant at atmospheric and at pressure p respectively
γ	Pressure coefficient of viscosity
σ	Yield stress of the deforming material
σ_o	A constant in the stress-strain relationship
σ_A	Billet augmenting stress
σ_p	Product augmenting stress
τ	The shear stress at the deforming material surface

3. ANALYSIS

The lubrication model assumed in the analysis is shown in Figure 1. The stress-strain relationship for a work-hardening material is closely approximated by:

$$\sigma = \sigma_o + (H + \beta\varepsilon - \sigma_o)(1 - e^{-n\varepsilon}) \quad \text{.........................} \quad (1)$$

where the strain is

$$\varepsilon = 2 \ln (D_1/D) = 2 \ln (x_1/x) \quad \text{.........................} \quad (2)$$

Therefore,

$$\frac{d\sigma}{dx} = -\frac{2ne^{-n\varepsilon}}{x} (H + \beta\varepsilon - \sigma_o) - \frac{2\beta}{x} (1 - e^{-n\varepsilon})$$

$$= -\frac{2n}{x} X^{2n} (H - 2\beta \ln X - \sigma_o) - \frac{2\beta}{x} (1 - X^{2n}) \quad \text{.........} \quad (3)$$

where,

$$X = \frac{x}{x_1}$$

Considering the equilibrium of an element of the deforming metal as shown in Fig. 1,

$$X\left\{\frac{dp}{dx}-\frac{d\sigma}{dx}\right\} - 2\sigma - 2\tau/\tan\alpha = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For the lubrication model assumed in the deforming zone,

$$v = \frac{1}{2\eta}\frac{dp}{dx}(y^2 - yh) - \frac{U_1}{h}\left\{\frac{x_1}{x}\right\}^2 y \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$Q = -x\left[\frac{h^3}{12\eta}\frac{dp}{dx} + \frac{U_1}{2}\left\{\frac{x_1}{x}\right\}^2 h\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\frac{dp}{dx} = -\frac{12\eta}{h^3}\left[\frac{Q}{x} + \frac{U_1}{2}\left\{\frac{x_1}{x}\right\}^2 h\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and

$$\tau = \frac{\eta U_1}{h}\left\{\frac{x_1}{x}\right\}^2 - \frac{h}{2}\frac{dp}{dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Substituting for τ from equation (8) in equation (4) and following the approximation procedure of Thiruvarudchelvan and Alexander {2}, we obtain

$$x\left\{\frac{dp}{dx} - \frac{d\sigma}{dx}\right\} - 2\sigma - \frac{2\eta_o U_1}{h_1\tan\alpha}\left\{\frac{x_1}{x}\right\}^3 e^{\gamma p} = 0 \quad \ldots\ldots\ldots\ldots (9)$$

Substituting for σ and $\frac{d\sigma}{dx}$ from equations (1) and (3) in the above equation,

$$x\frac{dp}{dx} + 2X^{2n}\{(H-\sigma_o)(n+1) - \beta - 2\beta(n+1)\ln X\}$$
$$+ 4\beta\ln X + 2(\beta - H) = \frac{2\eta_o U_1\gamma}{h_1\tan\alpha}\cdot\frac{e^{\gamma p}}{X^3} \quad \ldots\ldots\ldots (10)$$

Since $B = e^{\gamma p}$ and $X = \frac{x}{x}$, equation (10) becomes

$$\frac{dB}{dX} - 2\gamma B\left[X^{2n-1}.\{(n+1)(H - 2\beta\ln X - \sigma_o) - \beta\}\right.$$
$$\left.+ \frac{2\beta\ln X}{X} + \frac{\beta - H}{X}\right] = -\frac{2\eta_o U_1\gamma}{h_1\tan\alpha}\frac{1}{X^4} \quad \ldots\ldots (11)$$

Let $$2\gamma\left[(H - \sigma_o)(n + 1) - \beta\right] = A$$

$$2\gamma\ (H - \beta) = C$$

$$\frac{2\ \eta_o U_1 \gamma}{h_1 \tan\ \alpha} = D$$

$$4\gamma\beta(n + 1) = E$$

and $$4\gamma\beta = F$$

Then equation (11) becomes

$$\frac{dB}{dX} - B\left[X^{2n-1}\ (A - E\ \ln\ X) + \frac{F\ \ln\ X}{X} - \frac{C}{X}\right] = -\frac{D}{X^4} \quad \ldots\ldots (12)$$

Let $$\frac{A}{2n} + \frac{E}{4n^2} = L \quad \text{and} \quad \frac{E}{2n} = M$$

Then the solution of the differential equation (12) is

$$BX^{MX^{2n}+c}\ .\ e^{-LX^{2n} - \frac{F}{2}(\ln\ X)^2}$$

$$= -D\int_{\frac{1}{\sqrt{R}}}^{1} e^{-\ LX^{2n} - \frac{F}{2}(\ln\ X)^2}\ .\ X^{MX^{2n} + C - 4}\ dX$$

$$= -\ D.S\ \text{(say)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Substituting the boundary condition, at $X = 1,\ p = p_A + \sigma_A + \sigma_o$ and rearranging equation (13) we obtain

$$(p_A + \sigma_A - p_3 + \sigma_p) = \Big[\sigma_o\ \ln\ R + (H - \sigma_o)\left(\ln\ R + \frac{R^{-n}-1}{n}\right)$$

$$+ \frac{\beta}{n^2 R^n} + \frac{\beta\ \ln\ R}{R^n} + \frac{\beta}{2}(\ln\ R)^2 - \frac{\beta}{n^2}\Big]$$

$$+ p_{FD} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where

$$p_{FD} = \frac{1}{\gamma} \ln \left[1 + D.S.e^{\gamma(p_A + \sigma_A + \sigma_o)} + L \right] \quad \ldots\ldots\ldots\ldots\ldots (15)$$

4. DISCUSSION

The integral S in equation (13) is difficult to integrate analytically. However, this may be done by numerical methods. Once S and the constants defined in the text are evaluated, the driving stress given by equation (14) can be determined. Also, the die pressure, shear stress, the film thickness and the frictional contribution to the extrusion pressure can be determined. Since a more accurate stress-strain relationship is used in the analysis the quantities predicted by this analysis will be more accurate than those predicted earlier (2, 3).

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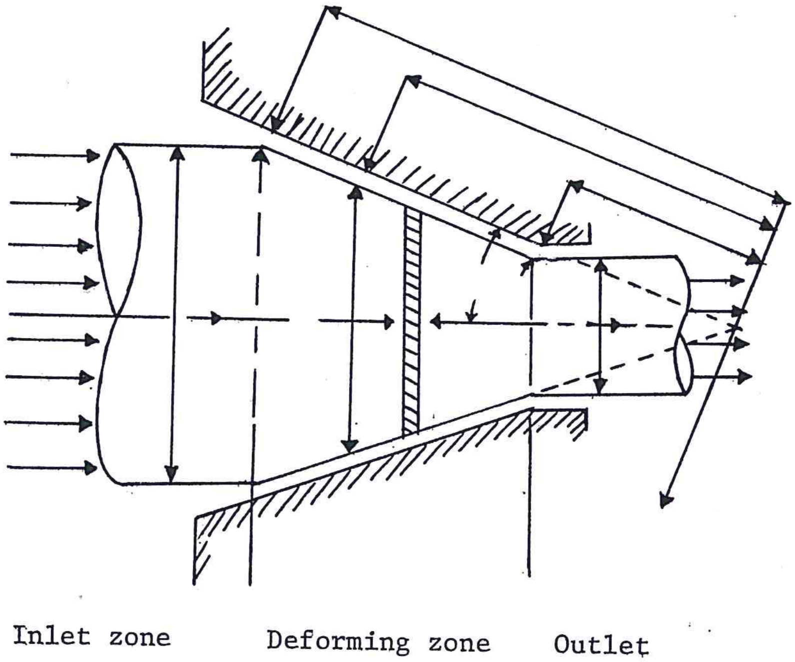

FIG.1

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These will describe original contributions to the advancement of engineering science.

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The submitted manuscript should be typed with double line spacing and a left hand margin. The whole text - apart from the Abstract - should preferably be sectionalised and numbered using the decimal system. Diagrams, tables and equations should be numbered in sequence.

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Brief titles are desirable, i.e. not more than 10 words.

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This should be as brief as possible (normally not more than 150 words) and should summarize the conclusions of the paper. It should be self-explanatory and not require reference to the paper itself.

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1. BARKER, J.J. "Heat Transfer in Fluidised Beds", Ind. Eng. Chem. 1965, 57(5), 33-39.

2. MORRIS, J.E. and GEWARTOWSKI, J.W. "A 1W 6GHz IMPATT Amplifier for Short Haul Radio Applications", Proceedings of the IEEE International Conference on Communication, Washington D.C., 1973, Vol. 1, pp. 8-27.

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4. CUTTERIDGE, O.P.D. "Computer Synthesis of Lumped Linear Networks of Arbitrary Structure" in SKWIRZYNSKI, J.K. and SCANLON, J.O. (Eds.): Network and Signal Theory (Peter Peregrinus, 1973), pp. 105-111.

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