AN EMPIRICAL FORMULA FOR DETERMINING THE EDGE CAPACITANCE OF A RECTANGULAR PARALLEL PLATE CAPACITOR

by

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SUMMARY

An empirical formula for determining the edge or fringing capacitance of a rectangular parallel plate capacitor system containing a dielectric between the plates is found for the case where the dielectric does not extend beyond the edges of the plates. The experiments were carried out using Alumina and polyguide dielectrics of different thicknesses. The results indicate that for the dielectrics and thicknesses used, the effect of the dielectric was negligible in determining this edge capacitance.

1. INTRODUCTION

The dielectric constant of a material can be found by using a parallel plate system and determining the capacitance of the system with the material between the plates. However, because of the fringing field along the edge of the plates, an edge correction is often required, as the equation for determining the dielectric constant does not take into account the effect of this fringing field.

2. BACKGROUND

From electromagnetic theory the capacitance C_T of a parallel plate system, as shown in Figure 1, is given by:-

$$C_T = \frac{A \varepsilon_o \varepsilon_R}{h} \qquad (1)$$

where A is the area of one of the plates.

h is the distance between the plates.

ε_o is the permittivity of free space.

and ε_R is the dielectric constant of the material.

This equation assumes that the fringing field does not extend beyond the edges of the plates.

In practice, the electric field will extend beyond the edges of the plates and it is this fringing field which creates an edge capacitance. Although methods are available for eliminating this field (e.g. when guard rings are used), it is not always convenient or possible to use these methods, so that a method involving an edge correction is sometimes necessary. Such a case arose, when the author had to determine the dielectric constant of Alumina which was metallized on both sides, and on which part of one side would eventually be etched away to form microstrip transmission lines.

Theoretical values for determining the edge capacitance have been calculated for specific shapes of parallel plates[1,2,3], but in all these cases free space or a dielectric was assumed which existed both within and outside of the plates, and in some cases involved complicated formulae. No formula could be found for the case of parallel plate system with dielectric between the plates and which did not extend beyond the edges of the plates.

An empirical formula was therefore determined which allowed the fringing field capacitance of a rectangular parallel plate system to be found for such a case.

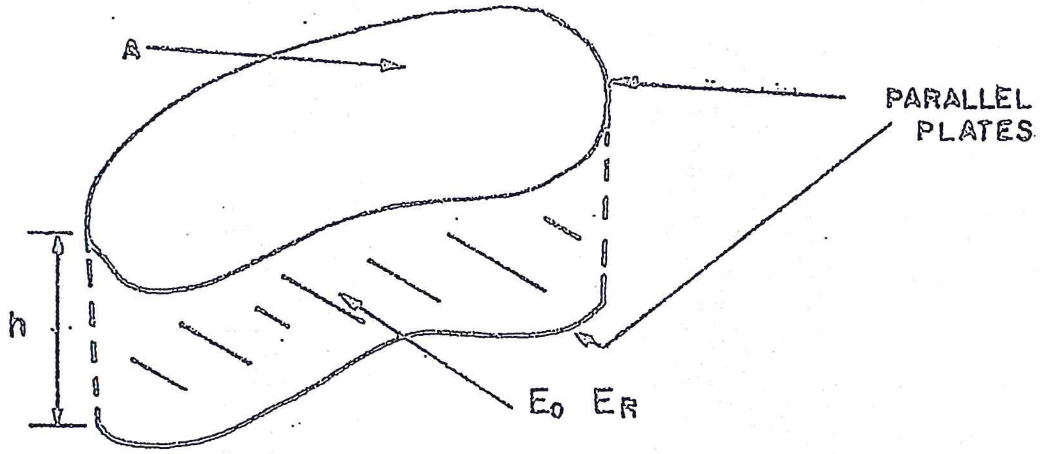


FIGURE 1

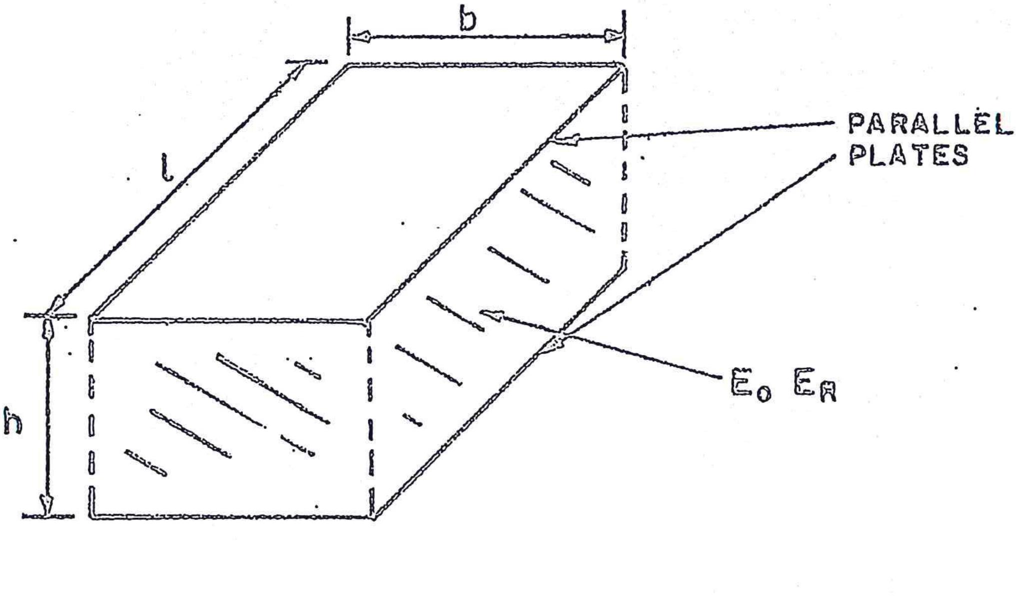


FIGURE 2

3. DETERMINATION OF EDGE CAPACITANCE

If C_M is the measured capacitance and C_E is the edge capacitance for a parallel plate system as shown in Figure 2, then

$$C_M = C_T + C_E \tag{2}$$

where C_T is defined in equation (1).

The possible parameters on which C_E could depend are h, ε_R, ε_o and P, the perimeter around the system where fringing occurs. In this analysis the dependence of C_E on P is considered to be linear. It is possible that at the corners, where two sides meet, this might affect such a linear relationship, but Scott and Curtis[4] found in their experiments, that the corners had a negligible effect on the edge capacitance.

Equation (2) can then be rewritten as:

$$C_M = \frac{A\ \varepsilon_o\ \varepsilon_R}{h} + PK_E \tag{3}$$

where $A = \ell \times b$

$P = 2(\ell + b)$

$PK_E = C_E$

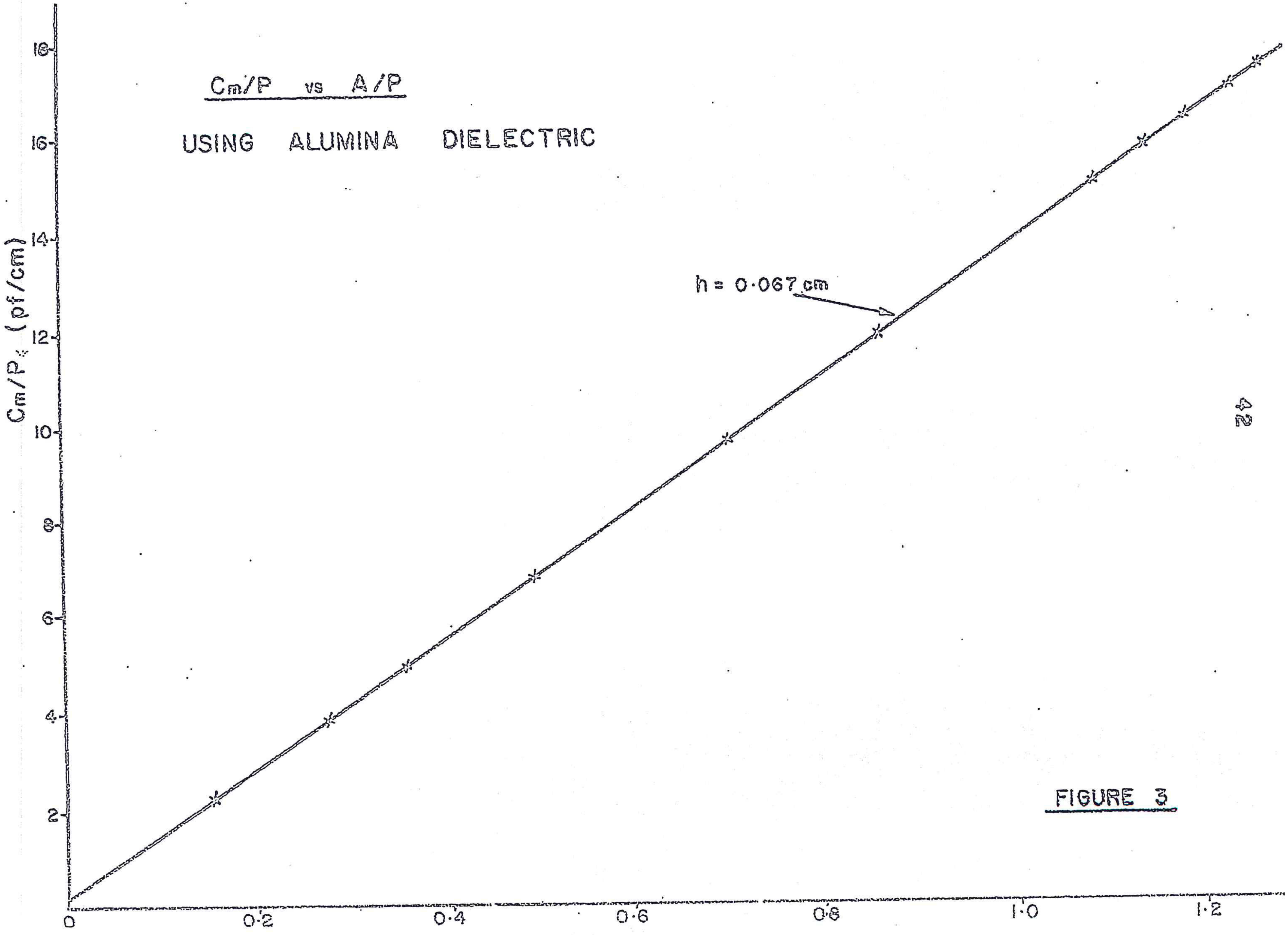
i.e. $$\frac{C_M}{P} = \frac{A}{P}\ \frac{\varepsilon_o\ \varepsilon_R}{h} + K_E \tag{4}$$

By varying ℓ and/or b and measuring the corresponding values of C_M, a plot of $\frac{C_M}{P}$ vs $\frac{A}{P}$ should give a straight line of gradient $\frac{\varepsilon_o\ \varepsilon_R}{h}$ and intercept K_E.

This was done for the following dielectrics:-

a) Alumina	($h = .067$ cm)
b) Polyguide	($h = .161$ cm and $.325$ cm)

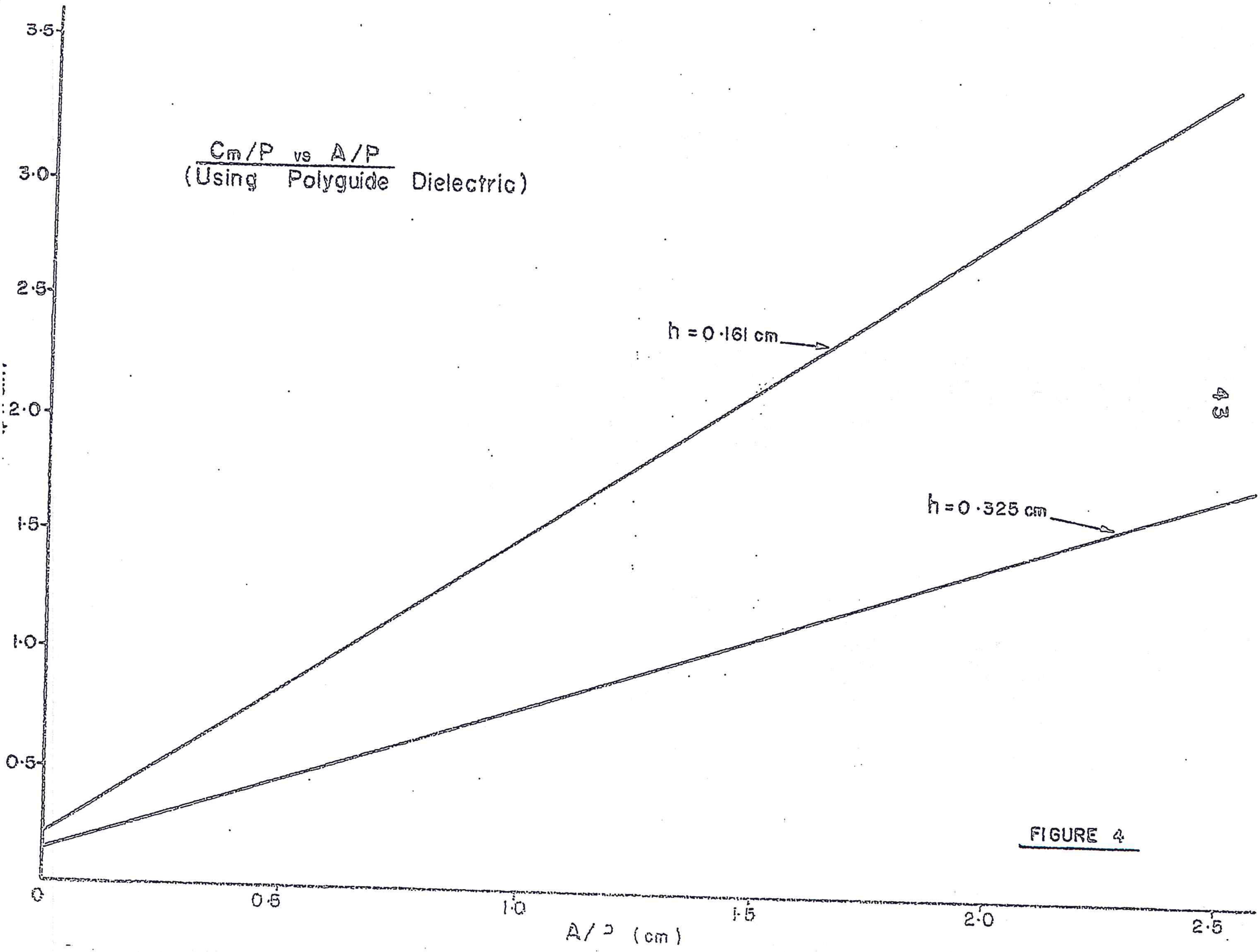
and the corresponding graphs of $\frac{C_M}{P}$ vs $\frac{A}{P}$ are shown in Figures 3 and 4, and K_E was found for each case. As there were only two types of dielectrics used, the dependence of K_E on ε_R could not be determined. However, intuitively, one might expect that this dependence would be very small or negligible by consideration of the fringing field at the edge of the substrate as shown in Figure 5.



Cm/P vs A/P

USING ALUMINA DIELECTRIC

FIGURE 3



C_m/P vs A/P
(Using Polyguide Dielectric)

FIGURE 4

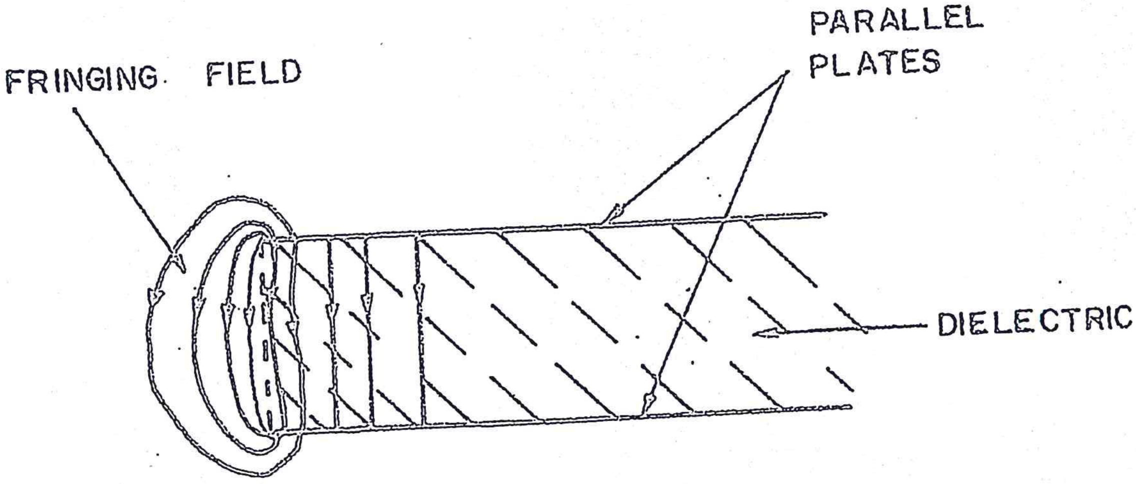


FIGURE 5

Practically all the fringing field in the air region would begin and end on the parallel plates, without passing through the dielectric. Those fields starting within the dielectric would tend to be confined to that region because of the higher permittivity of the dielectric.

On this assumption, that the fringing field would not depend on ε_R, a graph of K_E vs h was plotted and the relationship was found to be logarithmic as shown in Fig. 6.

Although there were only three points, these were found to lie on a straight line given by the equation:-

$$K_E = .08872 \text{ Ln } \left(\frac{1.6}{h}\right) \text{ pF/cm} \qquad (5)$$

where h is in centimetres.

This result shows that the gradient is virtually equal to ε_o and equation (5) can be rewritten as:-

$$K_E = \varepsilon_o \text{ Ln } \left(\frac{1.6}{h}\right) \text{ pF/cm} \qquad (6)$$

The fact that K_E is linearly dependent on ε_o supports the assumption made above that the fringing field lies in the air region and the fact that the points lie on a straight line, despite the wide difference in dielectric constants used, seems to confirm this point.

4. EXPERIMENTAL PROCEDURE

The experiment was carried out at a frequency of $1KH_z$ using a calibrated capacitance bridge, with the capacitive effects of the leads taken into account. The straight line fits for the points in Figures 3, 4 and 6 were obtained with a computer using the method of least squares. In all cases the coefficients of correlation were greater than 0.9996.

5. DISCUSSION

In Figure 6, a straight line equation was obtained which fitted the points for the graph. However, it must be remembered that this straight line is valid only for the ranges covered in the experiment. From an intuitive point of view, one would expect that this edge capacitance would gradually decrease to zero as h increased, and so the line in Figure 6 might follow a path as shown by the dashed line, where it is asymptotic with the horizontal axis.

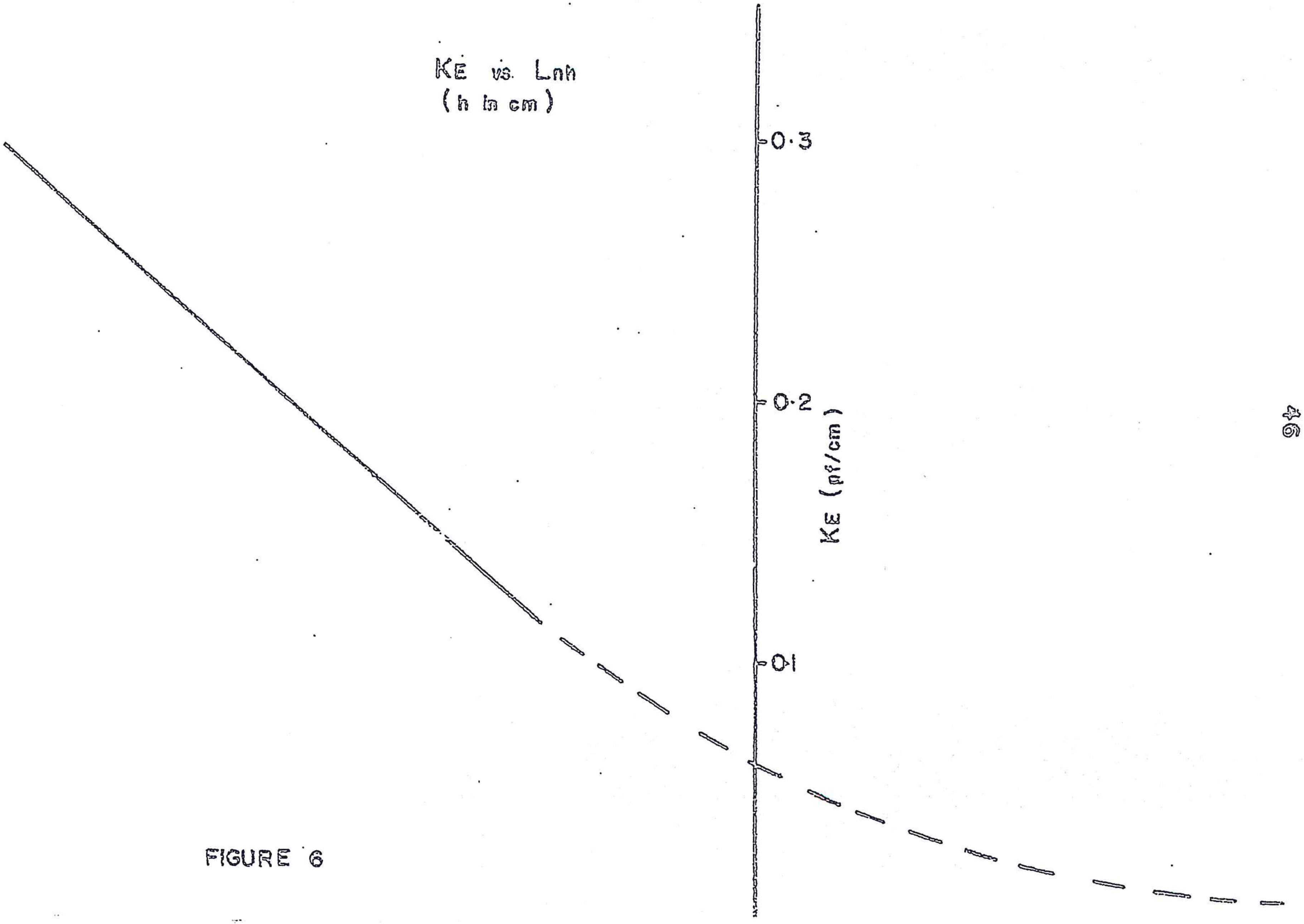


FIGURE 6

6. CONCLUSION

An empirical formula (Equation 6) is found for determining the edge capacitance for a rectangular parallel plate capacitor with a dielectric between the plates which did not extend beyond the edges of the plates. The formula was obtained for dielectric constants from 2.3 to 10.5 and for distances between the plates from 0.067 cm to 0.325 cm.

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