ANALYTICAL STUDY OF HEAT TRANSFER AND MELTING FRONT IN A SPHERICAL PCM CAPSULE

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ABSTRACT

An analytical method has been proposed considering uni-directional heat conduction to estimate the temperature field, motion of the phase-front and heat flux during heat storage in the Phase Change Material contained in a spherical capsule. The analysis is based on the time-dependent surface temperature variation as boundary condition, and can be used with reasonable accuracy.

1.0 INTRODUCTION

Latent heat thermal energy storage (TES) by Solar Energy may be achieved mainly by the capsule-type and the shell and tube-type units (1-3). For heat storage purpose, phase change material (PCM) is contained within the capsules, whereas the PCM is contained within a shell in the latter type.

Both spherical and horizontal capsules are used in a latent heat TES system. The advantage of spherical-shaped capsule is that of its larger surface area per unit volume and the tortuous path through the bed which accelerate heat exchange.

The PCM melts during the charging mode by taking heat from the hot environmental fluid. Exact solution for heat transfer calculation for a latent heat TES capsule is not available, as the melting or the solidification is associated by the non-linear boundary condition at the moving solid-liquid interface and the transient behaviour of heat exchange. The two representative methods are those by Riley (4) and Boley (5). Riley (4) applied a perturbation technique for the inward solidification of spheres. The sphere is considered to be initially molten and at the fusion temperature, when the outside surface is suddenly cooled. Basic series solution are first derived and these are then followed by a two-region analysis. Boley (5) presented a short-time analytical solution which is explicitly constructed by means of the embedding technique.

In this paper, the author proposes an analytical solution to formulate uni-directional heat conduction during melting in spherical co-ordinate system using time-dependent surface temperature variation from experimental data as boundary condition.

2.0 ANALYSIS

2.1 Physical Model and Assumptions

A spherical capsule of radius R containing solid PCM is considered (**Figure 1**) to be at an initial temperature T_{in}. The surface temperature T_w gradually increases to attain finally a fluid environmental temperature T_e($>T_m$, where T_m is melting point of the PCM). At any time $t>0$, the capsule has both solid and liquid phases.

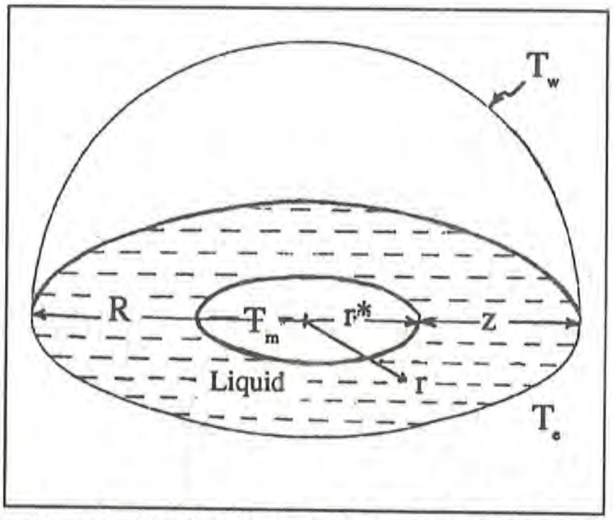

Figure 1: Physical Model and Co-ordinate System

2.2 Mathematical Formulation

The one-dimensional heat conduction problem with melting can be formulated in the spherical co-ordinate system as given below:

$$\frac{\partial^2 T_1}{\partial r^2} + \frac{2}{r}\frac{\partial T_1}{\partial r} = \frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} (r^* < r < R, t > 0) \qquad (1)$$

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Boundary conditions

$$T_s = T_1 = T_m \ (r = r^*, t > 0) \qquad (2)$$

The heat balance of
Heat by Conduction = Latent Heat Supplied
gives

$$-\lambda_1 \frac{\partial T_1}{\partial r} = L\rho \frac{dr^*}{dt} (r = r^*, t > 0) \qquad (3)$$

$$T_1 = T_w \ (r = R, t > 0) \qquad (4)$$

Initial conditions

$$r^* = R \ (t = 0) \qquad (5)$$

$$T_s = T_{in} \ (0 < r < R, \ t = 0) \qquad (6)$$

Solution to equation (1) is given by

$$T_e = \frac{-(T_w - T_m)YR + T_m(1-Y)r + (T_w - T_m)r}{(1-Y)r} \qquad (7)$$

where Y is the dimensionless radius (r*/R).

Equation (3) can be expressed as

$$-\lambda_1 (T_w - T_m)dt = L\rho(R - r^*)(r^*/R)dr$$

By integration,

$$\lambda_1 \int_o^t (T_w - T_m)dt = L\rho \int_R^{r^*} (R - r^*)(r^*/R)dr^*$$

$$\text{or,} \quad \lambda_1 \left[\Delta t \sum_{i=1}^{N} \left\{ (T_w)_t = t_i = T_m \right\} \right]$$

$$= L\rho R^2 \left[(1/2)(1 - Y^2) - (1/3)(1 - Y^3) \right] \qquad (8)$$

where i = 1 to N are the numbers when T_w is considered at time interval Δt.

Equation (8) is the expression for the motion of the phase-front.

Now, heat flux q may be obtained as

$$q = \lambda_1 (4\pi r^2) \frac{\partial T_1}{\partial r}$$

$$\text{or,} \quad q = 4\pi\lambda_1 r^* (T_w - T_m) / R(1 - Y) \qquad (9)$$

And, surface heat flux q" is

$$q'' = q / (4\pi R^2) = \lambda_1 Y (T_w - T_m) / R(1 - Y) \qquad (10)$$

3.0 DISCUSSION AND CONCLUSION

To ensure the validity of the proposed method, the discharging mode when solidification occurs is to be considered. It has been experimentally established that in case of a horizontal cylindrical PCM capsule, the solidification phenomenon follows the law of conduction (2). The same thing must be expected in case of a spherical PCM capsule also. However, experiments may be carried out to verify this. There are two errors concerning the proposed method. One is the assumption of the initial temperature and the other is the assumption of steady state heat transfer. In actual practice, it is not possible to keep the PCM at its melting point. The error in the amount of heat flux due to this assumption is found negligible (below 3%) by calculating with Neumann's solution for semi-infinite flat plate during solidification when the PCM (Napthalene) is kept only a little (1.5°C) above the melting point(6). Again, the assumption of steady state heat transfer is also negligible (below 3%) if calculated by considering semi-infinite flat plate with steady state heat transfer(6). The assumption of steady state heat transfer is acceptable if the variation of surface temperature is not large. However, experiments may be performed to further clarify by comparing the time-dependent surface heat flux variation by the proposed method with the measured values. It is obvious that melting is associated with convection (1, 2). But the purpose of the proposed solution is to clarify the limit where the rate of heat transfer can be explained by conduction only.

The proposed method of solution is very simple. It can be used to predict the heat transfer characteristics of the latent heat TES capsule when there is non-occurrence of convection within the molten region.

4.0 NONMENCLATURE

L	latent heat of phase change, J kg^{-1}
q	heat flux, W
q"	surface heat flux, W m^{-2}
r*	co-ordinate of location, m
r	the radius of boundary, R
t	time, sec

T temperature, °C
Y dimensionless radius, (r/R)
α thermal diffusivity, $m^2 sec^{-1}$
λ thermal conductivity, $W m^{-1} K^{-1}$
ρ density, kg m-3
equ. equation
PCM phase change material
TES thermal energy storage

Subscripts

e environment
i numbers (1 to N) when surface temperature T_w is considered at time interval Δt
in initial
l liquid
s solid
m melting
w surface

5.0 REFERENCES

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