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The Driving Point Impedance Method and Its Application to Operational Amplifier Gain Calculation

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The Driving Point Impedance (DPI) method is used to calculate the voltage gain of a typical operational amplifier circuit. The method simplifies the calculation and shows how the answer can be written by inspection without the need to draw an equivalent circuit.

1. Introduction

The driving point impedance method provides an easy way of calculating voltages in linear analog electronic circuits. This method is based on representing transistors by the approximate equivalent circuit [1,2]. In this paper, the method is explained and then applied to the calculation of the output voltage of a typical operational amplifier circuit, which shows an interconnection of many bipolar transistors.

2. Driving Point Impedance (DPI)

It is assumed that the active devices in the circuits considered are bipolar transistors only, and are represented by the approximate equivalent circuit as shown in **Figure 1**.

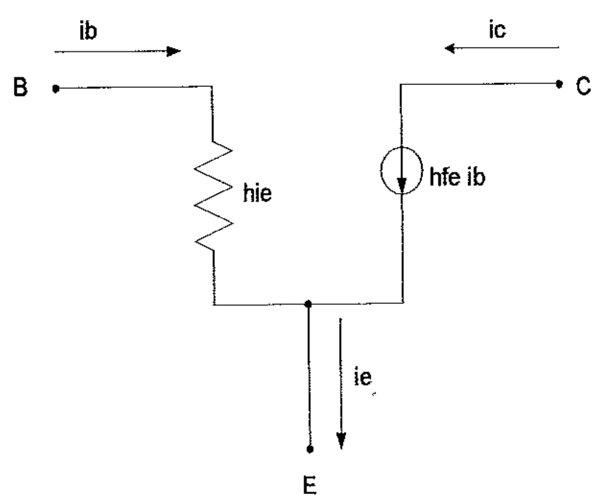

FIGURE 1: *Approximate Transistor Equivalent Circuit*

Definition: The driving point impedance at the emitter, base, and collector terminals of the bipolar transistor are defined, respectively, as follows:

$$(DPI)_{emitter} = \left(\frac{E_i}{I_i}\right)_{emitter} \tag{1}$$

$$(DPI)_{base} = \left(\frac{E_i}{I_i}\right)_{base} \tag{2}$$

$$(DPI)_{collector} = \left(\frac{E_i}{I_i}\right)_{collector} \tag{3}$$

where E_i is a voltage applied to the appropriate terminal and I_i is the resulting current.

$\mathbf{(DPI)_{emitter}}$: The circuit shown in **Figure 2a** is used to obtain the Driving Point Impedance at the emitter. The equivalent circuit is shown in **Figure 2b**.

It can be seen that the emitter - base equation is given by

$$E_i = -i_e R_E - \left(h_{ie} + R_B\right) i_b$$

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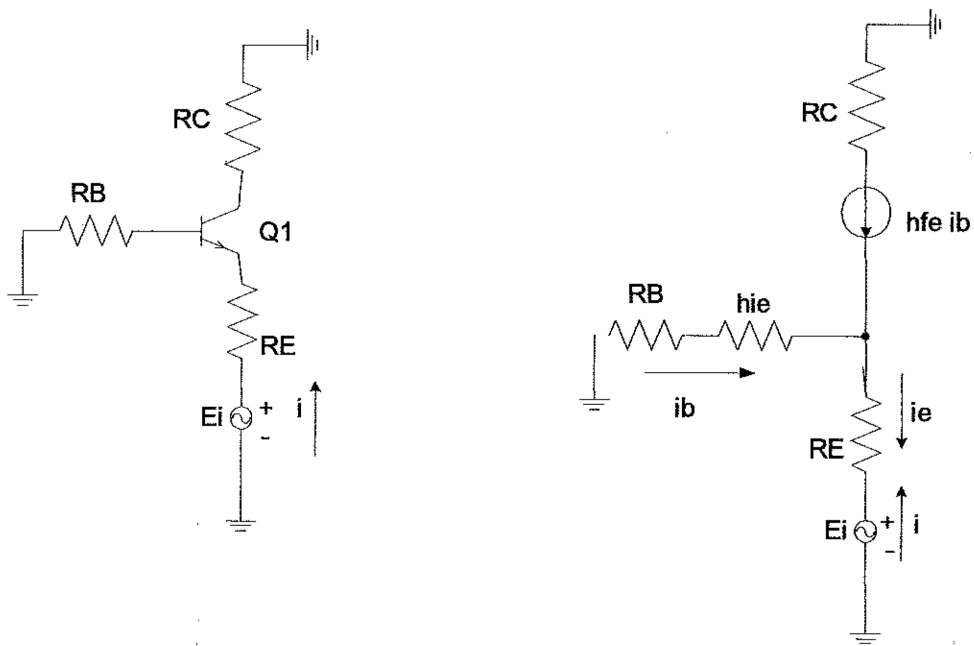


(a) (b)

FIGURE 2: *a): Circuit for calculating $(DPI)_{emitter}$. (b) Equivalent Circuit*

$$= -i_e R_E - \left(\frac{h_{ie} + R_B}{1 + hf_e}\right) i_e \tag{4}$$

where $i_b = \left(\dfrac{i_e}{1 + h_{fe}}\right)$

Since $i = -i_e$, the Driving Point Impedance at the emitter is given by

$$(DPI)_{emitter} = \frac{E_i}{i} = R_E + \frac{h_{ie} + R_B}{1 + h_{fe}} \tag{5}$$

$(DPI)_{base}$: The circuit shown in **Figure 3a** is used to obtain the Driving Point Impedance at the base. The equivalent circuit is shown in **Figure 3b**.

It can be seen that the Driving Point Impedance at the base can be obtained from the loop equation

$$E_1 = \left(R_B + h_{ie}\right) i_b + R_E i_e \tag{6}$$

but $i_e = \left(1 + h_{fe}\right) i_b$. Hence, substituting for i_e, the Driving Point Impedance at the Base is given by

$$(DPI)_{base} = \left(\frac{E_i}{i_b}\right) = R_B + h_{ie} + (1 + h_{fe}) R_E \tag{7}$$

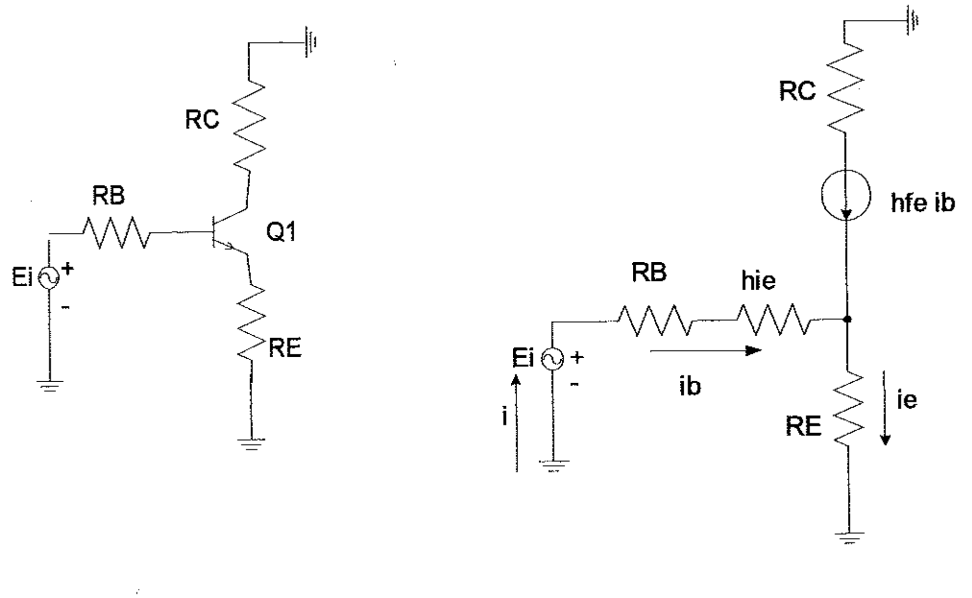
$(DPI)_{collector}$: In this case, the circuit is as shown in **Figure 4**. The voltage E_i cannot excite the emitter - base junction in order to produce the base current on which the collector current depends. Hence, the collector current is zero.

The Collector Driving Point Impedance is thus given by

$$(DPI)_{collector} = \left(\frac{E_i}{i_c}\right) = \left(\frac{E_i}{zero}\right) = \left(\frac{E_i}{0}\right) = \infty \tag{8}$$

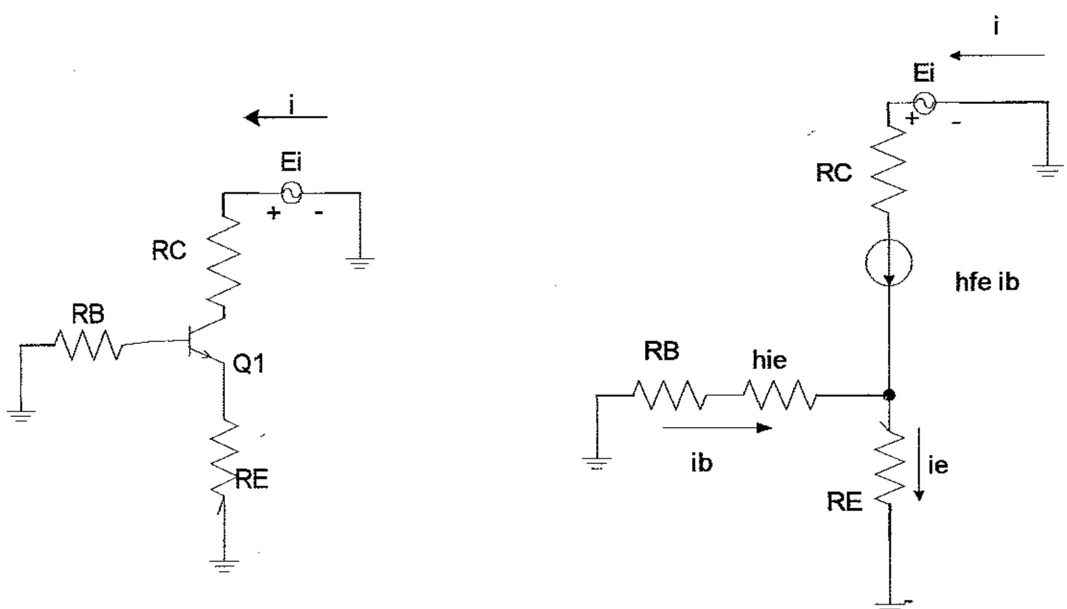
3. The $(DPI)_{out}$ Equation

The $(DPI)_{out}$ equation, which is used for the calculations, is another way of stating Norton's Theorem. It states that the output voltage V_{out} at any terminal in a circuit relative to ground is given by



(a) (b)

FIGURE 3: *a): Circuit for Calculating* $(DPI)_{base}$. *(b) Equivalent Circuit*



(a) (b)

FIGURE 4: *a): Circuit for Calculating* $(DPI)_{collector}$. *(b) Equivalent Circuit*

$$V_{out} = (I_{sc})((DPI)_{out}) \quad (9)$$

Where I_{sc} is the current flowing in a short circuit at the terminal, and $(DPI)_{out}$ is the output impedance at the terminal. The major advantage of the method is that the expressions for I_{sc} and $(DPI)_{out}$ can be written without resort to node or loop analysis, and it is not necessary to draw an equivalent circuit before obtaining these expressions; the two terms can be written by inspection by using the Driving Point Impedance equations together with the voltage and current divider principles. A simple emitter follower circuit is used to illustrate how the Driving Point Impedance method is used, after which the operational amplifier calculations will be carried out.

3.1 Emitter Follower Amplifier:

Consider the emitter follower circuit and the equivalent circuit as shown in **Figure 5**. It is desired to obtain V_{out}.

It can be seen that

$$V_{in} = i_b\left(R_B + h_{ie}\right) + i_e R_E \quad (10)$$

If the emitter current is expressed in terms of the base current, i.e., $i_e = \left(1 + h_{fe}\right)i_b$, the base current is given by

$$i_b = \frac{V_{in}}{R_B + h_{ie} + (1 + h_{fe})R_E} \quad (11)$$

V_{out} is given by

$$V_{out} = i_e R_E$$

$$= (1 + h_{fe})R_E i_b$$

$$= \frac{V_{in} R_E (1 + h_{fe})}{R_B + h_{ie} + (1 + h_{fe})R_E}$$

$$= \frac{V_{in} R_E}{R_E + \left(\frac{R_B h_{ie}}{1 + h_{fe}}\right)} \quad (12)$$

This is obtained after a number writing a number of equations.

The Driving Point Impedance (DPI) method is used to obtain V_{out} as

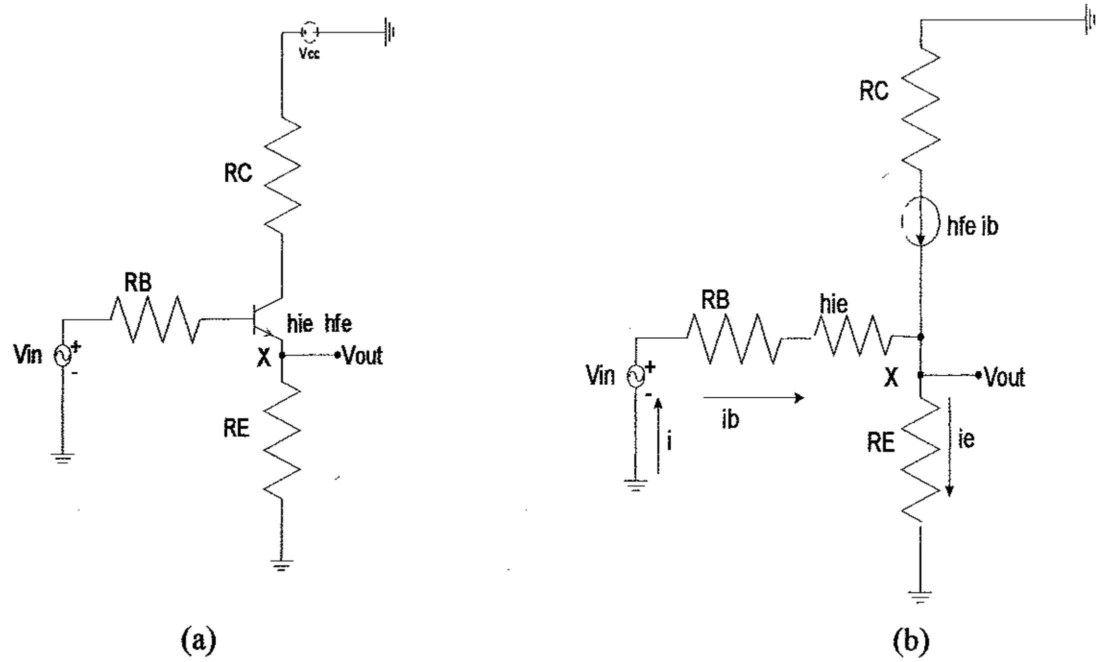


FIGURE 5: *a) Emitter Follower Circuit. b) Equivalent Circuit*

$$V_{out} = (I_{sc})((DPI)_{out}) \qquad (13)$$

at the emitter, where I_{sc} is the short circuit current at the point 'X', and the $(DPI)_{out}$ is the output impedance measured at the same point.

I_{sc} can be written by inspection, and is given by

$$I_{sc} = \frac{V_{in}}{R_B + h_{ie}}(1 + h_{fe}) \qquad (14)$$

The output impedance can also be written by inspection and is given by

$$(DPI)_{out} = R_E \left\| \left(\frac{R_B + h_{ie}}{1 + h_{fe}}\right)\right. \qquad (15)$$

Substitution in equation (13) gives

$$V_{out} = \left\{\frac{V_{in}(1 + h_{fe})}{R_B + h_{ie}}\right\}\left\{R_E \left\| \left(\frac{R_B + h_{ie}}{1 + h_{fe}}\right)\right.\right\}$$

$$= \left\{\frac{V_{in}(1 + h_{fe})}{R_B + h_{ie}}\right\}\left\{R_E\left(\frac{\frac{(R_B + h_{ie})}{1 + h_{fe}}}{R_E + \frac{R_B + h_{ie}}{1 + h_{fe}}}\right)\right\}$$

$$= \frac{V_{in} R_E}{R_E + \frac{R_B + h_{ie}}{1 + h_{fe}}} \qquad (16)$$

The result is the same as equation (12). However, equation (16) is written by inspection. It is sufficient to give the answer as the product of the short circuit current and the output impedance, without carrying out further simplification. That is the elegance of the Driving Point Impedance method.

4. Application of the DPI Method to an Operational Amplifier Circuit

Figure 6 shows a typical operational amplifier circuit; the small-signal circuit is shown in **Figure 7**.

In order to obtain the output voltage V_o, V_{c6} is calculated first, V_{c6} can be written down by inspection as (17).

The equation states that

$$v_{c6} = (i_{sc1} + i_{sc2})(DPI_{out}) \qquad (18)$$

where i_{sc1} is the short circuit current from i_{c1} and $-i_{sc2}$ is the short circuit current from i_{c2}. It can be seen from the circuit that $DPI_{out} = (R_4 + 3r_d)$.

The output voltage V_o is calculated in terms of V_{c6} using the DPI method.

Only one of the output transistors T_7 and T_8 is on at a time since they operate in Class B, mode v_o is calculated by the DPI method. Assume T_7 is on. Thus

$$v_{c6} = \left\{ \left(\frac{i_{c1} R_2}{R_2 + h_{ie6} + (1 + h_{fe6})\left(R_3 \left\| \frac{(h_{ie5} + R_2)}{(1 + h_{fe5})}\right.\right)} \right)(h_{fe6}) - \left(\frac{i_{c2} R_2}{R_2 + h_{ie5} + (1 + h_{fe5})\left(R_3 \left\| \frac{(R_2 + h_{ie5})}{(1 + h_{fe5})}\right.\right)} \right)(1 + h_{fe5})\left(\frac{R_3}{R_3 + \frac{(R_2 + h_{ie6})}{(1 + h_{fe6})}} \right)\left(\frac{1}{1 + h_{fe6}}\right)(h_{fe6}) \right\}\{R_4 + 3r_d\} \qquad (17)$$

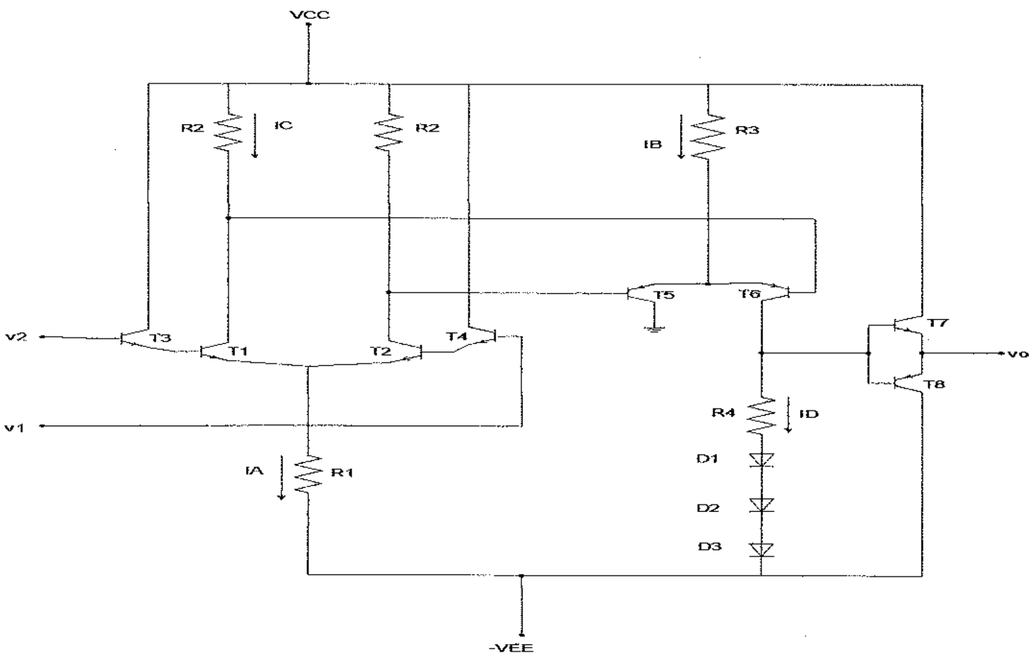


FIGURE 6: *An IC Operational Amplifier [3]*

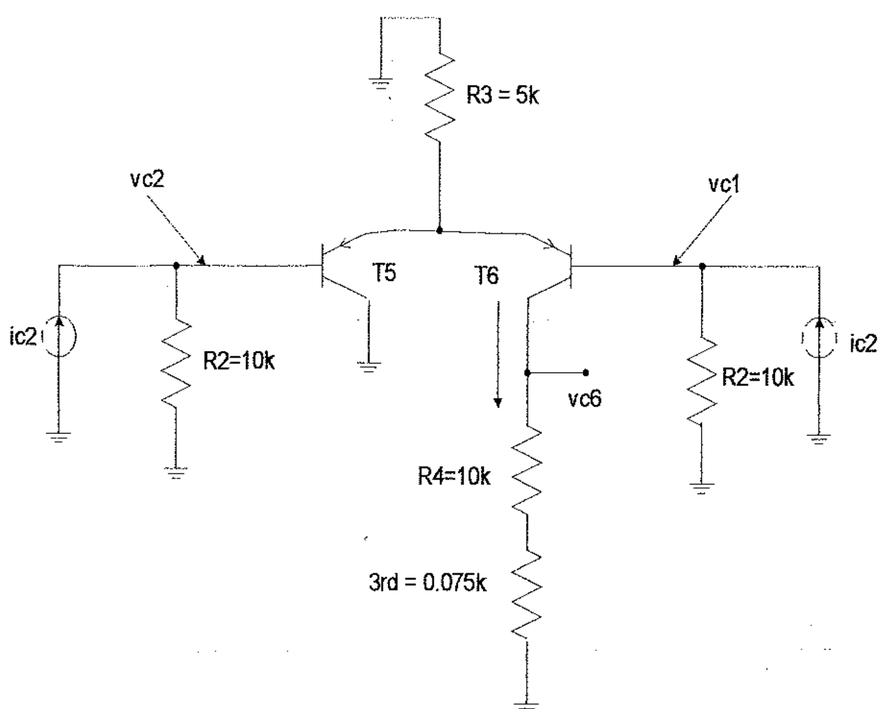


FIGURE 7: *Small-Signal Circuit for Operational Amplifier [3]*

$$v_o = \left(I_{sc}\right)\left(DPI\right)_{out}$$

$$= \left(\frac{v_{c6}}{h_{ie7}}\right)\left(1 + hfe_7\right)\left(\frac{hie_7}{1 + hfe_7}\right)$$

$$= v_{c6}$$

Substitution of numerical values gives

$$v_{c_6} =$$

$$\left\{ ic_i \frac{10\left(10^3\right)}{10\left(10^3\right) + 2.5(10^3) + 100\left(5(10^3) \middle\| \frac{(10(10^3) + 2.5(10^3))}{100}\right)} \right. \quad (99)$$

$$-ic_2\left(\frac{10x10^3}{10(10^3) + 2.5(10^3) + 100\left(5(10^3) \middle\| \frac{(10(10^3) + 2.5(10^3))}{100}\right)}\right) \quad (100)$$

$$\left.\left(\frac{5(10^3)}{5(10^3) + \frac{(10(10^3) + 2.5(10^3))}{100}}\right)\left(\frac{99}{100}\right)\right\}\left\{10(10^3) + 0.075(10^3)\right\}$$

$$= (40.09 i_{c1} - 39.11 i_{c2})(10.075x10^3)$$

Assume $i_{c1} = i_{c2} = i_c$

Then $v_{c6} \cong 10^4 i_c = v_o$

This agrees with calculations in [3].

5. Conclusion

The Driving Point Impedance method has been used to obtain the output voltage of a relatively complex circuit. It has been shown that the equations can be written down by inspection and in a step-by-step fashion as we 'travel' through the circuit from the input to the point at which the output is required. All that are required are the basic DPI equations for the transistor, and the application of the current divider principle at the nodes in the path of the current.

No assumptions have been made in writing the equations, apart from representing the transistors by the approximate equivalent circuit, which is adequate for most practical calculations.

The expressions obtained are rather long and no effort is made to simplify them. The reason is that each term in an equation can be written by inspection.

An example of a typical operational amplifier has been worked out. The result of the numerical calculation for the circuit agrees with that obtained using other methods. The appeal of the DPI method is that it avoids the need to write and solve simultaneous mesh or loop equations for the circuit.

References

[1] Kelly, R. (1980). *Lecture Notes on DPI Techniques.* University of New Mexico.

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[3] Schilling, D.I. and Belov, C. (1989). *Electronic Circuits, Discrete and Integrated.* McGraw Hill. pp. 363-371.