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Confidence Interval Approach to Consistency Ratio Rule in the Applications of Analytic Hierarchy Process

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Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach which employs a pairwise comparison procedure based on a scale of preferences among sets of alternatives. The pairwise comparisons of the alternatives/attributes are entered in a matrix. The consistency of the judgement is revealed by consistency ratio (CR). This paper deals with the CR rule in AHP. Satty's 10% rule provides a judgement in AHP in which, if the CR for a particular judgement matrix is 0.10 or less, it is considered to be acceptable. If the CR is more than 0.10, then the decision-maker may be required to revise the pairwise comparison. This is simply a guideline. This may be a time-consuming procedure, especially in case of group decision-making. An attempt is made in this paper with the help of a case study to demonstrate the combined use of a heuristic approach – confidence interval approach and Saaty's 10% rule to evaluate the consistency of the individuals in the decision-making group.

Notations and Explanations of Statistical Terms used

CR = Consistency ratio of the alternatives estimated by individuals.

x = Average consistency ratio of the alternatives estimated by individuals.

s = Standard deviation of consistency ratio of alternatives estimated by individuals.

n = Number of individuals, i.e., sample size.

α = Significance level (taken as 1% and 5%).

(1- α) = Confidence level (99% and 95%).

$t_{\alpha/2}$ = the value of the t distribution with (n-1) degrees of freedom, leaving an area $\alpha/2$ to the right.

1. Introduction

Rating of alternatives can be done using Analytic Hierarchy Process (AHP) by a single decision-maker or by a group of decision-makers. Generally, in an AHP exercise, a decision-maker or a single group of decision-makers are asked to give pairwise comparison of an attribute at a level, with respect to attribute at a level immediately above. This approach can be used to compute the composite rating of the alternatives to rank the alternatives. In the above system, there is a possibility of a decision-maker committing an error in the pairwise comparison, besides the bias in estimation (**Lockett *et al*, 1986**). If the decision is made by the group, again there may be a possibility of committing an error since the group decision process may have deficiencies such as:

(a) Principle of anonymity may not be followed.

(b) The whole group may be dominated by one decision-maker.

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(c) Key decision-makers in the group may not be available at the time of making the decision, and the decisions may be postponed for later sessions.

Because of the above points coupled with high subjectivism involved in the problem, the ranking of the alternatives can be improved by involving more than one decision-maker individually, and following the principle of anonymity.

2. Consistency Ratio Rule in AHP

In literature, one may find several methods for judging whether or not a set of preference measurements is sufficiently consistent. These consistency thresholds and inconsistency measures are useful tools for the decision-maker to move towards his/her true preferences (**Dadkhah** and **Zahedi 1993**). The decision-maker's consistency is not verified by most of the multi-objective methods (**Olson *et al.* 1986**). It is one of the advantages of the AHP that it is equipped with such a measure. Consistency implies that if A is preferred to B by a ratio 2:1, and if B is preferred to C by a ratio of, say 3:1, then one expects A to be preferred to C by a ratio of 6:1. AHP provides a measure of consistency through CR. In AHP, the decision-maker's rank 'n' alternatives using n(n-1)/2 pairwise comparisons using a 9-point semantic scale. The relative weights of the alternatives are computed from the judgement matrix with the eigen vector method. This method provides a numerical index for consistency. The consistency index of the matrix is defined as $\mathrm{CI}=(\lambda_{\max}-n)/(n-1)$. It measures the difference between the maximum eigen value $(\lambda_{\max})$ of the judgement matrix and the eigen value (n) of a perfectly consistent matrix (**Saaty** and **Wong 1983**). Consistency does not imply that a quality decision has been made. However, all quality decisions must be necessarily consistent. The consistency of decision-makers is often difficult to conceptualise and measure. There is actually no way to determine whether a quality decision has been made until it is implemented (**Madu 1994**). The judgements applied in the AHP models are often based on a personal construct of the decision-maker's perceptions, experience, assessment and biases (**Boucher** and **MacStravic 1991**). Inconsistencies may arise due to differences in participant's backgrounds in case of group decision-making (**Willet** and **Sharda 1991**), and it is observable in human judgement (**Lee** and **Ahn 1991; Weiss** and **Rao 1987**). **Murphy (1993)** demonstrated that the bounded 9-point scale inherently gives results which are outside the accepted consistency standard. The only standard that we can insist upon from the decision-maker is that the judgements applied be logically consistent with each other.

Saaty (1980) has developed a concept of "tolerable inconsistency". He recommends that a consistency ratio of 0.1 or less be considered consistent; a ratio greater than 0.1 shows inconsistencies and it should be definitely below 0.2 (**Saaty 1977**). This is simply a guideline (**Boucher *et al* 1993**). When the number of objects being developed exceeds 7 (±2), the consistency can be expected to be very poor – a theoretical confirmation of **Miller's** (1956) psychological observation (**Saaty 1980**). However, in engineering problems, it is pragmatic to take into account several attributes/alternatives for quality decision-making. **Isli** and **Lockett (1988)** argue that Saaty's definition of a consistency index/ratio provides a crude measure with limited statistical properties. They further argue whether empirical evidence justifies the assumption that judgements are reciprocal in real life situations. **Karapetrovic** and **Rosenbloom (1999)** argue that there are many situations where the decision-maker has been reasonable, logical and non-random in making the pairwise comparison and yet will fail the consistency check. They have recommended a quality control approach for the consistency test.

In order to get the consistent matrices (whenever CR > 0.1) on the basis of the expert's judgement, the judgements leading to this evaluation should be reconsidered and quality of judgements should be improved, perhaps by revising the manner in which questions are asked in making pairwise comparisons. If this should fail to improve consistency, then it is likely that the problem should be more accurately structured, i.e., it is advisable to group similar elements under more meaningful criteria. Obviously, these methods are very time-consuming, more so in the case of group decision-making and especially in cases where Delphi method is combined with AHP. In such situations, the combination of the heuristic confidence interval approach with **Saaty's** 10% ratio rule may lead to a logically consistent approach.

The confidence interval approach proposed identifies the CR that is beyond the permissible level set by the management in terms of confidence limit (**Madu 1994**). The comparison matrix for which the CR is above the confidence limit is only revised, to bring back the consistency within the revised confidence limit. This proposed approach reduces the time of the experts, especially in case of group decision-making and more specifically where Delphi mehtod is integrated with AHP.

3. Methodology of Confidence Limit Chart

The step-by-step procedure of the confidence limit chart approach is given below. The flow chart describing various steps of the confidence interval approach to CR is explained in **Figure 1**. The proposed methodology in this model is to develop a confidence limit chart based on the estimation by a group on an individual basis following the principle of anonymity (**Muralidharan *et al.* 1999**).

Step 1: Identify the Active Participants to be Involved in Decision-making

The various individual members may be drawn from various functions within an organisation. **Radford (1980)** refers to this group as the "active" participants. The term signifies that these people can influence or be influenced by the organisational decision-making process. These people should be selected based on their experience, knowledge of the organisation, etc.

Step 2: Identify the Significant Factors/attributes Involved in Decision-makiug

Brainstorming sessions (**Nair 1996**) or nominal group technique (NGT) (**Sink 1983**) involving various executives drawn from different functions could be used for this purpose. The participants (individuals) must identify those factors/criteria that will enable the organisation to achieve the management's objectives, i.e., to select the best alternative. Several factors may be identified at this stage and a diversity of views may be represented. However, some of the factors may not be significant in achieving the overall objective, and a way to filter out the insignificant factors is required (**Madu** and **Georgantzas 1991**). Since there are bound to be conflicts in what participants perceive as important to achieving the objective, a group leader may resolve these conflicts. The group leader can apply several strategies such as brainstorming, NGT, etc., to control the process and identify and eliminate the insignificant factors. As the number of factors grows, it becomes difficult to comprehend and compare all the choices at one time. The essence of this was to avoid information overload (**Madu *et al.* 1991**).

Step 3: Identify the Alternatives to be Rated.

This can be obtained from data base. Here, also to limit the number of alternatives, the similar procedure as explained in Step 2 may be followed.

Step 4: Use AHP Model to Obtain the Ratings of the Alternatives

The sequential procedure for understanding the general structure of the AHP model is explained below (**Mohanty** and **Deshmukh 2001**).

(i) Create a hierarchy of the factors and alternatives by breaking down the problem into a hierarchy of decision elements.

(ii) Provide judgements about the relative importance of each of the factors and alternatives using the pairwise comparison of decision elements.

(iii) Determine whether the judgements satisfy a "consistency test". If it does not, go back to (ii) and redo the pairwise comparisons.

(iv) Calculate the relative weights of the decision elements.

(v) Aggregate the relative weights to obtain composite scores and hence rankings of the decision alternatives.

Step 5: Establish Coufidence Interval for the Rating done by the Individuals

The confidence interval for the mean rating of CR can be calculated by using the following formula (**Miller *et al.* 1990**)

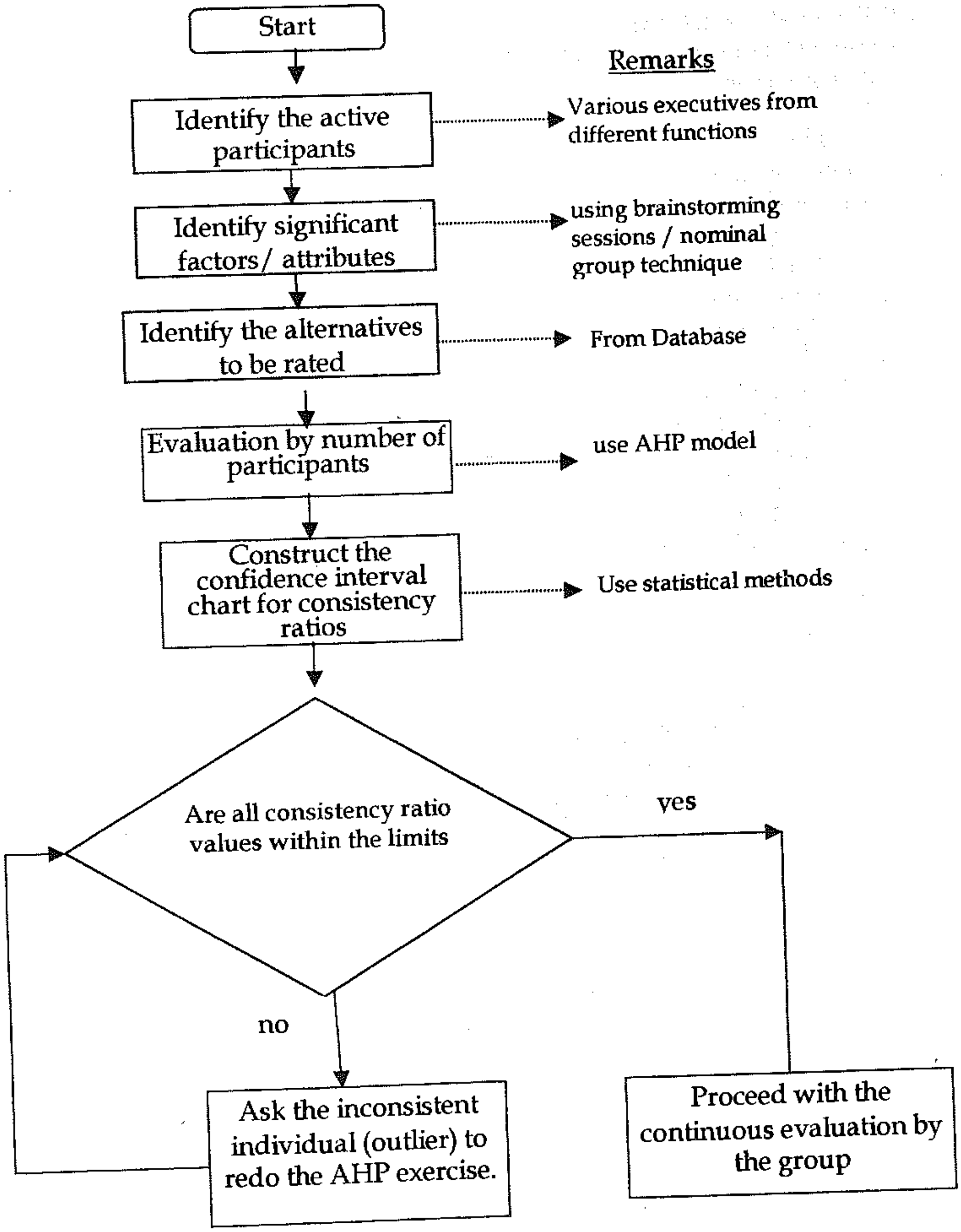

FIGURE 1: *Flow Chart describing Various Steps in Confidence Interval Approach for Consistency Ratio*

$$\mu < x + t_{\alpha/2}.s/\sqrt{n} \qquad ....(1)$$

Step 6: Operate the Confidence Interval Chart
Plot the individuals CR values with respect to established confidence limit and identify the points outside the upper confidence limit.

4. An Illustrative Case

A case study was carried out by a group of six executives in the engineering workshop and were asked to evaluate five jobs namely helical gear milling, spur gear milling, T-slot milling, pantograph milling, and plain milling and rank them in ascending order beginning with the minimum and ending with maximum relative worth in the workshop considering three significant attributes, namely effort, skill and knowedge (**Figure 2**). The description of the attributes are given in **Table 1**. It is essential to ascertain the consistency of the members in the group who are assigned the task of ranking the jobs. Though **Saaty's** 10% consistency ratio rule can be used as a guideline for this purpose, an attempt is made to demonstrate the effectiveness of the combination of confidence interval approach and **Saaty's** 10% rule. For level 2 in the hierarchy, six decision-makers were asked to do the pairwise comparison individually. The pairwise comparison matrix and the CR is given in **Table 2**. The consistency of the judgements is checked as follows (**Wabalickis 1988; Canada** and **Sullivan 1989**):

(i) Let the pairwise comparison matrix be denoted by $M1$ and the principle vector be denoted by $M2$. Pairwise comparison matrix ($M1$) for the individual 2 is **Table 2**.

$$\begin{bmatrix} 1 & 5 & 7 \\ 1/5 & 1 & 5 \\ 1/7 & 1/5 & 1 \end{bmatrix}$$

and principle eigen vector ($M2$) = $\begin{bmatrix} 0.697 \\ 0.232 \\ 0.071 \end{bmatrix}$

(ii) Define $M3 = M1 * M2$ and $M4 = M3 / M2$. Carrying out the calculations:

$$M3 = \begin{bmatrix} 2.354 \\ 0.726 \\ 0.217 \end{bmatrix} \text{ and } M4 = \begin{bmatrix} 3.377 \\ 3.129 \\ 3.056 \end{bmatrix}$$

The average of the elements of $M4$ is 3.19.

(iii) The consistency index is given by:

C.I. = (3.19 - n) / (n – 1)

= 0.095

From the table of random consistency index, R1 (**Saaty 1980**), at n = 3, a value of 0.58 is found.

(iv) Consistency Ratio CR =

C.I./R1 = 0.095 / 0.58 = 0.164.

5. Computation of Confidence Interval

The confidence interval for the attribute can be calculated by using the formula (1). The following assumptions are made in using the above formula:

(i) The individual values of the CR are independent of each other, i.e., there is normal variability in decision-making (**Karapetrovic** and **Rosenbloom 1999**).

(ii) Sample size (number of individuals) is adequate to use statistical confidence limits.

The "student's t" distribution may be used, if the sample comes from a normal population (to within a reasonable degree of approximation). The assumption that the sample must come from normal population [assumption (i)] is not so severe. The normality assumption holds in many practical situations. Consequently, the required sample size 'n' must be determined through trial and error, using a prior estimate of σ (this estimate might be based on previous experience). Another possibility would be to take a preliminary sample of 'n' observations to provide an

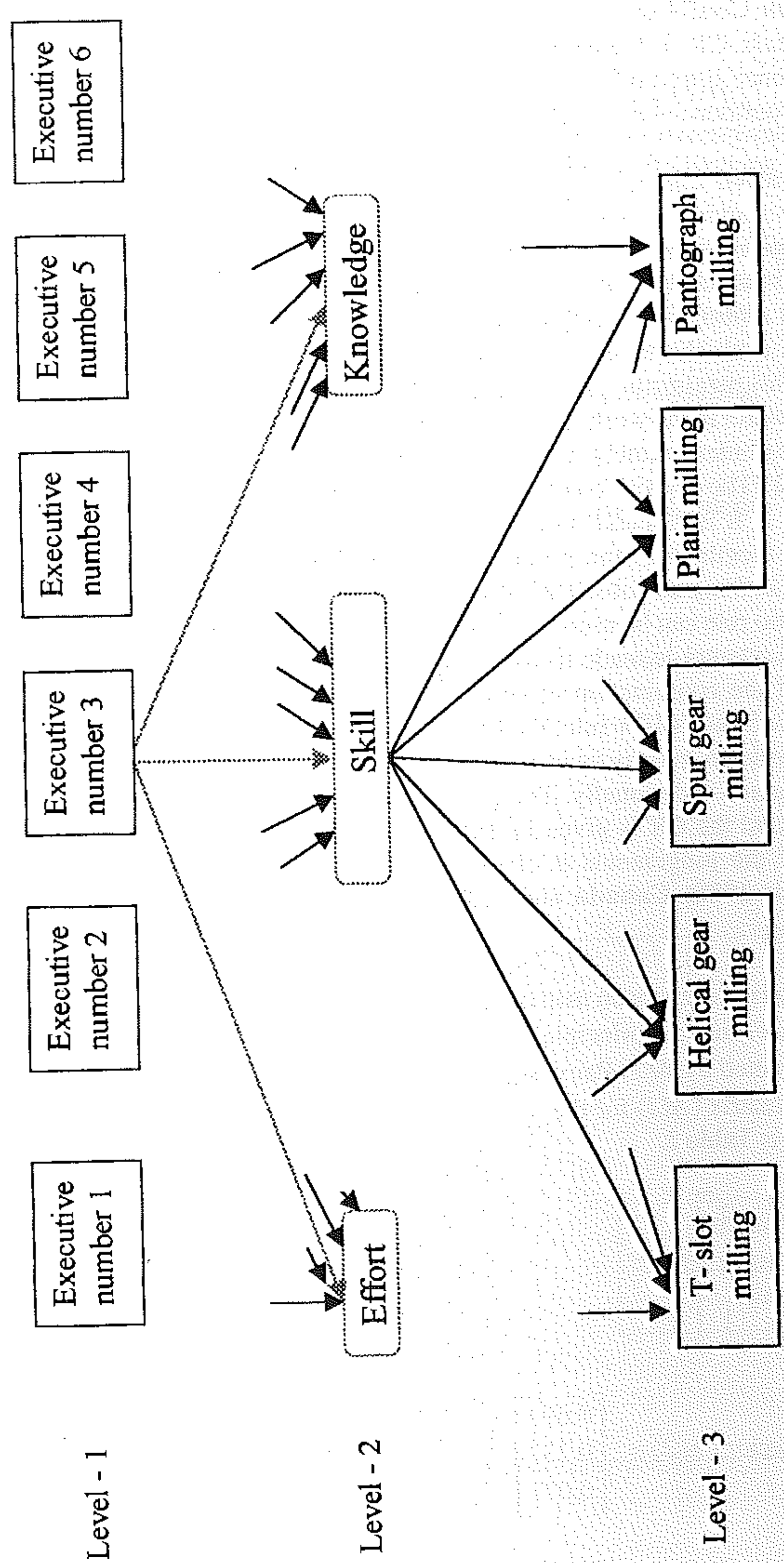

FIGURE 2: *Description of the Case*

TABLE 1: *Description of Attributes*

Attributes		Description
Effort (E)	-	Amount or volume of work
		Job complexity or job difficulty
Skill (S)	-	Dexterity required
		Ingenuity required
Knowledge (K)	-	Experience required
		Education level required
		Training required

TABLE 2: *Pairwise Comparison Matrix and Consistency Ratio of Individuals*

Individual 1 (CR = 0.06)

	E	S	K
E	1	1/5	1/7
S	5	1	3
K	7	3	1

Individual 4 (CR = 0)
[Perfectly consistent case

	E	S	K
E	1	1	5
S	1	1	5
K	1/9	1/5	1

Individual 2 (CR = 0.164)

	E	S	K
E	1	5	7
S	1/5	1	5
K	1/7	1/5	1

Individual 5 (CR = 0.06)

	E	S	K
E	1	1/7	1/3
S	7	1	5
K	3	1/5	1

Individual 3 (CR = 0.187)

	E	S	K
E	1	7	9
S	1/7	1	5
K	1/9	1/5	1

Individual 6 (CR = 0.187)

	E	S	K
E	1	7	9
S	1/7	1	5
K	1/9	1/5	1

estimate of σ. Then, using s computed from this sample as an approximation for σ, to calculate the required value of 'n' to provide the desired accuracy and level of confidence. Studies have shown that the distribution of random variables with the values of 't' is fairly close to a 't' distribution even for samples from certain non-normal populations (**Miller** and **Freund 1990; Montgomery** and **Runger 1994**). Here, single confidence limit is set (upper confidence limit only). Smaller the value of the CR, smaller will be the inconsistency of the group member in evaluating the alternatives. The value of CR equal to zero indicates a perfect consistent case (individual 4). Two confidence levels were chosen (90% and 99%). The assumed confidence level depends on the accuracy required in evaluation of the alternatives.

Average CR = 0.1097 and Standard deviation of CR = 0.079

(i) For 90% confidence level (**Figure 3**)

$$\mu < 0.1097 + 2.015 * 0.079 / \sqrt{6}$$

$$\mu < 0.175$$

upper confidence limit = 0.175

(ii) For 95% confidence level (**Figure 4**)

$$\mu < 0.1097 + 2.591 * 0.079 / \sqrt{6}$$

$$\mu < 0.193$$

upper confidence limit = 0.193

For 90% confidence level, the executive number 2 and 6 have to repeat the AHP procedure, since the consistency ratio is greater than upper confidence limit. For 95% confidence level, no individuals in the group have to revise the AHP exercise, since all the consistency ratios of the individuals fall within the confidence limit. However, if **Saaty's** 10% CR rule is followed, then the executive numbers 2, 3 and 6 have to repeat the AHP exercise. These comparisons are presented in **Table 3**. If all the CR values form a cluster, it may be suspected that there is bias in estimation. In such situations, it is better to repeat the AHP exercise again by closely guarding the principle of anonymity. If CR values of most of the decision-makers are above 0.1, this will lead to higher value of average CR and confidence limit. In such conditions, even though all the points may lie within the confidence limit, it may not refer a consistent decision, since there is an upward shift of average CR and confidence limit. This may be due to lack of understanding of the problem by the decision-makers in forming the hierarchy levels or in the questioning process. As a guideline, we may fix the maximum confidence limit value as 0.2 as recommended by **Saaty**.

6. Conclusion

The confidence interval should be agreed upon before the process is begun. If this is not done, when large differences arise in the CRs of participating individuals, the temptation will always be to increase the confidence interval to accommodate the outliers and thereby facilitate agreement where real differences and inconsistencies may exist. It is always possible to accommodate real difference by increasing the confidence interval. This proposed approach will be very much useful in a group's decision-making and especially where Delphi technique is used with AHP. This approach is a sort of modified Delphi. In a strict Delphi, the results of each iteration would be disclosed to the participants, who would then be allowed a second or third opportunity to submit new estimates. This would be done with the idea of obtaining gradually converging results. But with the approach in this paper, the whole idea of treating the results statistically is to avoid having to engage in a lengthy process, which is what the classical Delphi can be. The use of **Saaty's** 10% rule alone does not consider various decision situations that may arise in group decision-making. This approach gives a pictorial representation of the data for better clarity. It also saves time and cost to arrive at group consensus. Similar charts can be developed for other attributes/alternatives at various levels.

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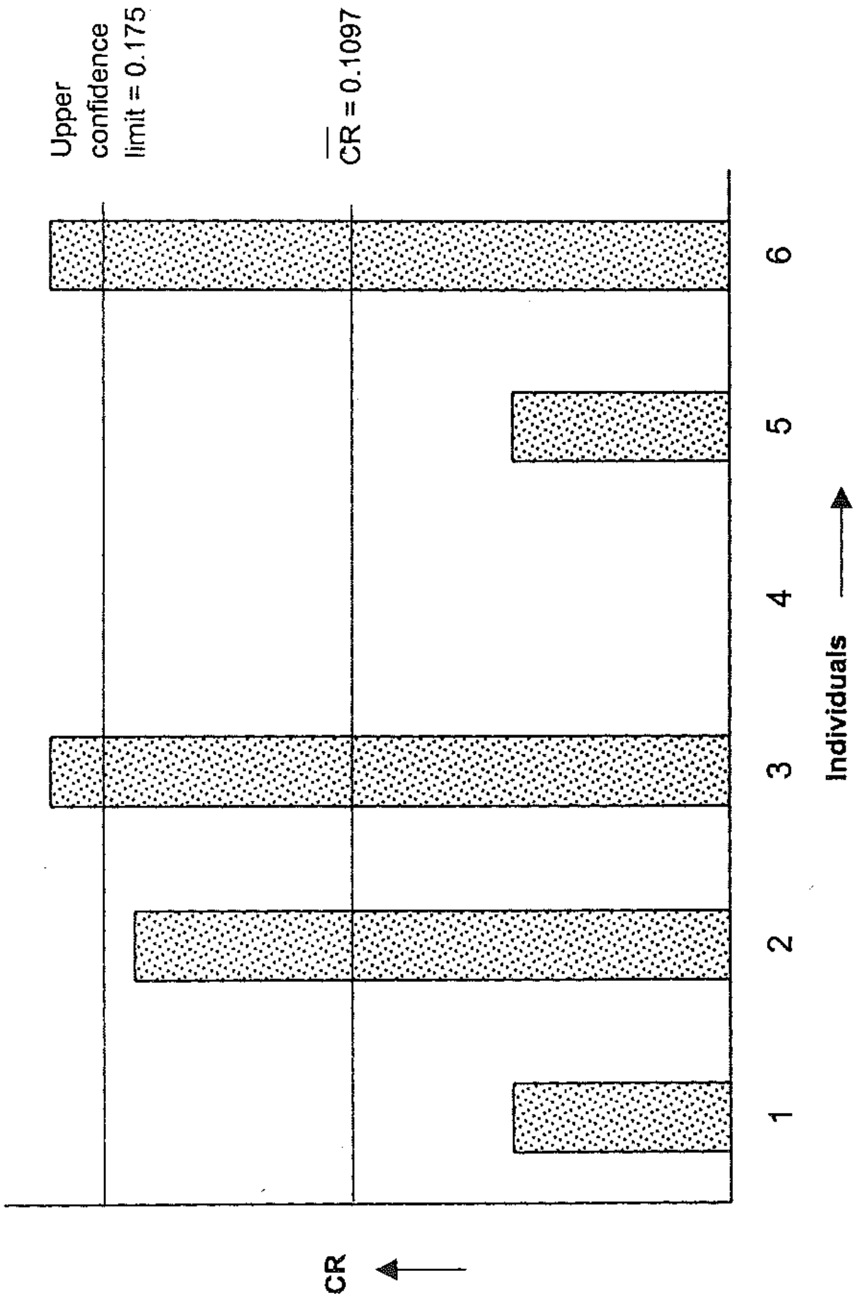

FIGURE 3: *90% Confidence Interval Consistency Ratio Chart*

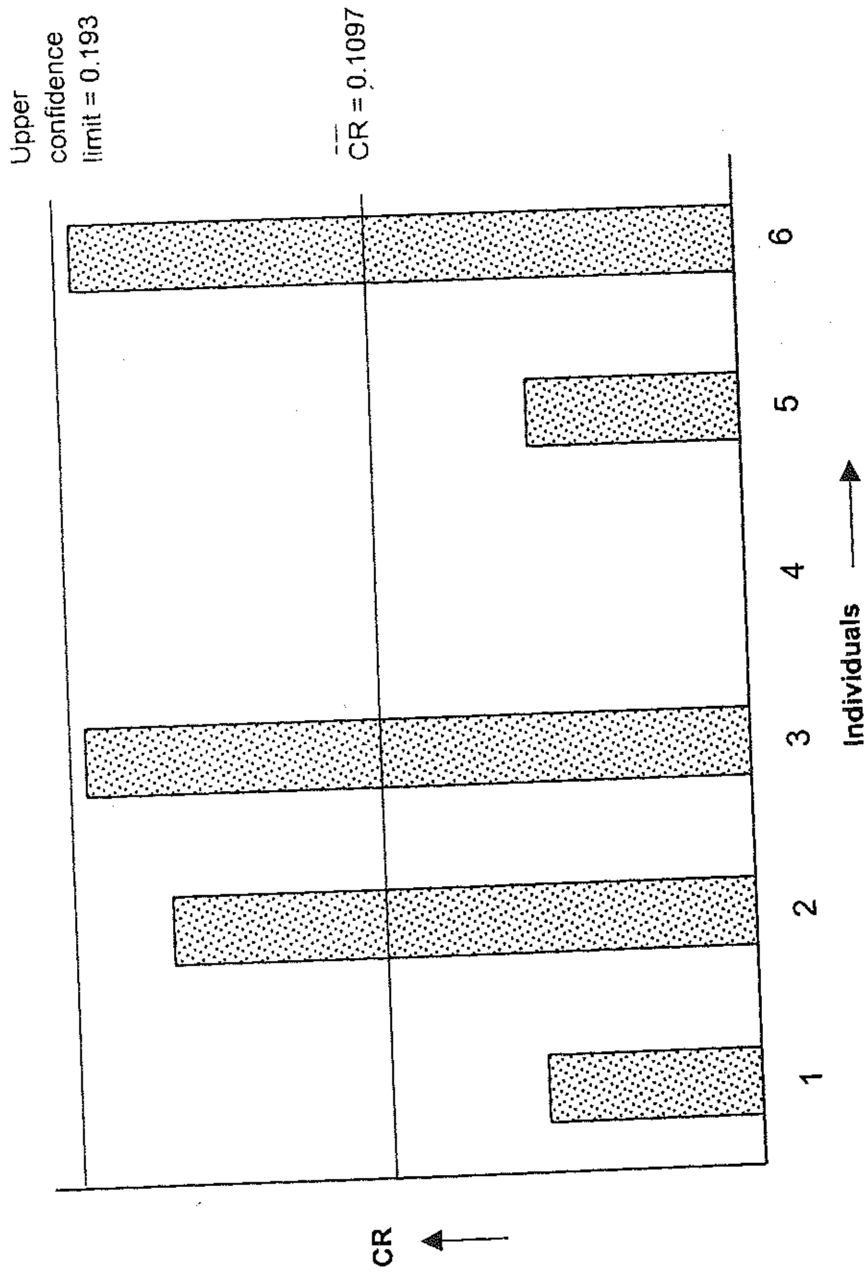

FIGURE 4: *95% Confidence Interval Consistency Ratio Chart*

TABLE 3: *Comparison of Confidence Interval Approach and Satty's 10% Rule*

95% Confidence Level	90% Confidence Level	Satty's 10% Rule
The group is consistent. No individual is required to redo the AHP exercise.	The group is inconsistent. Executive number 2 and 6 have to repeat the AHP exercise.	The group is inconsistent. Executive numbers 2, 3 and 6 have to repeat the AHP exercise.

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