

**THE EFFECTS OF CONFINEMENT AND STRAIN HARDENING ON THE  
PERFORMANCE AND DESIGN OF SHORT CIRCULAR COLUMNS**

**Sema NOYAN ALACALI<sup>1</sup> , Bilge DORAN<sup>2</sup>**

- 1 Ph.D. Assistant Professor of Civil Engineering, Faculty of Civil Engineering, Yıldız Technical Univ., Main Campus, 80750 Yıldız/ISTANBUL/TURKEY, Phone: (+90212) 259 70 70/2257.  
e.mail: noyan@yildiz.edu.tr.
- 2 Ph.D. Lect.Dr. of Civil Engineering, Faculty of Civil Engineering, Yıldız Technical Univ., Main Campus, 80750 Yıldız/ISTANBUL/TURKEY, Phone: (+90212) 259 70 70/2257.  
e.mail: doran@yildiz.edu.tr.

# **THE EFFECTS OF CONFINEMENT AND STRAIN HARDENING ON THE PERFORMANCE AND DESIGN OF SHORT CIRCULAR COLUMNS**

## **ABSTRACT**

In limit design of reinforced concrete structures, the design bending moment distribution is related to the ductility at plastic hinges. The past research has shown that the ductility and energy dissipation capacity of a reinforced concrete member can be improved significantly by confining the concrete by circular spirals. The ultimate curvatures of reinforced concrete sections cannot be calculated accurately by neglecting strain hardening (if strain hardening is formed) in steel. In such a case, the reliability of the limit design and seismic design may be affected unfavourably. In this context, based upon an appropriate steel behaviour model including strain hardening, an algorithm can be developed for confined circular column sections.

**KEYWORDS:** Short Columns, Design, Circular Section, Strain Hardening

## **1.INTRODUCTION**

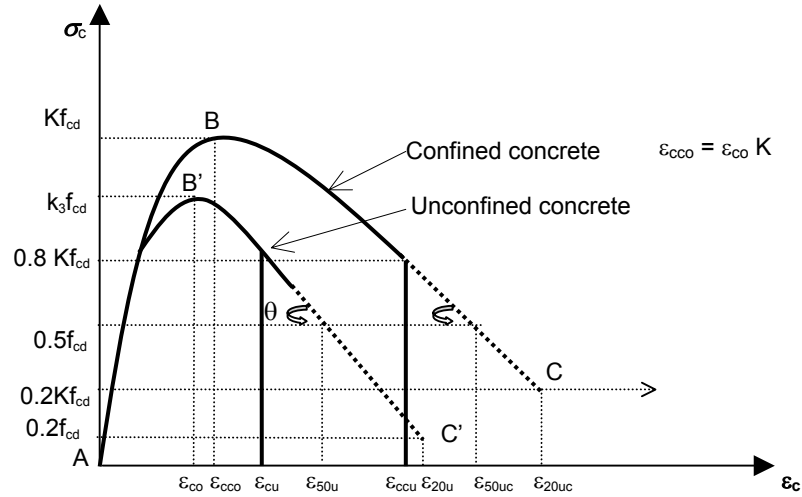
The analysis and design of circular short columns subjected to flexural bending with axial load have been examined in this paper. Confined circular short columns, with compression and tension which have the effect of strain hardening or not, can be analyzed in an accurate way. In the limit design, the curvature ductilities of reinforced concrete cross-sections, and the rotation capacities of the plastic hinges can be increased significantly by confining the sections. In other words, the capacities of potential plastic hinge rotation which are functions of these curvatures are estimated greater than the real values because of the negligence of the strain hardening effect [9].

Bearing all these facts in mind, idealized stress-strain behaviour models are proposed to use in the design for unconfined and confined concrete, considering the effect of strain hardening of the steel. Depending on the afore mentioned models, algorithms for the analysis and design of reinforced concrete columns subjected to flexural bending with axial load have been proposed. By using these algorithms, the confined and strain hardening circular short columns which will reach the ultimate state by compression or tension failure, with steel arrayed in a circle, can be analyzed and designed. Also a numerical example related with the study is given.

## 2. BEHAVIOUR MODELS FOR CONCRETE AND STEEL

### 2.1. Behaviour Model for Concrete

On the basis of the existing experimental evidence, stress-strain behaviour models have been proposed for concrete unconfined and confined by circular spirals [3,5,6,11,13,15].



**Figure 1.** Idealized stress-strain behaviour model for concrete confined by circular spirals

As indicated in Fig.1, characteristic compressive cylinder strength of the concrete confined with circular spirals is given by [8]

$$f_{cck} = f_{ck} + 2.05\rho_h f_{ywk} = Kf_{ck}; \quad K = f_{cck} / f_{ck} = f_{ccd} / f_{cd} \quad (1)$$

K can also be expressed as function of  $\rho_h$ ;

$$K = 1 + 2.05\rho_h (f_{ywk} / f_{ck}) \quad \text{for } C < C50 \quad (2a)$$

or

$$K = 1 + 1.5375\rho_h (f_{ywk} / f_{ck}) \quad \text{for } C \geq C50 \quad (2b)$$

where K is the confinement coefficient,  $f_{ck}$  is the characteristic compressive cylinder strength of the unconfined concrete,  $f_{ywk}$  is the characteristic yield strength of the spiral bar,  $f_{cd}$  and  $f_{ccd}$  are the design compressive strength of the concrete unconfined and confined, respectively.

The ratio of volume of spiral bar to volume of concrete core measured to center lines of spirals is

$$\rho_h = 4A_{sh} / (R_h s_h) \quad (3)$$

where

$$R'_h = R_h - D_h \quad (4)$$

$$R_h = 0.85R \quad (5)$$

$$A_{sh} = 0.25\pi D_h^2 \quad (6)$$

where  $A_{sh}$  is the area of the spiral bar,  $R$  is the diameter of circular column section,  $R'_h$  is the diameter of circle through centre of reinforcement,  $D_h$  is the diameter of spiral,  $s_h$  is pitch of the spiral,  $R_h$  is the diameter of circle through outside of reinforcement. The characteristics of the suggested curve in Figure 1 are as follows [14]:

**For region AB** ( $\varepsilon_c \leq \varepsilon_{cc0}$ ):

$$\sigma_c = Kf_{cd} \left[ (2\varepsilon_c / \varepsilon_{cc0}) - (\varepsilon_c / \varepsilon_{cc0})^2 \right] \quad (7a)$$

**For region BC** ( $\varepsilon_{cc0} < \varepsilon_c \leq \varepsilon_{20uc}$ ):

$$\sigma_c = f_{cd} [K - \psi_c (\varepsilon_c / \varepsilon_{cc0})] \quad (7b)$$

**For region AB'** ( $\varepsilon_c \leq \varepsilon_{c0}$ ):

$$\sigma_c = k_3 f_{cd} \left[ (2\varepsilon_c / \varepsilon_{c0}) - (\varepsilon_c / \varepsilon_{c0})^2 \right] \quad (7c)$$

**For region B'C'** ( $\varepsilon_{c0} < \varepsilon_c \leq \varepsilon_{20u}$ ):

$$\sigma_c = k_3 f_{cd} [1 - \psi (\varepsilon_c / \varepsilon_{c0})] \quad (7d)$$

The parameters of the stress-strain behaviour model (Figure 1) are defined below :

$$\varepsilon_{ccu} = K(0.2 / \psi_c + \varepsilon_{c0}) \quad (8)$$

$$\psi_c = \tan \theta_c / f_{cd} = (K - 0.5) / (\varepsilon_{50u} + \varepsilon_{50h} - \varepsilon_{c0} K) \quad (9)$$

$$\psi = \tan \theta / f_{cd} = 0.5 / (\varepsilon_{50u} - \varepsilon_{c0}) \quad (10)$$

where

$$\varepsilon_{50u} = (3 + 0.29k_3 f_{cd}) / (145k_3 f_{cd} - 1000) \quad (11)$$

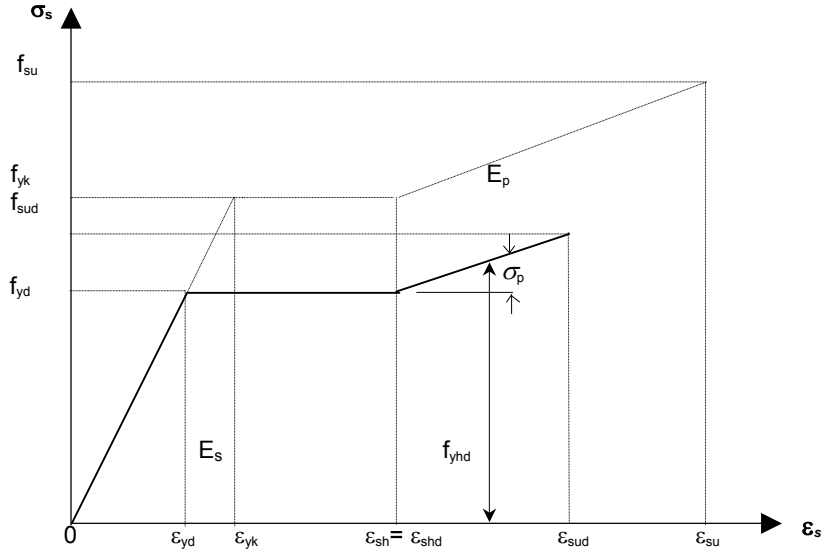
$$\varepsilon_{50h} = 0.75 \rho_h \sqrt{(R_h / s_h)} \quad (12)$$

The strain at the maximum stress  $\varepsilon_{co}$  is approximately 0.002 or 0.0022 [3, 7],  $\gamma_{mc}$  is the material coefficient (safety factor) for concrete,  $k_3$  is the ratio of concrete maximum strength to cylinder strength of the concrete,  $\varepsilon_{ccu}$  is the concrete strain at the extreme compression fiber of confined concrete,  $\varepsilon_{cu}$  is the concrete strain at the extreme compression fiber of unconfined cover concrete [1,2,13, 14].

For any given strain  $\varepsilon_{cm}$  in the extreme compression fiber, and a given concrete stress-strain curve, the compressive stress block parameters  $k_{1c}$ ,  $k_{2c}$ ,  $k_1$ ,  $k_2$  can be determined for unconfined and confined concrete, respectively [7].

## 2.2. Behaviour Model for Steel

Stress-strain behaviour models are shown for steel including the strain hardening effect for analysis and design in figure 3 [9].



**Figure 2.** Idealized stress-strain behaviour model for steel, including the effect of strain hardening

It is assumed trilinear approximate, considering the upper yield strength and the increase in strain due to strain hardening (Fig.2). The slope of the ascending linear region described as plastic behaviour is,

$$E_p = (f_{su} - f_{yk}) / (\epsilon_{su} - \epsilon_{sh}) \quad (13)$$

where  $E_p$  = modulus of plasticity of steel,  $f_{su}$  = the failure strength for steel,  $f_{yk}$  = the characteristic yield strength for steel,  $\epsilon_{su}$  = the ultimate strain for steel,  $\epsilon_{sh}$  = the initial value of strain hardening for steel.

The ultimate strain for steel is,

$$\epsilon_{sud} = (f_{sud} - f_{yd}) / E_p + \epsilon_{sh} \quad (14)$$

For  $\epsilon_s > \epsilon_{shd}$  ( $\epsilon_{shd} = \epsilon_{sh}$ ) in figure 2, the design value of the upper yield strength is,

$$f_{yhd} = f_{yd} + (\epsilon_s - \epsilon_{sh})E_p \leq f_{sud} \quad (15)$$

where  $f_{sud} = f_{su} / 1.3$

### 3. DESIGN FOR CIRCULAR SHORT COLUMNS

In multi-storey buildings, the end moments of the column can change in sign due to different loadings. If the column is bent in double curvature, the same face of the column can be subjected to compression or tension. When the end moments change sign, compressive and tension effects reverse. For this reason, the columns are designed with symmetrical reinforcement. In this part, design algorithms are suggested for eccentrically short circular columns with steel arrayed in a circle [4,10].

In this paper, the equilibrium equations which define the mechanical behaviour of the confined concrete for short circular columns are given as a function of the unknown parameter  $D_v$ , for certain configuration of the longitudinal reinforcement (for example  $n = 10$ ) as shown in figure 3. The algorithms suggested are also convenient for different reinforced sections.

The input data of the problem are the design axial load capacity  $N_d$ , design bending moment capacity  $M_d$  and the geometric parameters,  $D_h$ ,  $s_h$ ,  $n$ . The output data are area of longitudinal reinforcement  $A_{sv}$  and the ultimate curvature  $\phi_{uc}$  respectively.

#### 3.1. Algorithm

Geometric parameters are shown in figure 3.

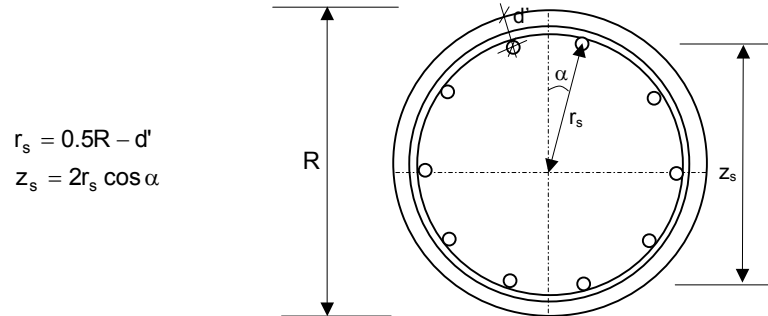


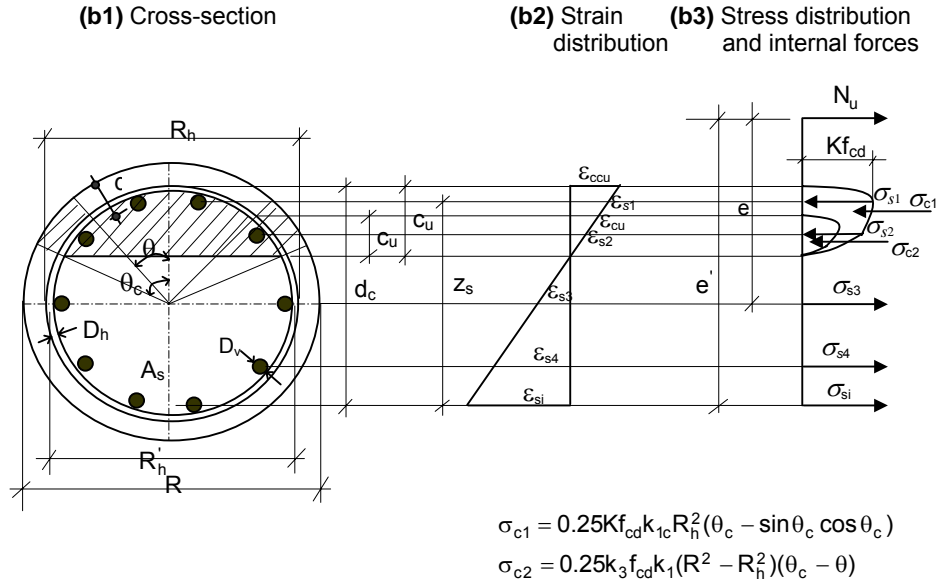
Figure 3. Geometric parameter

Area of the circular spirals for  $n=10$ ,  $\alpha$  can be calculated from the geometry shown in figure 3. In this study,  $\alpha = 18^\circ$  and  $z_s = 2(0.951)r_s$  are computed.

The neutral axis for the cover concrete ( $c_u$ ) can be written as,

$$c_u = \varepsilon_{cu} c_{uc} / \varepsilon_{ccu} \quad (16)$$

where  $c_{uc}$  shows that the depth of the neutral axis for the confined core concrete.



**Figure 4.** The analysis of short columns

The cross-section, strain and stress distribution and internal forces are shown in figure 4. The angles  $\theta$  and  $\theta_c$  for the unconfined and confined concrete respectively can be expressed as [12];

For  $c_{uc} \leq 0.5R_h$  ;

$$\theta = \cos^{-1}[(0.5R_h - c_{uc} + c_u)/(0.5R_h)] \quad (17a)$$

$$\theta_c = \cos^{-1}[(0.5R_h - c_{uc})/(0.5R_h)] \quad (17b)$$

For  $c_{uc} > 0.5R_h$  ;

$$\theta = \pi - \cos^{-1}[(0.5R_h - c_{uc} + c_u)/(0.5R_h)] \quad (18a)$$

$$\theta = \pi - \cos^{-1}[(c_{uc} - 0.5R_h)/(0.5R_h)] \quad (18b)$$

The stresses  $\sigma_{si}$  can be expressed according to strain  $\varepsilon_{si}$ ,  $\sigma_{si}$  and  $\varepsilon_{si}$  have positive signs if the longitudinal reinforcement is located in compression zone. There are three states which may be represented by the following equations:

$$\varepsilon_{si} < \varepsilon_{yd} \Rightarrow \sigma_{si} = \varepsilon_{si} E_s \quad (19a)$$

$$\varepsilon_{yd} < \varepsilon_{si} < \varepsilon_{sh} \Rightarrow \sigma_{si} = f_{yd} \quad (19b)$$

$$\varepsilon_{sh} < \varepsilon_{si} < \varepsilon_{sud} \Rightarrow \sigma_{si} = f_{yd} + [(\varepsilon_{si} - \varepsilon_{sh}) E_p] \quad (19c)$$

Otherwise,  $\sigma_{si}$  and  $\varepsilon_{si}$  have negative signs if the longitudinal reinforcement is located in tension. Similarly, the stresses are as follows:

$$|\varepsilon_{si}| < \varepsilon_{yd} \Rightarrow \sigma_{si} = \varepsilon_{si} E_s \quad (20a)$$

$$\varepsilon_{yd} < |\varepsilon_{si}| < \varepsilon_{sh} \Rightarrow \sigma_{si} = -f_{yd} \quad (20b)$$

$$\varepsilon_{sh} < |\varepsilon_{si}| < \varepsilon_{sud} \Rightarrow \sigma_{si} = -f_{yd} + [(\varepsilon_{si} + \varepsilon_{sh}) E_p] \quad (20c)$$

In this study, compression zone in cover concrete is assumed to be sector of a circular section. Therefore, the area of circular section can be written as,

$$A_{shell} = 2 \frac{\pi}{4} (R^2 - R_h^2) (\theta_c - \theta) \frac{1}{2\pi} \quad (21)$$

Eq.21 simplifies to

$$A_{shell} = 0.25(R^2 - R_h^2)(\theta_c - \theta) \quad (22)$$

The compression force in cover concrete is

$$F_{shell} = 0.25k_3 f_{cd} k_1 (R^2 - R_h^2) (\theta_c - \theta) \quad (23)$$

The compression force in core concrete is then

$$F_{core} = 0.25K f_{cd} k_{1c} R_h^2 (\theta_c - \sin \theta_c \cos \theta_c) \quad (24)$$

Thus the ultimate load of column may be written as

$$N_u = F_{core} + F_{shell} + F_s \quad (25)$$

where  $F_s$  is the summation of the tension forces. The equilibrium equation obtained from the sum of the internal forces is



$$N_u = [0.25 K f_{cd} k_{1c} R_h^2 (\theta_c - \sin \theta_c \cos \theta_c)] + [0.25 k_3 f_{cd} k_1 (R^2 - R_h^2) (\theta_c - \theta)] + \sum_{i=1}^n \sigma_{si} A_{si} \quad (26)$$

and the expression obtained from taking moments about the tension steel is

$$M_u = 0.25 K f_{cd} k_{1c} R_h^2 (\theta_c - \sin \theta_c \cos \theta_c) [0.5(R_h + z_s) - k_{2c} c_{uc}] + 0.25 k_3 f_{cd} k_1 (R^2 - R_h^2) (\theta_c - \theta) [0.5(R_h + z_s) - c_{uc} + c_u - (k_2 c_u)] + \sum_{i=1}^n \sigma_{si} A_{si} x_i \quad (27)$$

where  $M_u$  is the moment of resistance

The moment equilibrium equation given by Eq.27 may be also written as

$$N_u e' = N_u (e + 0.5z_s) = 0.25 K f_{cd} k_{1c} R_h^2 (\theta_c - \sin \theta_c \cos \theta_c) [0.5(R_h + z_s) - k_{2c} c_{uc}] + 0.25 k_3 f_{cd} k_1 (R^2 - R_h^2) (\theta_c - \theta) [0.5(R_h + z_s) - c_{uc} + c_u - (k_2 c_u)] + \sum_{i=1}^n \sigma_{si} A_{si} x_i \quad (28)$$

where  $e'$  is the eccentricity of ultimate load  $N_u$  from the centroid of the tension steel.

Substituting the value of  $D_{v1}^2$  obtained from Eq.26 into Eq.28,  $N_u$  is calculated. Until values of  $N_u$  and  $N_d$  are equal, the depth of the neutral axis is changed. If the values of  $N_u$  and  $N_d$  are equal,  $D_{v1}$  is the diameter of the longitudinal steel. Thus, total area of longitudinal steel in the section is

$$A_{sv} = n A_{si} \quad (29)$$

The ultimate curvature is given by

$$\phi_{uc} = \varepsilon_{ccu} / c_{uc} \quad (30)$$

### 3.2 Balanced Eccentricity

A "balanced failure" occurs when the tension steel reaches the yield strength and the extreme fiber concrete compressive strain reaches the ultimate strain at the same time. In the general case when  $e$  or the section is different from  $e_b$ , the type of failure that occurs will depend on whether  $e$  is less than or greater than  $e_b$ . If  $e < e_b$  (or  $e/h < e_b/h$ ), compression failure occurs. Tensile failure occurs if  $e > e_b$  (or  $e/h > e_b/h$ ). The subscript "b" has been added to all parameters concerned with balanced failure. For balanced failure,  $\varepsilon_s = \varepsilon_{yd}$  and  $\sigma_s = f_{yd}$ . Balanced eccentricity  $e_b$  is derived from similar triangles

of the strain diagram, force and moment equilibrium equations (Figure 4). For a balanced failure, the neutral axis depths  $c_b$  and  $c_{bc}$  is given by the following relationships.

$$c_b = [ \varepsilon_{cu} / ( \varepsilon_{cu} + \varepsilon_{yd} ) ] 0.5 ( R + z_s ) \quad (31a)$$

$$c_{bc} = [ \varepsilon_{ccu} / ( \varepsilon_{ccu} + \varepsilon_{yd} ) ] 0.5 ( R_h + z_s ) \quad (31b)$$

Substituting  $c_{uc} = c_{bc}$ ,  $c_u = c_b$  and  $e = e_b$  into Eq.28, the following equation is obtained:

$$\begin{aligned} N_b(e_b + 0.5z_s) = & 0.25Kf_{cd}k_{1c}R_h^2(\theta_{bc} - \sin\theta_{bc} \cos\theta_{bc})[0.5(R_h + z_s) - k_{2c}c_{bc}] \\ & + 0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_{bc} - \theta_b)[0.5(R_h + z_s) - c_{bc} + c_b - (k_2c_b)] \\ & + \sum_{i=1}^n \sigma_{si}A_{si}x_i \end{aligned} \quad (32)$$

Solving for  $e_b$ , balanced eccentricity becomes

$$\begin{aligned} e_b = & \left\{ \left[ 0.25Kf_{cd}k_{1c}R_h^2(\theta_{bc} - \sin\theta_{bc} \cos\theta_{bc})[0.5(R_h + z_s) - k_{2c}c_{bc}] \right. \right. \\ & \left. \left. + 0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_{bc} - \theta_b)[0.5(R_h + z_s) - c_{bc} + c_b - (k_2c_b)] + \sum_{i=1}^n \sigma_{si}A_{si}x_i \right] / \right. \\ & \left. \left[ 0.25Kf_{cd}k_{1c}R_h^2(\theta_{bc} - \sin\theta_{bc} \cos\theta_{bc}) \right] + \left[ 0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_{bc} - \theta_b) \right] \right. \\ & \left. + \sum_{i=1}^n \sigma_{si}A_{si} \right\} - 0.5z_s \end{aligned} \quad (33)$$

#### 4. NUMERICAL EXAMPLE

i) Calculate  $A_{sv}$  and  $\phi_{uc}$  for circular column confined by circular spirals.

$$N_d = 1200 \text{ kN}, M_d = 115 \text{ kNm}, R = 400 \text{ mm}$$

$$n = 10, f_{ck} = 25 \text{ MPa}, f_{yk} = 220 \text{ MPa}, f_{ywk} = 220 \text{ MPa}, D_h = 10 \text{ mm}, s_h = 100 \text{ mm},$$

$$\gamma_c = 1.5, \gamma_s = 1.15,$$

$$E_s = 2 \cdot 10^5 \text{ MPa}, E_p = 750 \text{ MPa}, \varepsilon_{sud} = 0.114, \varepsilon_{sh} = 0.02, \varepsilon_{co} = 0.0022, \varepsilon_{cu} = 0.0035,$$

$$k_1 = 0.754, k_2 = 0.443, k_3 = 1, d' = 50 \text{ mm}$$

$$f_{cd} = f_{ck} / \gamma_c = 16.67 \text{ MPa}, f_{yd} = f_{yk} / \gamma_s = 191.3 \text{ MPa}, \varepsilon_{yd} = f_{yd} / E_s = 0.0009565,$$

$$e = M_d / N_d = 96 \text{ mm}$$

$$R_h = 0.85 \cdot R = 0.85 \cdot 400 = 340 \text{ mm}, R'_h = R_h - D_h = 330 \text{ mm}, A_{sh} = 0.25\pi D_h^2 = 78.54 \text{ mm}^2,$$

$$r_s = 0.5R - d' = 0.5 \cdot 400 - 50 = 150 \text{ mm}, \rho_h = 4A_{sh} / (R'_h \cdot s_h) = 0.00952,$$

$$\varepsilon_{50h} = 0.75\rho_h(R_h / s_h)^{1/2} = 0.013166, K = 1 + (2.05\rho_h f_{yk} / f_{ck}) = 1.17,$$

$$\varepsilon_{50u} = (3 + 0.29k_3 f_{cd}) / (145k_3 f_{cd} - 1000) = 0.005528,$$

$$\Psi_c = (K - 0.5) / (\varepsilon_{50u} + \varepsilon_{50h} - \varepsilon_{co}K) = 41.56$$

$$\varepsilon_{ccu} = K(0.2/\Psi_c + \varepsilon_{co}) = 0.0082, k_{1c} = 0.827, k_{2c} = 0.469$$

### Solution

For  $n=10$  bars,

$$z_s = 2r_s \cos \alpha = 285.32 \text{ mm}$$

Assume that  $c_{uc} = 231.99 \text{ mm}$

$$c_u = \varepsilon_{cu} c_{uc} / \varepsilon_{ccu} = 99.02 \text{ mm}$$

$$c_{uc} = 231.99 \text{ mm} < 0.5R_h = 170 \text{ mm}$$

$$\theta = \pi - \cos^{-1}[(0.5R_h - c_{uc} + c_u)/(0.5R_h)] = 1.790 \text{ rad}$$

$$\theta_c = \pi - \cos^{-1}[(c_{uc} - 0.5R_h)/(0.5R_h)] = 1.944 \text{ rad}$$

The values  $\sigma_{si}$  may be determined from the strain diagram:

$$\varepsilon_{s1} = [c_{uc} - 0.5(R_h - z_s)]\varepsilon_{ccu} / c_{uc} = 0.00723$$

$$\varepsilon_{s1} > 0, \varepsilon_{yd} < \varepsilon_{s1} < \varepsilon_{sh}, \sigma_{s1} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{s2} = [c_{uc} - 0.5R_h + 0.588r_s]\varepsilon_{ccu} / c_{uc} = 0.005308$$

$$\varepsilon_{s2} > 0, \varepsilon_{yd} < \varepsilon_{s2} < \varepsilon_{sh}, \sigma_{s2} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{s3} = [c_{uc} - 0.5R_h]\varepsilon_{ccu} / c_{uc} = 0.00219$$

$$\varepsilon_{s3} > 0, \varepsilon_{yd} < \varepsilon_{s3} < \varepsilon_{sh}, \sigma_{s3} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{s4} = [c_{uc} - 0.5R_h - 0.588r_s]\varepsilon_{ccu} / c_{uc} = -0.000926,$$

$$\varepsilon_{s4} < 0, |\varepsilon_{s4}| < \varepsilon_{yd}, \sigma_{s4} = \varepsilon_{s4} E_s = -0.000926 * 2 * 10^5 = -185.2 \text{ N/mm}^2$$

$$\varepsilon_{s5} = [c_{uc} - 0.5(R_h + z_s)]\varepsilon_{ccu} / c_{uc} = -0.002851,$$

$$\varepsilon_{s5} < 0, \varepsilon_{yd} < |\varepsilon_{s5}| < \varepsilon_{sh}, \sigma_{s5} = -f_{yd} = -191.3 \text{ N/mm}^2$$

And from Eq.26 we write

$$D_{v1}^2 = 2[N_d - 0.25K_f k_{cd} k_{1c} R_h^2 (\theta_c - \sin \theta_c \cos \theta_c) - 0.25k_3 f_{cd} k_1 (R^2 - R_h^2) (\theta_c - \theta)] / [\pi(\sigma_{s1} + \sigma_{s2} + \sigma_{s3+} + \sigma_{s4} + \sigma_{s5})] = 367.85$$

From Eq.28 we have

$$N_u = 1202.21 \text{ kN}. \text{ Because of } N_u \cong N_d$$

Total area of longitudinal steel in the section is

$$A_{sv} = nA_{si} = n0.25\pi D_{v1}^2 = 10 * 0.25 * \pi * 367.85 = 2889.09 \text{ mm}^2 \text{ (10 } \phi 20)$$

The ultimate curvature is

$$\phi_{uc} = \varepsilon_{ccu} / x_{uc} = 0.0082 / 231.99 * 2 * 10^5 = 0.03535 \text{ rad/m}$$

### Balanced Eccentricity

$$c_b = \left[ \varepsilon_{cu} / (\varepsilon_{cu} + \varepsilon_{yd}) \right] 0.5(R + z_s) = 269.11 \text{ mm}$$

$$c_{bc} = \left[ \varepsilon_{ccu} / (\varepsilon_{ccu} + \varepsilon_{yd}) \right] 0.5(R_h + z_s) = 280 \text{ mm}$$

$$c_b = 269.11 \text{ mm} > 0.5R_h = 170 \text{ mm}$$

$$c_{bc} = 280 \text{ mm} > 0.5R_h = 170 \text{ mm}$$

$$\theta_b = \pi - \cos^{-1} \left[ (0.5R_h - c_{uc} + c_u) / (0.5R_h) \right] = 2.781 \text{ rad}$$

$$\theta_{bc} = \pi - \cos^{-1} \left[ (c_{uc} - 0.5R_h) / (0.5R_h) \right] = 2.275 \text{ rad}$$

$$\varepsilon_{sb1} = \left[ c_{bc} - 0.5(R_h - z_s) \right] \varepsilon_{ccu} / c_{bc} = 0.007399 \text{ N/mm}^2$$

$$\varepsilon_{s1} > 0, \quad \varepsilon_{yd} < \varepsilon_{s1} < \varepsilon_{sh}, \quad \sigma_{s1} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{sb2} = \left[ c_{bc} - 0.5R_h + 0.588r_s \right] \varepsilon_{ccu} / c_{bc} = 0.005804$$

$$\varepsilon_{s2} > 0, \quad \varepsilon_{yd} < \varepsilon_{s2} < \varepsilon_{sh}, \quad \sigma_{s2} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{sb3} = \left[ c_{bc} - 0.5R_h \right] \varepsilon_{ccu} / c_{bc} = 0.003221$$

$$\varepsilon_{s3} > 0, \quad \varepsilon_{yd} < \varepsilon_{s3} < \varepsilon_{sh}, \quad \sigma_{s3} = f_{yd} = 191.3 \text{ N/mm}^2$$

$$\varepsilon_{s4} = \left[ c_{uc} - 0.5R_h - 0.588r_s \right] \varepsilon_{ccu} / c_{bc} = 0.0006384,$$

$$\varepsilon_{s4} > 0, \quad \varepsilon_{s4} < \varepsilon_{yd}, \quad \sigma_{s4} = \varepsilon_{s4} E_s = 0.0006384 * 2 * 10^5 = 127.68 \text{ N/mm}^2$$

$$\varepsilon_{s5} = \left[ c_{bc} - 0.5(R_h + z_s) \right] \varepsilon_{ccu} / c_{bc} = -0.0009565,$$

$$\varepsilon_{s5} < 0, \quad \varepsilon_{yd} < |\varepsilon_{s5}| < \varepsilon_{sh}, \quad \sigma_{s5} = -f_{yd} = -191.3 \text{ N/mm}^2$$

$$A_{si} = 0.25 * \pi * 400 = 314.16 \text{ mm}^2$$

Substituting the values calculated into Eq.33 gives

$$e_b = 55.12 \text{ mm}$$

$$e = 96 \text{ mm} > e_b = 55.12 \text{ mm}$$

Therefore, a tension failure occurs.

ii) Calculate  $A_{sv}$  and  $\phi_u$  for unconfined circular column.

The total area is  $A_{sv} = 1533.02 \text{ mm}^2$  ( $10\phi14$ ) and the ultimate curvature is

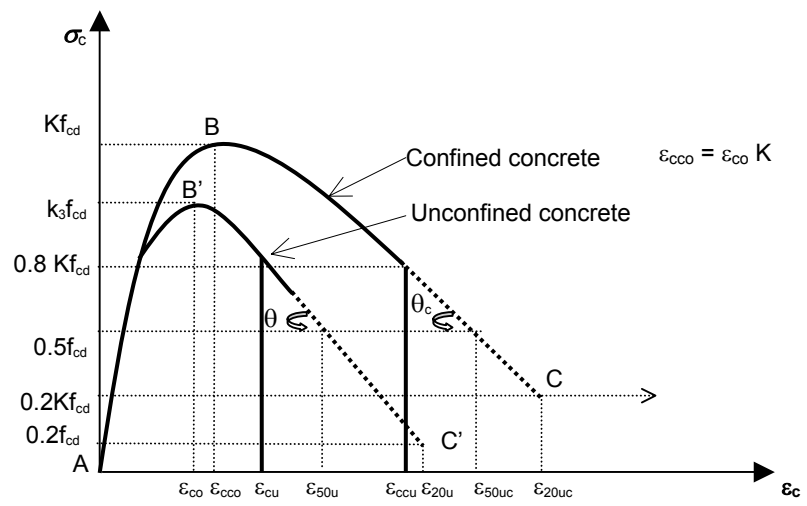
$$\phi_{uc} = 0.0132 \text{ rad/m} .$$

## 5. CONCLUSIONS

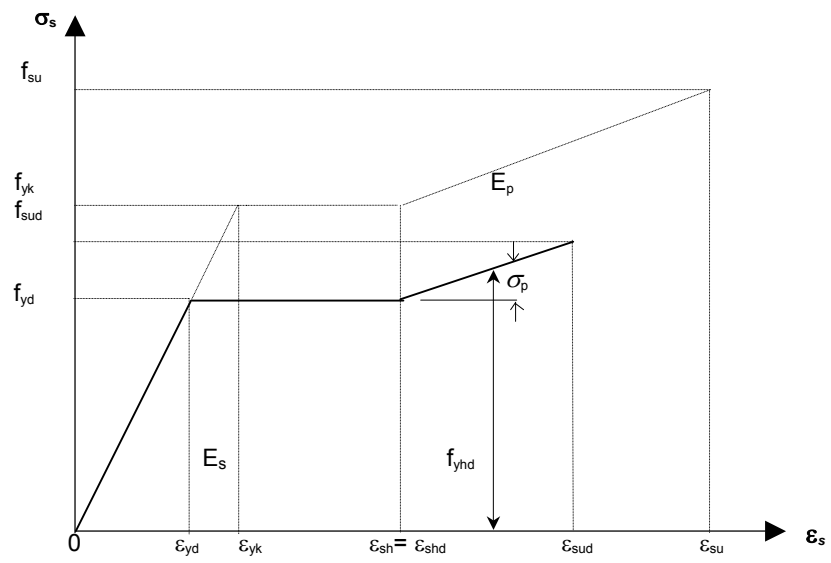
The bending moment distribution is related to the ductility at plastic hinges of reinforced concrete structures. Also, structures subjected to seismic action must be ductile enough to absorb and dissipate energy. Past research has shown that the ductility enough to absorb and dissipation capacity of a reinforced concrete member can be improved significantly by confining the concrete by circular spirals. In this context, the confined circular short columns can be accurately designed, including consideration of the strain hardening in steel. Besides, the capacities of potential plastic hinge rotation are estimated greater than the real values because of the negligence of the strain hardening effect.

## 6. REFERENCES

- 1.ACI Committee 318, (1984a), Building Code Requirements for Reinforced Concrete, ACI Publication, Detroit.
- 2.BSI, (1980), Code of Practice for Structural Use of Concrete (CP 110, Part 1: Design, Materials and Workmanship), British Standards Institution, London.
- 3.CEB, (1991a), "CEB – FIP Model Code 1990 Final Draft Chapter 1-3", Bulletin d'Information, 203.
- 4.Ersoy, U., (1985b), Betonarme Temel İlkeler ve Taşıma Gücü Hesabı Cilt 1, Bizim Büro Basımevi, Ankara.
- 5.Ersoy, U., Tankut, T. ve Uzumeri, S.M., (1987), "The Influence of Strain History and Strain Gradient of Confined Concrete", Canadian Journal of Civil Engineering, 14 (3).
- 6.FIP/CEB, (1990), "High Strength Concrete, State of the Art Report", Bulletin d'Information, 197.
- 7.Gündüz, A., (1986) "Kuşatılmamış Betonlu Dikdörtgen Kiriş Kesitlerinde Moment-eğrilik İlişkilerinin Belirlenmesiyle İlgili Bir Tasarım Algoritması" YTU Dergisi, Sayı 1.
- 8.Gündüz, A., (1991a), "Yüksek ve Normal Mukavemetli Betonların Davranışının Yanal Donatıyla Yetkinleştirilmesi", TMMOB İnşaat Mühendisleri Odası İstanbul Şubesi 2. Ulusal Beton Kongresi, 27-30 Mayıs, İstanbul.
- 9.Gündüz, A. ve Noyan, S., (1988), "Kuşatılmış Kesitli Betonarme Kirişlerde Son Limit Momentinin ve Eğriliğinin Donatıdaki Pekleşme Göz Önüne Alınarak Belirlenmesi", Yıldız Üniversitesi Dergisi Sayı 2 , İstanbul, 13-20.
- 10.Gündüz, A. ve Noyan, S., (1993 ), "Kuşatılmamış Betonlu Kısa Kolonlar İçin Geliştirilmiş Tasarım Algoritmaları", Yıldız Teknik Üniversitesi Dergisi Sayı 4, İstanbul, 17-26.
- 11.Kent, D.C. ve Park, R., (1971), "Flexural Members With Confined Concrete", Proceedings ASCE, 97 (7).
- 12.Mac Gregor, J.G., (1997), Reinforced Concrete Mechanics and Design, Prentice – Hall International Inc., New Jersey.
- 13.Park, R., Priestley, M.J. ve Gill, W.D., (1982), "Ductility of Square – Confined Concrete Columns", Proceedings ASCE, 108 (4).
14. Park, R., and Paulay, T., (1975), "Reinforced Concrete Structures, Wiley, New York, 769 pp.
- 15.Sheikh, S.A., (1982), "A Comparative Study of Confinement Models", Journal of the American Concrete Institute, 79 (3): 296-305.

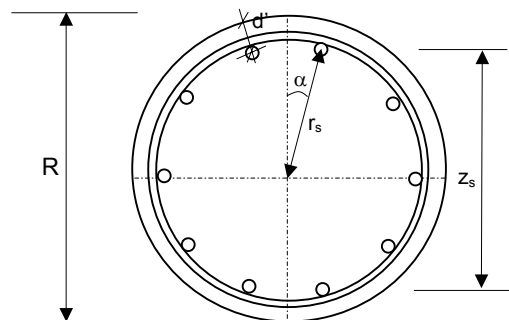


**Figure 1.** Idealized stress-strain behaviour model for concrete confined by circular spirals



**Figure 2.** Idealized stress-strain behaviour model for steel, including the effect of strain hardening

$$r_s = 0.5R - d'$$
$$z_s = 2r_s \cos \alpha$$



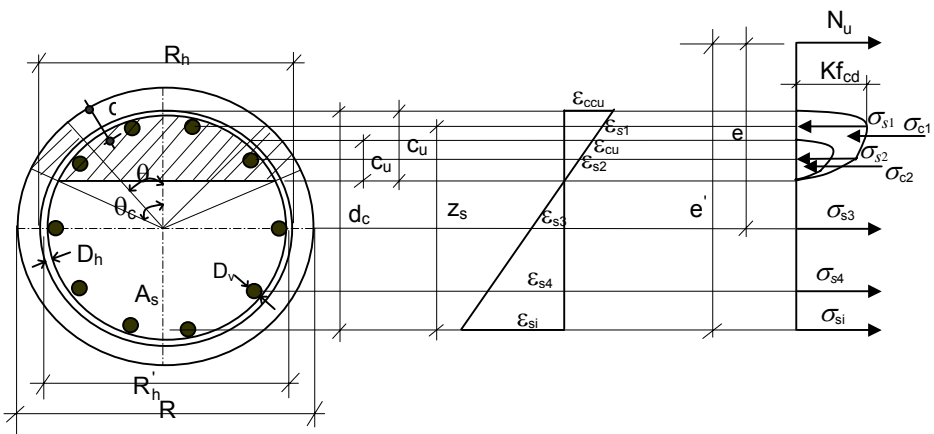
**Figure 3.** Geometric parameter



(b1) Cross-section

(b2) Strain distribution

(b3) Stress distribution and internal forces



$$\sigma_{c1} = 0.25Kf_{cd}k_1cR_h^2(\theta_c - \sin\theta_c \cos\theta_c)$$

$$\sigma_{c2} = 0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_c - \theta)$$

Figure 4. The analysis of short columns