THE MHD FLOW IN A REGION PARTIALLY FILLED WITH POROUS MEDIUM AND BOUNDED BY TWO PERIODICALLY HEATED OSCILLATING PLATES

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Abstract

The flow of a conducting liquid between two parallel periodically heated oscillating plates has been studied. The space between the plates has been divided into two regions (i) clear fluid region and (ii) porous medium region. It is assumed that in region (i) the flow is governed by Navier-Stokes equations while in the region (ii) by Brinkman equations. At the interface the velocity, temperature and skin-friction are assumed to be continuous. A transverse uniform magnetic field is applied normal to the plane. Method of separating of variables is used to solve the resulting equations. The expressions for velocities and temperature fields are obtained. The effects of permeability and magnetic field on the flow characteristic have been studied through several graphs.

Key words: Magnetohydrodynamics, Porous medium, Oscillating plates, Periodic heating, Heat transfer.

1. Introduction

Flows of fluids through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholar due to their applications in many branches of science and technology, viz. in the fields of agriculture engineering to study the underground water resources, seepage of water in river beds, in petroleum technology to study the movement of natural gas, oil, and water through the oil reservoirs, in chemical engineering for filtration and purification processes. Schlichting [1] discussed well-known Stokes second problem in classical hydrodynamics. Rudraiah [2] discussed this problem in magnetohydrodynamics. Tokis [3] further studied this oscillatory plate problem subjected to uniform suction or injection in the presence of a uniform magnetic field. The Stokes first and second problem in porous medium has been investigated by Murthy [4]. Chauhan and Vyas [5] discussed the Stokes second problem in porous medium in the presence of a magnetic field. Viscous flow formation in Couette motion has been discussed in the book by Schlichting [1]. Mishra and Mishra [6] considered the flow of a viscous elastic liquid due to a plate, which suddenly starts oscillating in the presence of another parallel stationary plate. The aim of this paper is to consider the flow in a region partially filled with porous medium of finite thickness bounded by two parallel plates in the presence of a magnetic field. The flow is due to the oscillations of the plates. The plates are kept with oscillating wall temperatures. The problem already defined here is a particular case of MHD that has attracted the attention of many investigators because of its applications to astrophysics, geophysics and engineering (Crammer and Pai [7]).

2. Mathematical analysis

The viscous incompressible electrically conducting flow in a region that is half filled with porous medium is considered. This region is bounded by two parallel plates, which are oscillating with constant amplitude U_0 and frequency ω^*. This region is divided into two regions, (i) clear fluid region ($0 \le y^* \le h$) and (ii) porous medium region ($-h \le y^* \le 0$). The x^*-axis is taken along the interface and y^*-axis is normal to it. The pressure is constant through out the flow field. The temperature of both the plates are T_1^* and T_2^* respectively. A uniform magnetic field B_0 is applied in the direction normal to flow (along y^*-axis). We assume that all fluid properties are constant, magnetic dissipation effects are neglected in the energy equation, the magnetic Reynolds number is small so that the induced magnetic field can be neglected.

In the clear fluid region ($0 \le y^* \le h$) the flow is governed by the following equations of motion and energy

$$\left.\begin{aligned} &\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^*}{\rho} \\ &\rho C_p \frac{\partial T_1^*}{\partial t^*} = \kappa \frac{\partial^2 T_1^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*}\right)^2 \end{aligned}\right\} \qquad \ldots (1)$$

The boundary conditions at the plate are

$$y^* = h \; : \; u^* = U_0 \cos \omega^* t^* \; , \; T_1^* = T_0 \; (1 + \cos \omega^* t^*) \qquad \ldots (2)$$

In the porous region ($-h \le y^* \le 0$) the flow is governed by the following Brinkman equations [8] and the equation of energy

$$\left.\begin{aligned} &\frac{\partial U^*}{\partial t^*} = \nu_p \frac{\partial^2 U^*}{\partial y^{*2}} - \nu_p \frac{U^*}{k^*} - \frac{\sigma B_0^2 U^*}{\rho} \\ &\rho C_p \frac{\partial T_2^*}{\partial t^*} = \kappa_p \frac{\partial^2 T_2^*}{\partial y^{*2}} + \mu_p \left(\frac{\partial U^*}{\partial y^*}\right)^2 + \mu \frac{U^{*2}}{k^*} \end{aligned}\right\} \qquad \ldots (3)$$

The boundary conditions at this plate are

$$y^* = -h \; : \; U^* = U_0 \cos \omega^* t^* \; , \; T_2^* = T_0 \; (1 + \cos \omega^* t^*) \qquad \ldots (4)$$

At the interface of the porous medium and clear fluid y* = 0, we assume the velocity components, the temperature and the shearing stresses are continuous. The boundary conditions at the interface of porous medium and clear fluid have been investigated and discussed by Kim and Russel [9]. These assumptions in our notation can be written as

$$y^* = 0 : u^* = U^* \ , T_1^* = T_2^* \ , \ \nu \frac{\partial u^*}{\partial y^*} = \nu_p \frac{\partial U^*}{\partial y^*} \ , \ \kappa \frac{\partial T_1^*}{\partial y^*} = \kappa_p \frac{\partial T_2^*}{\partial y^*} \qquad \ldots (5)$$

Various physical variables are defined in the appendix. We introduce the following non-dimensional quantities as

$$u = u^*/U_o \ , \ U = U^*/U_0 \ , \ y = y^*/h \ , \ t = t^*\nu/h^2 \ , \ \omega = \omega^* h^2/\nu \ , \ k = k^*/h^2$$

$$\phi_1 = \nu/\nu_p \ , \ \phi_2 = \kappa/\kappa_p \ , \theta_1 = (T_1^* - T_0)/T_0 \ , \ \theta_2 = (T_2^* - T_0)/T_0$$

$$\alpha \text{ (Thermal diffusivity)} = \frac{\kappa}{\rho C_p} \ , \ \text{Pr (Prandtl number)} = \nu/\alpha$$

$$\text{M (Hartmann number)} = \left(\frac{\sigma B_0^2 h^2}{\rho \nu}\right)^{1/2} \ , \ \text{Ec (Eckert number)} = \frac{U_0^2}{C_p T_0}$$

The dimensionless form of the equations of motion and energy are

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial y^2} - M^2 u \\ \frac{\partial \theta_1}{\partial t} &= (\text{Pr})^{-1} \frac{\partial^2 \theta_1}{\partial y^2} + \text{Ec} \ \left(\frac{\partial u}{\partial y}\right)^2 \end{aligned}\right\} \qquad \ldots (6)$$

$$\left.\begin{aligned} \frac{\partial U}{\partial t} &= (\phi_1)^{-1} \left(\frac{\partial^2 U}{\partial y^2} - \frac{U}{k}\right) - M^2 U \\ \frac{\partial \theta_2}{\partial t} &= (\phi_2 \,\text{Pr})^{-1} \frac{\partial^2 \theta_2}{\partial y^2} + \text{Ec}(\phi_1)^{-1} \ \left(\frac{\partial U}{\partial y}\right)^2 + \text{Ec}\frac{U^2}{k} \end{aligned}\right\} \qquad \ldots (7)$$

The boundary conditions (2), (4) and (5) reduce to

$$\left.\begin{aligned} & y = 1 \ : \ u = \cos \omega t \ , \ \theta_1 = \cos \omega t \\ & y = 0 \ : \ u = U \ , \ \theta_1 = \theta_2 \ , \ \phi_1 \frac{\partial u}{\partial y} = \frac{\partial U}{\partial y} \ , \phi_2 \frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y} \\ & y = -1 : \ U = \cos \omega t \ , \ \theta_2 = \cos \omega t \end{aligned}\right\} \qquad \ldots (8)$$

In order to solve the equations (6) and (7), we assume the solutions of the following form

$$\left.\begin{aligned} u\ (y,t) &= \text{Real part of } [\, f(y)\, e^{i\omega t} \,] \\ U\ (y,t) &= \text{Real part of } [\, F(y)\, e^{i\omega t} \,] \\ \theta_1\ (y,t) &= \text{Real part of } [\, h(y)\, e^{i\omega t} \,] \\ \theta_2\ (y,t) &= \text{Real part of } [\, H(y)\, e^{i\omega t} \,] \end{aligned}\right\} \qquad \ldots (9)$$

Substituting (9) in equation (6) and (7) and solving under the corresponding boundary conditions (8), we get the solutions as

$$\begin{aligned} u\ (y,t) = {} & e^{a_1 y}\,[\, g_9 \cos(\omega t + b_1 y) - g_{10} \sin(\omega t + b_1 y) \,] \\ & + e^{-a_1 y}\,[\, h_3 \cos(\omega t - b_1 y) + h_4 \sin(\omega t - b_1 y) \,], \end{aligned}$$

$$\begin{aligned} U\ (y,t) = {} & e^{a_2 y}\,[\, h_5 \cos(\omega t + b_2 y) - h_6 \sin(\omega t + b_2 y) \,] \\ & + e^{-a_2 y}\,[\, h_7 \cos(\omega t - b_2 y) - h_8 \sin(\omega t - b_2 y) \,], \end{aligned}$$

$$\begin{aligned} \theta_1\ (y,t) = {} & e^{k_{11} y}\,[\, t_7 \cos(\omega t + k_{12} y) - t_8 \sin(\omega t + k_{12} y) \,] \\ & + e^{-k_{11} y}\,[\, t_{12} \cos(\omega t - k_{12} y) - t_{13} \sin(\omega t - k_{12} y) \,] \\ & - e^{2a_1 y}\,[\, k_{16} \cos(\omega t + 2b_1 y) - k_{17} \sin(\omega t + 2b_1 y) \,] \\ & - e^{-2a_1 y}\,[\, k_{18} \cos(\omega t - 2b_1 y) - k_{19} \sin(\omega t - 2b_1 y) \,] \\ & - 2\, Ec\, [\, k_{10} \cos\omega t + k_9 \sin\omega t \,] / \omega\ , \end{aligned}$$

$$\begin{aligned} \theta_2\ (y,t) = {} & e^{r_1 y}\,[\, t_{14} \cos(\omega t + r_2 y) - t_{15} \sin(\omega t + r_2 y) \,] \\ & + e^{-r_1 y}\,[\, t_{16} \cos(\omega t - r_2 y) - t_{17} \sin(\omega t - r_2 y) \,] \\ & - e^{2a_2 y}\,[\, r_6 \cos(\omega t + 2b_2 y) - r_7 \sin(\omega t + 2b_2 y) \,] \\ & - e^{-2a_2 y}\,[\, r_8 \cos(\omega t - 2b_2 y) - r_9 \sin(\omega t - 2b_2 y) \,] \\ & - 2\, Ec\, [\, p_{22} \cos\omega t + p_{21} \sin\omega t \,] / \omega\ . \end{aligned}$$

Now after knowing the temperature fields, we can calculate the rate of heat transfer in dimensionless form as

$$\begin{aligned} -q_1 = \left(\frac{\partial\theta_1}{\partial y}\right)_{y=0} = {} & [k_{11} t_7 - k_{12} t_8 - k_{11} t_{12} + k_{12} t_{13} - 2a_1 k_{16} + 2b_1 k_{17} + 2a_1 t_{18} \\ & - 2b_1 k_{19}] \cos\omega t - [k_{11} t_8 + k_{12} t_7 - k_{11} t_{13} - k_{12} t_{12} \\ & - 2a_1 k_{17} - 2b_1 k_{16} + 2a_1 t_{19} + 2b_1 k_{18}] \sin\omega t \end{aligned}$$

$$-q_2 = \left(\frac{\partial\theta_2}{\partial y}\right)_{y=0} = [r_1 t_{14} - r_2 t_{15} - r_1 t_{16} + r_2 t_{17} - 2a_2 r_6 + 2b_2 r_7 + 2a_2 r_8 - 2b_2 r_9\,] \cos\omega t$$

$$-[r_1 t_{15} + r_2 t_{14} - r_1 t_{17} - r_2 t_{16} - 2a_2 r_7 - 2b_2 r_6 + 2a_2 r_9 + 2b_2 r_8]\sin \omega t$$

Constants involved in the solutions are not given due to sake of brevity.

3. Discussion

The flow of an incompressible conducting viscous fluid in a region bounded by two parallel periodically heated oscillating plates has been studied. The region is half filled with porous material. The velocity field, temperature distribution and rate of heat transfer have been obtained and shown in figures for various values of parameters. The following conclusions have been drawn. An examination of Fig.1 shows that the velocity decreases exponentially as the fluid moves towards the interface, in both the regions. By increasing the strength of the magnetic field the decay is greater, over all for the same values of k. At any instance the particle velocities attain a maximum either on the plate or somewhere within the fluid in both the regions. Furthermore the maximum velocities are located mostly on the oscillating plates (i.e. they are the velocities of the plates itself). When $\omega t = \pi/2$, the maximum velocities are in the fluid not far from the plates. The Fig.2, gives the temperature distribution which is evenly distributed in both the regions. It is found that it increases near the plates and decreases away from the plates. When $\omega t = \pi/2$ the temperature increases near the plates and decreases as the distance from the surface increases. The temperature in the case of water (Pr = 7) is less than that of air (Pr = 0.71) in both the situations ($\omega t = 0$ and $\omega t = \pi/2$). There is a temperature wave, which is rapidly fading away with increasing depth inside the region for Pr = 7. Like velocity distribution, the temperature attains its maximum values either on the plates or somewhere within the fluid in both the regions. When $\omega t = 0$, the maximum temperature occurs on the oscillating plates, or they are the temperature of the plates. However, for $\omega t = \pi/2$ the temperature is maximum in the fluid not far from the plates. Figures 3 and 4 give the rate of heat transfer against M for various values of parameters at the interface. Fig.3, ($\omega t = 0$) shows that the heat transfer is greater in clear fluid region than that in porous region for water (Pr = 7), while the reverse effect is observed in the case of air (Pr = 0.71). However, over all the heat transfer remains constant with large values of M and it is greater in water than air. If we take $\omega t = \pi/2$, here we found that in the case of air, the heat transfer is greater in clear fluid region than that of porous matrix, which is a reverse phenomena caused when $\omega t = 0$, again reverse effect is observed for Pr = 7 (water). For increasing M the heat transfer almost remains constant, but when $M < 2$ it decreases slightly in water. It is observed that periodicity in the boundary temperature does not affect rate of heat transfer, however, the thermal characteristics are altered.

4. Conclusions

The above studies on the flow of a conducting liquid between two parallel periodically heated oscillating plates in the presence of magnetic field lead to the following conclusions.

1) The velocity decreases with the increase in strength of magnetic field in both the situations ($\omega t = 0$ and $\omega t = \pi /2$).
2) When $\omega t = 0$, the maximum velocity and temperature occur at the oscillating plates. For $\omega t = \pi/2$, the maximum velocity and temperature occur between the plates.
3) The temperature increases near the plates and decreases away from the plates.
4) In both the situations ($\omega t = 0$ and $\omega t = \pi /2$) the temperature in the case of water (Pr =7) is less than that of air (Pr = 0.71).
5) When $\omega t = 0$, the rate of heat transfer is greater in clear fluid region than that of porous medium for water (Pr = 7).
6) The rate of heat transfer almost remains constant with increasing strength of magnetic field.

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Captions of the figures

Figure .1 : Velocity profiles for ω = 8, k = 0.5 and ϕ_1 = 0.4

Figure .2 : Temperature profiles for ω = 8, M = 5, ϕ_1 = 0.4, ϕ_2 = 0.6 and Ec = 0.01

Figure .3 : The rate of heat transfer for ω = 8, ωt = 0, Ec = 0.01, ϕ_1 = 0.4, ϕ_2 = 0.6 and k = 0.5

Figure .4 : The rate of heat transfer for ω = 8, ωt = π / 2, Ec = 0.01, ϕ_1 = 0.4, ϕ_2 = 0.6 and k = 0.5

Appendix- Nomenclature

B_0 = magnetic field,

C_p = specific heat at constant pressure,

h = distance between the interface and the plate,

k* = permeability parameter,

k = dimensionless permeability parameter,

M = Hartmann number,

Pr = Prandtl number,

t* = time,

t = dimensionless time,

T_0 = mean temperature,

T_1^* = temperature of upper plate,

T_2^* = temperature of lower plate,

u* = velocity in clear fluid region,

u = dimensionless velocity in clear fluid region,

U* = velocity in porous region,

U= dimensionless velocity in porous region,

U_0 = mean velocity,

x* = direction along the plate,

x = dimensionless direction along the plate,

y* = direction perpendicular to the plate,

y = dimensionless direction perpendicular to the plate,

Greek symbols

α = thermal diffusivity,

ϕ_1 = ratio of kinematic viscosity,

ϕ_2 = = ratio of thermal conductivity,

κ= thermal conductivity,

κ_{p} = thermal conductivity of porous medium,

μ = viscosity,

μ_p = viscosity of the porous medium,

ν = kinematics viscosity,

ν_p = kinematics viscosity in the porous medium,

θ_1 = dimensionless temperature of upper plate,

θ_2 = dimensionless temperature of lower plate,

ρ = density,

σ = electrical conductivity of the fluid,

ω = frequency of the flow variables,

ω* = dimensionless frequency of the flow variables.