# The Performance of Heaving Bodies

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The Caribbean region has been identified as a favourable area for the exploitation of wave energy. A cost effective approach to the development of this resource is to use hard skinned heaving bodies. A mathematical model for heaving bodies is developed and used to determine the performance for such bodies under the Eastern Caribbean wave regime for rectangular, circular and elliptical geometries. The results indicate that performance is a strong function of the dimensionless frequency parameter. As the wave regime is seasonal, active tuning is indicated for extraction optimization.

### 1. Introduction

Wave energy possesses the highest energy density of the renewable energy sources. The islands of the Eastern Caribbean all have coastlines that are exposed to the North Atlantic Ocean. Furthermore, the wave climate of the region is dominated by the Trade Winds that approach with great consistency from the north-east [1].

This renewable resource is yet to be exploited despite the fact that the Caribbean Islands have been identified as favorable sites for the application of wave extraction systems [2].

Compared with other forms of renewable sources, the exploitation of wave energy is lagging. This is due, in the main, to the high cost of wave extraction systems. This can be attributed to two main factors. These are the high costs of the supporting infrastructure and the extraction device itself. It should be noted that the cost of the extraction device is a function of the technical complexity of the device. To facilitate the feasibility of conducting research on this significant potential source of renewable energy, an appropriate strategy is required, one that reduces costs by a significant amount.

In the eastern seaboard of Trinidad, where the wave energy potential is the highest, are located several platforms for the exploitation of natural gas. These are all located in deep water, which is required for wave energy exploitation, and can provide necessary infrastructure for the the systems. Secondly, conversion the technology for the fabrication of hard skinned heaving devices is available locally  $\{3, 4, 5, 6\}$ . Thus the approach adopted was to focus on hard skinned heaving bodies, which can be located on existing offshore platforms, thereby facilitating a cost effective approach. The present research thrust in the development and performance evaluation of heaving-body wave extraction systems, operating in the wave regime of the Caribbean.

Other types of wave extraction systems are described elsewhere [7, 8, 9, 10, 11] and are not considered here for the reasons noted above.

In this paper, a theoretical evaluation of the performance of heaving bodies under the influence wave conditions of the Eastern Caribbean is presented.

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## 2. Analysis

The analysis assumes only heaving motion of a hard skinned device located in deep water and hence linear wave can be utilized. It is further assumed that the body is always stable and it does not leave the surface of the water or becomes fully submerged.

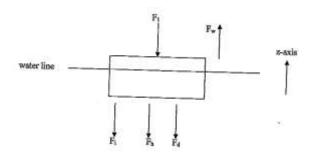


FIGURE 1: Free Body Diagram

In Figure 1, the free body diagram with the forces acting of the heaving device is shown. The linearized equation for heaving motion is given by:

$$M_{f} \{ d^{2}z / dt^{2} \} = F_{w} - (F_{l} + F_{i} + F_{h} + F_{d})$$
(1)

Where

- *z* displacement of heaving body from the neutral position.
- $F_w$  wave induced force due to scattering.
- $F_i$  inertial reaction on body due to the induced motion of the added mass.
- $F_h$  hydrostatic restoring force due to changes in the buoyancy forces.
- $F_d$  wave induced force that accounts for losses due to radiation of the waves.
- $F_l$  external force induced by fluid on the body.

And

 $M_f$  mass of the heaving body.

The forces  $F_w$ ,  $F_i$ ,  $F_h$ ,  $F_d$ , and  $F_1$  were evaluated, the details of which are given elsewhere[1,] and substituted into equation (1) to obtain the equation of motion for the heaving body, which is given in equation (2):

 $(M_{\rm f} + M_{\rm a}) \{ d^2 z / dt^2 \} + (C_{\rm w} + C_{\rm l}) \{ dz / dt \} + K_{\rm w} z = F_0 \cos \omega t$ (2)

Where

 $M_f$  mass of the heaving body.

 $M_a$  total added mass.

- $C_w$  wave damping force coefficient.
- $C_l$  load damping coefficient.

 $K_w$  stiffness coefficient

And

 $F_0$  total exciting force amplitude.

Equation (2) is a second order differential equation for forced harmonic oscillation and thus yields a closed form solution, which gives the response of the heaving body.

# 2.1. Power Extraction

The power of the heaving body is the mean rate of work that is being done on the body by the by the working fluid, the water wave. This is equivalent to the force of energy extraction multiplied by the velocity of the body. Assuming a viscously damped load and a regular wave and using the velocity of the heaving body obtained from equation (2), the extraction power of the heaving body, P, is thus given by:

$$P = \{C_L \omega^2 Z_0^2\} / 2$$
 (3)

Where

 $Z_0$  amplitude of heave oscillation.

And

 $\omega$  wave frequency.

### 2.2. Efficiency of Power Extraction

The efficiency of the extraction device is defined as the ratio of the power, P, absorbed by the heaving body per unit length to the power available per unit frontage of the incident wave,  $P_w$ . The maximum value for this efficiency is 50 %.

$$P_w = (\rho g^2 a^2)/4\omega$$

The efficiency of the device,  $\eta$ , is defined as the ratio of the power absorbed by the heaving body per unit length to the power available per unit frontage of the incident wave and is given by:

 $\eta = (2\omega^{3}C_{L} Z_{0}^{2}) / (\rho g^{2}a^{2})$ (4)

Where

- a amplitude of the wave
- $\rho$  density of sea water
- g accelerant due to gravity

## 3. Results

The maximum extraction efficiencies of circular, rectangular and elliptical heaving bodies were determined using wave data for the Eastern Caribbean [1]. These were plotted against a dimensionless frequency parameter,  $\varepsilon_0$ , with m' and  $H_0$  as parameters where

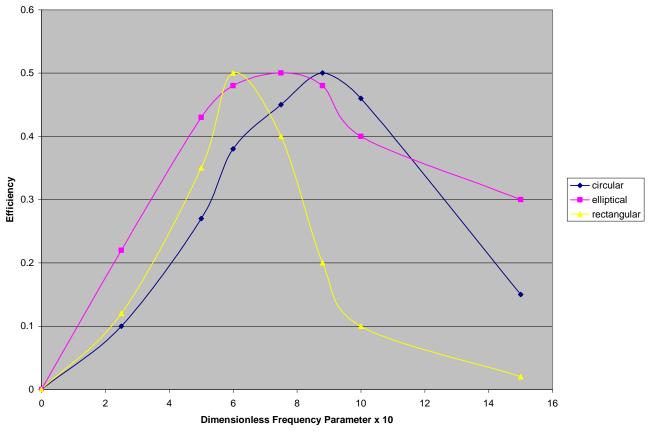
- $\begin{array}{ll} m' & dimensionless \mbox{ mass factor parameter} \\ \{M_f \, / \, M\} \end{array}$
- H<sub>0</sub> half breath to draft ratio

and

 $\varepsilon_0 = (\omega^2 b / 2g)$ 

In Figure 2, results are presented for m' = 1 and  $H_0 = 2$ . It can be seen that all three geometries achieve the maximum extraction efficiency of 50% but at different values of the dimensionless frequency parameter. For the case shown, the rectangular heaving body reaches the maximum efficiency at the lowest value of the dimensionless frequency parameter. As the wave regime at any location would be characterized by particular seasonal zero crossing period the design of the heaving body will have to be tuned to maximize the extraction efficiency. In the Caribbean, the two dominant wave periods are 7 seconds and 9 seconds. For 80% of the year, the former is predominant and hence it would reasonable to design the extraction systems using this value.

This would mean that for the other 20% of the year the efficiency would drop. This points to two approaches to the optimization of energy extraction. One would be to optimize for seasonal efficiency and the other for year round efficiency. In both cases a need for active tuning is indicated.



 $(m' = 1, H_0 = 2)$ 

**FIGURE 2:** Efficiency vrs Dimensionless Paremeter,  $\varepsilon_0$ 

#### 4. Conclusions

A theoretical model was developed for predicting the performance of heaving bodies. It was used to determine the extraction efficiency of heaving bodies of rectangular. circular and elliptical geometries. It was found that they all achieve the maximum extraction efficiency of 50% of the wave power, though at different values of the dimensionless frequency parameter. This means that for every specific wave period value, the heaving body will need to be adjusted to achieve maximum extraction efficiency. It is thus recommended that active tuning be investigated in order to optimize seasonal extraction efficiency or optimization studies

be performed for maximum energy extraction over the year.

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