Modelling the Isothermic Heating Process in a Charcoal Bed of a Solar Powered Adsorption Cooling System

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(Received 26 March 2013; Revised 17 July 2013; Accepted 13 September 2013)

Abstract: This paper addresses a problem associated with Solar-Powered Adsorption Cooling (SPAC) systems. The problem is the difficulty in knowing if the complete charcoal bed reaches desorption temperature during the isosteric heating process. In addressing the issue, the process is modeled and the results are compared with those from experiments. The temperature-profile modelling of the isosteric-heating process establishes a time-related formulation that gives the temperature, at any radius, across a cylindrical shaped bed comprising of charcoal/methanol pair of adsorbent/adsorbate; and the results from modelling compare favorably with the measured temperature-profiles obtained from the experiments.

Keywords: Solar-powered adsorption cooling; isosteric heating; thermal conductivity; renewable energy technology

1. Introduction

Renewable energy technology applications are on the increase as the world continues to reduce its dependence on fossil fuel. One such application is Solar-Powered Adsorption Cooling (SPAC) which is a refrigeration process that utilises the sun’s energy as its power source. A problem with the SPAC system is the difficulty in predicting if the complete charcoal bed reaches the desorption temperature during the isosteric heating process; and if it does, how long does the bed take to reach this temperature which enables the refrigerant to desorb (evaporate) from the adsorbent. This problem arises due to the fact that installation of temperature probes in the system can compromise the system, especially for systems operating under vacuum.

Verifying, without being invasive, that the entire bed reaches the desorption temperature is of significance since the amount of refrigerant that desorbs from the bed is critical to the efficiency of the SPAC system, given that the cooling effect is highly dependent on the amount of refrigerant cycling through the system. Therefore, the desorption temperature/process which is a function of heat conduction through the bed constitutes a major theoretical engagement for SPAC systems (Li and Wang, 2003; Sakoda and Suzuki, 1984; Wang and Oliveira, 2005).

This investigation develops a mathematical model that predicts the temperature at any radius/time in a cylindrical copper-tube adsorption/desorption bed during the isosteric-heating process. The research also describes the attending experiments used to analyse the results of the mathematical model.

2. About Solar-Powered Adsorption Cooling-System

SPAC systems have no moving parts (see Figure 1). There are three main components: namely 1) adsorption/desorption bed that is usually built with flat rectangular sheets or cylindrical tubes; 2) a condenser; and 3) an evaporator. The system utilises simple controls, simple designs, and readily available materials such as charcoal, stainless steel, aluminum and copper.

![Figure 1. Schematics of a No Valve SPAC System](image-url)

The system is powered by an extensive temperature range: 50°C, to above 150°C (Wang and Oliveira, 2005) and comprises a 2-phase cycle (see Figure 2). The first phase is the heating-desorption-condensation phase (1-2-
3) and the second is the cooling-evaporation-adsorption
phase (3-4-1). The first phase utilises solar radiation for
the heating of the adsorbent bed. The bed is usually
made from metal tubes and contains activated charcoal
with an adsorbate, usually methanol, adsorbed into its
pores (Jing and Exell, 1993).

Evaporation. During this process the adsorbent
temperature continues decreasing, which induces
adsorption of vapour. This adsorbed vapour is
vapourised in the evaporator. The evaporation heat is
supplied by the heat source at low temperature.

3. Mathematical Modeling of Isothermic Heating
Process in a Charcoal Bed

The isothermic heating process takes place in a charcoal
bed which is comprised of activated granular charcoal
that has methanol adsorbed in its pores and is encased in
a 19mm diameter by 1 metre long copper-tube. The
mathematical model formulated in this investigation
differs from other models mainly in its treatment of the
effective thermal conductivity, \( K_e \), for the bed. The
formulation was developed in the following stages and
with the following assumptions.

Solar Energy (irradiation) over a period of time heats up
the copper tube which conducts heat through the
methanol-charcoal bed: the average temperature \( T_f \)
of the copper tube over the time period becomes the
boundary temperature of the bed (system); and energy
conducted increases the bed temperature to the
desorption temperature, \( T_{des} \) (isothermic/sensible heating
of the bed. No concentration change).

The main assumption is that the specific heat of the
adsorbed methanol is the same as that of bulk liquid
methanol.

For this investigation, the isothermic temperature-
profile of the bed, \( T_{des} \), (heating the bed to the point
where it reaches the desorption temperature, \( T_{des} \)), is
modelled as one-dimensional (radial) heat conduction in
an infinite cylinder.

The general heat equation for the bed is given as:

\[
\frac{\partial T}{\partial t} = \frac{k_e}{\rho C_p} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \dot{Q} \tag{1}
\]

where \( \dot{Q} = \dot{Q}_{solar} \) is the heat source term for the heat
of desorption (Li and Wang, 2003; Demir et al., 2008).
Anyanwu et al. (2001) pointed out that during isothermic
heating of the bed \( \frac{dT}{dt} = 0 \); hence equation (1) is
reduced to:

\[
\frac{\partial T}{\partial t} = \frac{k_e}{\rho C_f} \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \tag{2}
\]

where \( C_p \) is the combined charcoal/methanol specific
heat:

\[
C_p = \left[ \varepsilon C_p f \right] + \left[ \left( 1 - \varepsilon \right) C_p s \right] \tag{2a}
\]

where, subscripts \( f \) and \( s \) are fluid and solid, respectively
and \( \varepsilon = \) porosity of bed. Effective thermal conductivity,
\( k_e \), is developed as:

\[
k_e = k_f \left[ \frac{\varepsilon k_s / k_f}{\varepsilon k_s / k_f + (1 - \varepsilon) k_f / k_s} \right] \tag{2b}
\]

where, \( k_s \) and \( k_f \) are thermal conductivity of solid
(charcoal) and fluid (methanol), respectively.
This treatment of the effective thermal conductivity is adopted from Kaviany (1995) as one of the predictive equations for effective thermal conductivity for packed, porous, beds. Kaviany (1995) states that this particular formulation is given by Hashin and Shtrikman (1962) variational formulation (upper bound); and its effectiveness is dependent on the ratio of the thermal conductivities of the solid (charcoal), $k_s$, to that of the fluid (methanol), $k_f$, being equal to or greater than one ($k_s/k_f \geq 1$). This formulation is chosen because its configuration is dependent on the porosity, $\varepsilon$, of the charcoal bed. Also, the charcoal particles used are assumed to be spherical and also the arrangement of the particles in the bed is simple cubic. For such an arrangement, Kaviany (1995) gives the porosity, $\varepsilon$, as 0.476. Therefore, the effective thermal conductivity is reduced to simply a matter of geometry for a given solid/liquid combination.

4. Modeling of Bed Temperature-Profile
Regarding heat conduction in an infinite cylinder, with a length to diameter ratio of 1000/19, the heat flow through the tube can be assumed as one dimensional in the radial direction. This length, with order of magnitude 3, can be taken as an infinite cylinder (Trim 1990).

The non-dimensional form of Equation (2) gives:
\[ \frac{\partial \Theta}{\partial \tau} = \left[ \frac{\partial^2 \Theta}{\partial \xi^2} + \frac{1}{\tau} \frac{\partial \Theta}{\partial \tau} \right] \quad 0 < \xi < 1; \tau > 0 \]  
(3)

where: \( \Theta = \frac{T - T_{\infty}}{T_{\infty}} \); \( \tau = \frac{r^2}{k_s \rho C_p} \); with \( a^2 = k_s / \rho C_p \)

With boundary conditions:
\[ \left[ \begin{array}{c} 0 \lfloor (1, \tau) = 1 \\ 0 \lfloor \frac{\partial \Theta}{\partial \xi} \right. \right. \quad \tau > 0 \]  
(3a)

\[ 0 \lfloor (r, 0) = 0 \\ 0 \lfloor \frac{\partial \Theta}{\partial \xi} \right. \quad 0 < \xi < 1 \]  
(3b)

Transforming equation 3 to:
\[ \Theta(\xi, \tau) = \Psi(\xi, \tau) + \Psi^{*}(\xi) \]  
(4)

\( \Psi(\xi, \tau) \) is the transient portion of the temperature-profile and \( \Psi^{*}(\xi) \) is the steady state portion.

Developing the steady state solution:
\[ \frac{\partial^2 \Psi^{*}}{\partial \xi^2} + \frac{1}{\tau} \frac{\partial \Psi^{*}}{\partial \tau} = 0 \]  
(4a)

and applying boundary conditions of:
\[ \Psi^{*}(1) = 1 \]  
(4b)

\[ \frac{\partial \Psi^{*}}{\partial \xi}\bigg|_{\xi=0} = 0 \]  
(4c)

Therefore, at steady state the temperature-profile is:
\[ \Psi^{*}(\xi) = 1 \]  
(5)

The transient portion of the temperature-profile of the porous bed is developed as follows:
\[ \frac{\partial \Psi(\xi, \tau)}{\partial \tau} = \left[ \frac{\partial^2 \Psi(\xi, \tau)}{\partial \xi^2} + \frac{1}{\tau} \frac{\partial \Psi(\xi, \tau)}{\partial \tau} \right] \]  
(6)

with boundary conditions
\[ \Psi(1, \tau) = 0 \]  
(6a)

\[ \frac{\partial \Psi}{\partial \xi}\bigg|_{\xi=0} = 0 \]  
(6b)

\[ \Psi(1, 0) = 1 \]  
(6c)

Let:
\[ F(\xi) = \mathcal{F}(\xi) \mathcal{S}(\xi) \]  
(6d)

\( F(\xi) \) is a function of radius only, and \( \mathcal{S}(\tau) \) is a function of time only.

Differentiating Equation (6d) and substituting in equation (6) gives:
\[ \frac{\partial^2 \Psi}{\partial \xi^2} + \frac{1}{\tau} \frac{\partial \Psi}{\partial \tau} = -\lambda^2 \]  
(6e)

where \( \lambda \) is a non-zero constant

Analysing \( \mathcal{S} \) and \( F \) from equation (6e), gives
\[ \mathcal{S}(\tau) = c \exp(-\lambda^2 \tau) \]  
(7)

\( C \) is an arbitrary constant

\[ F(\xi) = \mathcal{F}(\xi) \mathcal{S}(\xi) \]  
(8)

Bessel equation of order 0
And from BC:
\[ G = 0 \]

yielding general solution for equation (6d):
\[ \Psi(\xi, \tau) = \Psi^{*}(\xi) + \int_0^\infty \mathcal{F}(\xi) c \exp(-\lambda^2 \tau) \mathcal{S}(\tau) d\tau \]  
(9)

with boundary conditions
\[ \Psi(1, \tau) = 0 \]  
(9a)

This implies that
\[ J_0(\lambda R) = 0 \]  
(9b)

So that
\[ \lambda = \mu \nu \omega R \]  
(9c)

Thus, the transient temperature-profile of the porous bed is:
\[ \Psi(\xi, \tau) = \int_0^\infty \mathcal{F}(\xi) c \exp(-\lambda^2 \tau) \mathcal{S}(\tau) d\tau \]  
(10)

and the normalised Eigen functions give:
\[ \Psi_0(\xi) = \int_0^\infty \mathcal{F}(\xi) c \exp(-\lambda^2 \tau) \mathcal{S}(\tau) d\tau \]  
(11)

\( J_0 \) and \( J_1 \) are Bessel Function of orders zero and one, respectively.

Using Trim’s (1990) formulation, \( A_n \) is derived as:
\[ A_n = \frac{-\frac{\lambda^2}{\lambda_n^2}}{\lambda_n^2} \]  
(11a)

Therefore, the predictive temperature-profile for the porous bed \( \theta(\xi, \tau) \) is:
\[ \theta(\xi, \tau) = 1 - 2 \int_0^\infty \Psi_0(\xi) c \exp(-\lambda^2 \tau) \mathcal{S}(\tau) d\tau \]  
(12)

The above system of equations was programmed in Matlab and the experimental model, described below, was used to validate the system of equations by comparing the experimental data with those derived from the equations.

5. Experiments with the SPAC System
The SPAC experimental system was modeled as a 19mm diameter x 1,000 mm long copper- tube bed with 12mm diameter copper- tube coil used to build both condenser and evaporator. Figure 3 shows the construction of the SPAC system. The system was designed with no valves and is similar to the no-valve, solar ice-maker found in the literature (Li et al., 2004), which requires minimum supervision. The bed comprised of activated granular charcoal/methanol pair. The condenser and evaporator
Step 2: The tube was placed in an oil bath and heated to 5.1°C while at the same time subjected to a vacuum of 710 mm (or 28”) Hg. This condition was held for one hour to ensure the removal of moisture from the charcoal.

Step 3: After one hour both the vacuum and oil bath were switched off and the charcoal was exposed to the methanol. The system was left overnight (for convenience) and the amount of methanol adsorbed was recorded. This procedure was carried out twice and in both instances the amount of methanol adsorbed was 11 grams. This gives a concentration ratio (adsorptive capacity) of 1:2 (kg methanol/kg charcoal).

5.2 Experimental Setup of SPAC System

The metre long copper tube was charged with 120 gram of the activated charcoal and placed under vacuum for four hours after which 60 gram of methanol was drawn into the tube. This amount of methanol was to satisfy the 1:2 ratio established as the adsorptive capacity of the charcoal. The system stabilised at 405 mm (or 16”) Hg vacuum. At this level, the boiling point (Tboil) of methanol drops from 65°C at one atmosphere to 52°C (Properties of Fuel, retrieved 2008).

The complete SPAC system (including bed, condenser, and evaporator) was placed on the roof of the lab. Figure 4 shows the experimental setup of the SPAC system. Thermocouple probes were attached to the system and connected to a data logger for recording of the various temperatures. The thermocouple probes were located:

- On the surface of the copper tube, shielded from direct sunlight (tube temperature, Tc)
- At the centre of the charcoal/methanol bed (bed temperature, Tbed)
- At the outlet water stream of the condenser (condenser temperature, Tcond)
- In the water which covered the evaporator coil (evaporator temperature, Tevp)
- In the ambient air, shielded from sunlight and wind velocity (Tamib)

A pyranometer was also attached to the logger to record solar irradiation.

5.1 Determination of the Affinity of the Charcoal for Methanol

In order to determine the adsorptive properties of the charcoal, the steps adopted from Jing and Exell (1993) were taken. These steps are:

Step 1: Twenty-two (22) grams of charcoal were encased in a 150 mm long by 19 mm diameter copper tube.

Step 2: The tube was placed in an oil bath and heated to 120°C while at the same time subjected to a vacuum of 710 mm (or 28") Hg. This condition was held for one hour to ensure the removal of moisture from the charcoal.

Step 3: After one hour both the vacuum and oil bath were switched off and the charcoal was exposed to the methanol. The system was left overnight (for convenience) and the amount of methanol adsorbed was recorded. This procedure was carried out twice and in both instances the amount of methanol adsorbed was 11 grams. This gives a concentration ratio (adsorptive capacity) of 1:2 (kg methanol/kg charcoal).

Table 1 shows typical temperature data recorded from the SPAC system.

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<th>Bed T</th>
<th>Tube T</th>
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For each day of the experiment, particular focus was placed on the bed during the isosteric heating period when the average temperature of the tube ranged from 54-56°C. The intent was to record the time it took the centre of the bed to reach the tube temperature. These results were compared to those generated by the mathematical model.

6. Results

Table 1 shows typical temperature data recorded from the SPAC system.

Figure 3. Construction of SPAC system

Figure 4. Experimental setup of SPAC System
Having established that the isosteric temperature profile of the bed at any radius and time can be modelled non-dimensionally as:

\[ q_{(\lambda S)} = 1 - 2 \sum_{i=1}^{n} \exp \left( - \frac{1}{\lambda_{i} S} \right) \left( \frac{Q_{(\lambda S)} \Delta T_{i}}{\lambda_{i} S} \right) \]  

Running the model in MATLAB demonstrated that with the outer (copper-tube) surface of the bed held at a temperature of 54°C, it took approximately four minutes for the centre of the bed to reach this temperature, starting at 49°C (see Figure 5).

![Figure 5. Time-Steps of temperature-profiles for charcoal/methanol bed with outer surface (copper tube) held at 54°C](image)

This demonstrates that the entire charcoal/methanol bed was capable of reaching the required temperature (\(T_{scc}\)) of 52°C for the methanol to desorb (evaporate) from the charcoal pores and migrate to the condenser, once there is sufficient solar irradiance to raise and keep the tube temperature at over 54°C. This is significant on cloudy days when the irradiance fluctuates rapidly within minutes.

When the results from the model were graphed and compared with those from the experiments, as shown in Figure 6 (a) and (b), it is observed that both sets of bed-temperature-profiles moved in-step over the same time period. That is, in (a) the model showed that with a constant tube temperature of 54°C, it took four minutes for the centre of the bed to approach 54°C starting from 49°C. Likewise in (b) within the same time period the bed centre moved from 48 to 55°C. The difference in the graphs in the first minute could be a result of thermal contact between tube and charcoal, where the model under-compensated for the contact. It is therefore reasonable to say that the results of the mathematical model seem to be in good agreement with the results from the SPAC experiments.

![Figure 6. Experiment and Mathematical Model Temperature Variations at Centre of Charcoal Bed](image)

4. Conclusions

This research set out to address a problem associated with SPAC systems. It is the difficulty in determining the temperature-profile across the complete charcoal bed during the isosteric-heating process. In addressing this issue, the research developed a model for the temperature-profile across a cylindrical copper-tube bed as:

\[ q_{(\lambda S)} = 1 - 2 \sum_{i=1}^{n} \exp \left( - \frac{1}{\lambda_{i} S} \right) \left( \frac{Q_{(\lambda S)} \Delta T_{i}}{\lambda_{i} S} \right) \]  

The close agreement between the values produced by this model and the results from experiments suggests that the model may be considered as a predictive “tool” to establish temperature conditions across charcoal adsorption/desorption beds of similar configurations (within the limitations stated earlier). It is therefore recommended that in order to establish the developed temperature-profile model as a truly predictive tool, further investigations are needed with tubes of different materials and diameters.

References:


Authors’ Biographical Notes:

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