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A Re-engineered Transmission Line Parameter Calculator

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Abstract: *This paper documents the development and testing of a Transmission Line Parameter Calculator (TLPC), which computes the impedance parameters for short and medium transmission lines. LPARA, an existing software at The Trinidad and Tobago Electricity Commission (T&TEC), has been taken as the standard for comparison, since it has been tested and proved consistent with Power World Software, as well as it has been satisfactorily employed for decades at T&TEC. Comparative testing of the newly developed TLPC with LPARA revealed a maximum percentage difference of 0.05%, 0.02% and 0.80% in Series Resistance, Series Reactance and Shunt Admittance Matrices, respectively. The package, when compared to its FORTRAN based predecessor, LPARA, has a user friendly Graphical User Interface (GUI) with an expandable database of support structures and conductors. The TLPC has interactive program help, error checking, and validation of all user inputs. It is tailored to T&TEC, but yet flexible enough for use by other similar electric utilities. The finished product has demonstrated a vast improvement in the overall speed of parameter calculations, the reduced susceptibility to input errors and it has addressed recent compatibility issues which LPARA experiences as T&TEC upgrades and transitions to 64-bit Operating Systems.*

Keywords: *Power transmission line, power system planning, transmission line theory, admittance impedance matrix*

1. Introduction

Transmission line oriented parameter software is readily available for most electric utilities to purchase, however many utilities opt for software which is custom developed to the requirements of their existing physical topological as well as software infrastructure. The Trinidad and Tobago Electricity Commission (T&TEC) employs a custom written command line interface which utilises a text file input and produces a text file output with the transmission line parameters for a given type of support structure. The input text file is manually created by the utility engineer and character spacing as well as character positioning is critical to prevent any errors from occurring when the input is supplied to the LPARA software. Moreover, the required inputs are only available after manual pre-processing of data to obtain the information required by the program. This renders the complete process for a line parameter calculation very tedious and the utility engineer exercise caution to prevent any errors from occurring when manipulating the input data. In recent times utility engineers at T&TEC have faced compatibility issues in the transition to 64-bit operating systems and thus T&TEC has expressed the need for a reengineered transmission line parameter calculation tool.

This paper details the theory, operation and validation of the developed TLPC to address the challenges presently faced. The primary transmission line parameters developed using TLPC include the line's series resistance matrix, the series reactance matrix and the shunt admittance matrix and using these matrices, several other line parameters, including sequence impedance parameters and sequence capacitance parameters can be derived. Studies by Galloway et al. (1964) allow these matrices to be calculated and quantities expressed in units per kilometer of overhead transmission line. LPARA, the existing software at T&TEC, has been verified using actual live line test data by Moorthy and Sharma (1988), as well as PSAF Software. LPARA has been employed by T&TEC for over 30 years with no reports of the unexpected triggering of protection relays. Thus, LPARA is used as the validation standard for comparison of the TLPC output.

2. Methodology

This section documents the processing done on the inputs supplied to the TLPC and the internal organisation of information. The main calculations which are performed utilise the work done by Galloway et al. (1964), and this theory is detailed in Appendix 1. Figure 1 shows a simple system overview of how information is processed by the TLPC and it also identifies the key processing modules present in the Line Parameter Package. The user must select a circuit configuration, after which the bundle parameters will be entered (only in the case of bundled circuits) and then the information is processed.

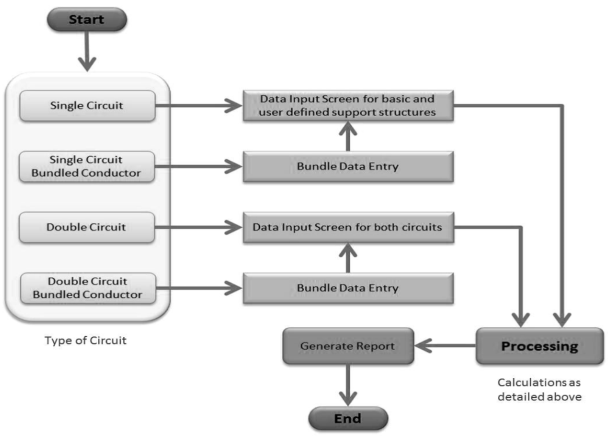

Figure 1. System flow diagram of the TLPC

Figure 2 identifies the series of processes which is done in the processing block of Figure 1, and upon completion; the TLPC will generate a Microsoft Excel 2010 Output Report. The report details both the user entered inputs and the calculated outputs. It shows the processing block of the TLPC consists of two separate processing paths, one for single conductor circuits and the other for bundled conductor circuits. Both paths are capable of handling both single circuits and double circuits. It shows that the Shunt Admittance Matrix is dependent only on the Conductor Coordinates, and the Series Impedance Matrix is calculated using Kron's Reduction on complex sum of the Self Impedance of the Conductor, the Impedance of the Earth Return Path and the Reactance due to Physical Geometry. When the Series Impedance Matrix is known, then further calculations can be performed to obtain the Derived Line Parameters. This is the general procedure for the Line Parameter Calculation, and if the transmission line consists of a bundled conductor, then an additional step must be performed to determine a revised Self Impedance of the Conductor. The Kron's Reduction procedure is then performed on the complex sum of the revised Self Impedance, the Impedance of the Earth Return Path, and the Reactance due to Physical Geometry matrices to determine the Series Impedance Matrix

3. Program Operation

It is evident that the system overview illustrated in Figure 1 and Figure 2 requires specific program inputs, and pre-processing of span length data, before the transmission line parameter calculations can be performed. These inputs include, but are not limited to:

- power system frequency,
- ambient and circuit operating temperatures,
- earth resistivity,
- line insulator length,
- aerial and phase line tensions,
- aerial and phase conductor types and specifications,
- sorted span lengths for each support structure, and
- support structure types and specifications

In the case where bundled conductors are selected, additional information is required which includes;

- number of conductors in the bundle, and
- conductor spacing.

All of these user inputs are validated by the program to ensure that the inputs are appropriate, before the user can proceed. Several, tips, warning, and error messages are also available to aid the user in navigating through the program and to rectify input errors.

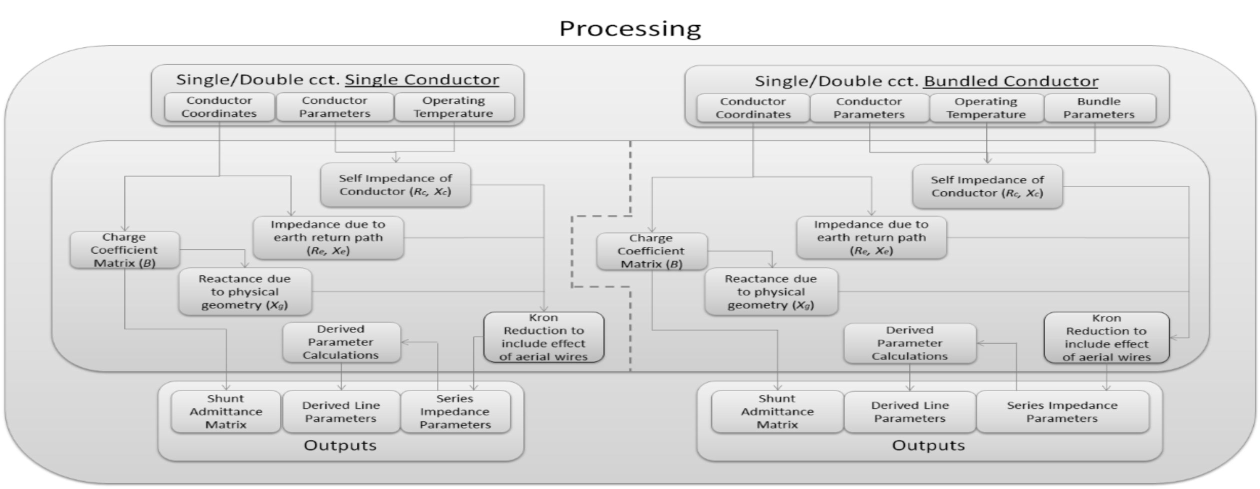

Figure 2. Processing block of the TLPC

4. Pre-processing

When the user has entered all of the required input data, the sequence of calculations documented in Equations 1 to 3 is performed. These Equations have been adapted from the Southwire Manual, (Southwire Company, 2007) and they allow the actual conductor coordinates to be determined using the user entered design coordinates. Firstly the conductor sag is calculated using Equation 1 and Equation 2.

Equation 1

$$Typical\ Span\ =\ Average\ Span\ +\ \frac{2}{3}\ (Max\ Span\ -\ Average\ span)$$

Equation 2

$$Sag = \left(\frac{Tension}{Weight}\right)\left(\cosh\left(\frac{Weight \times Typical\ Span}{2 \times Tension}\right) - 1\right)$$

Where

Tension = Tension of line conductor, and
Weight = Weight of line conductor

The conductor heights (y-coordinates) can then be determined using Equation 3, which is then paired with the x-coordinates to give the actual conductor coordinates.

Equation 3

$$Conductor\ Height\ =\ ConductorDesign\ Height\ -\ \frac{2}{3}(Sag)$$

In the event that the actual conductor coordinates are already known, the user shall enter these directly into the program with a high line tension and a short span length. Physically, the magnitude of these parameters represents the sag on the conductors, and such a combination of high tension and short span, results in negligible sag. Negligible sag hints that the actual conductor height is approximately equal to the user defined design height as can be seen from **Error! Reference source not found.**. This approach allows the program to handle calculated conductor coordinates and is illustrated in Figure 3.

These actual conductor coordinates are then passed to the main processing block of the program which performs the line parameter calculation as outlined in Galloway et al. (1964). This has been illustrated in Figure 1.

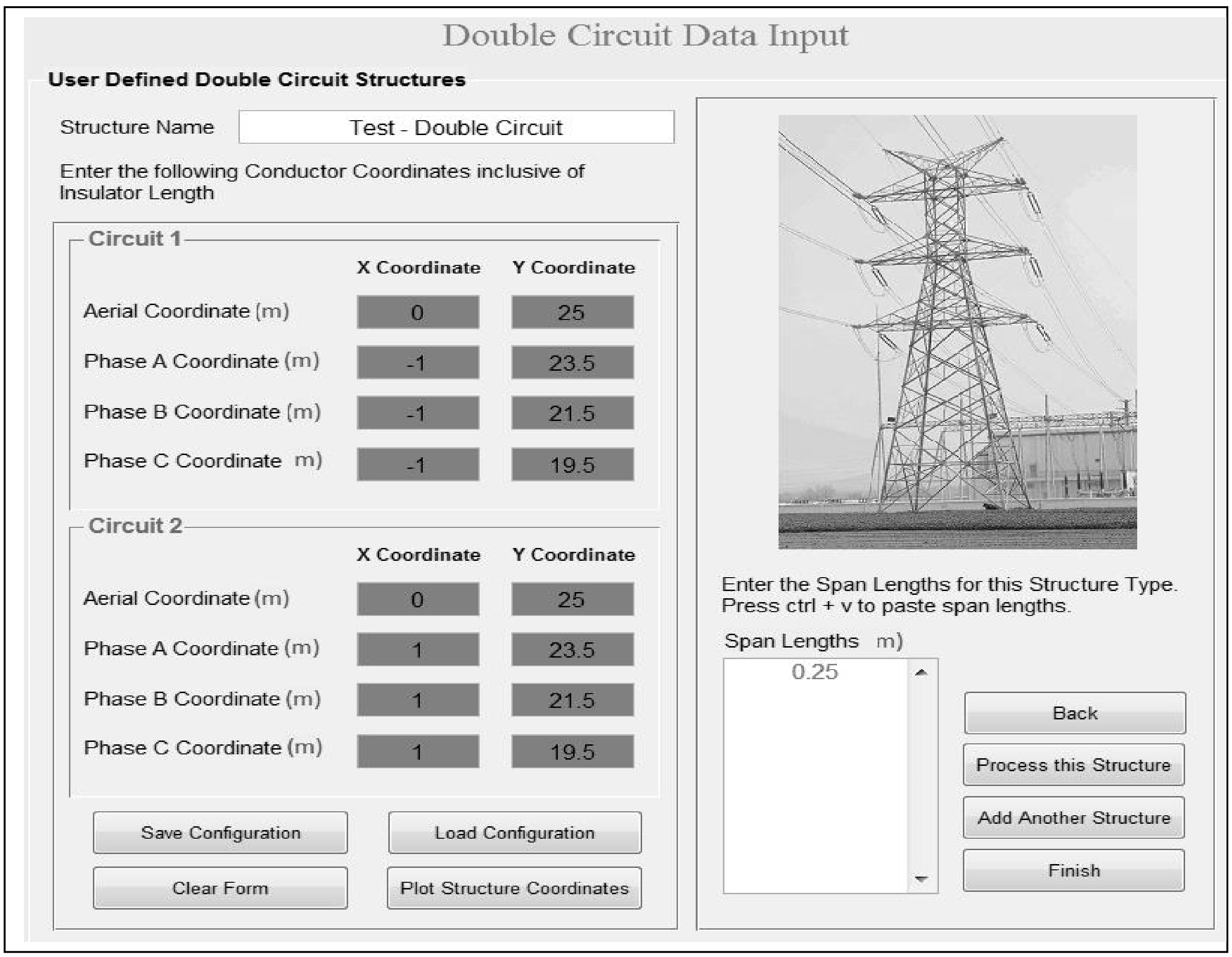

Figure 3. Entry of calculated conductor coordinates (with short span lengths)

5. Results

White box and black box testing was performed on each of the four separate processing paths (which each constituted a test case), *Single Circuit Single Conductor*, *Single Circuit Bundled Conductor*, *Double Circuit Single Conductor* and *Double Circuit Single Conductor*. The generated output of white box testing on a *Single Circuit Single Conductor* test case has been documented in this Test Results Section. The other three cases have not been included due to space restrictions but are available upon request. The inputs for Test Case 1 (*Single Circuit Single Conducto*r) are shown in Table 1 and Table 2 and the outputs, with comparison to that of LPARA are shown in Table 3.

Combined these four test cases demonstrates the total functionality and accuracy of the TLPC when compared to LPARA. The maximum percentage differences of all four Tests are presented in Table 4.

Table 1. Test Conductor Coordinates for Single Circuit, Single Conductor

	X Coordinate (m)	Y Coordinate (m)
Phase A	0.9	12
Phase B	0.9	11
Phase C	0.9	10
Aerial Wire	0.0	14

Table 2. Properties for Single Circuit, Single Conductor

	Aerial Wire	Phase Wire
Conductor Name	Raven	Osprey
Cable Diameter (cm)	1.1011	2.2330
Manufacture Unit Resistance (Ω/km)	0.5338	0.1323
Manufacture Unit Reactance (Ω/km)	0.0321	0.0192
Number of Conductors (per bundle)	1.0000	1.0000
GMD of Conductor (cm)	1.1011	2.2330

Table 3. Comparative output matrices for series resistance, reactance and shunt admittance for the Test 1

Series Resistance Matrix: Max. Percentage Difference = 0.0448%					
LPARA Software (Ω/km)			*TLPC (Ω/km)*		
0.246213	0.109317	0.105852	0.246161	0.109268	0.105805
0.109317	0.237372	0.101871	0.109268	0.237326	0.101828
0.105852	0.101871	0.231172	0.105805	0.101828	0.231131

Series Reactance Matrix: Max. Percentage Difference = 0.0117%					
LPARA Software (Ω/km)			*TLPC (Ω/km)*		
0.691079	0.342884	0.298102	0.691106	0.342921	0.298137
0.342884	0.710364	0.359314	0.342921	0.710387	0.359347
0.298102	0.359314	0.724175	0.298137	0.359347	0.724195

Shunt Admittance Matrix: Max. Percentage Difference = 0.1873%					
LPARA Software (mS/km)			*TLPC (mS/km)*		
0.003546	-0.001068	-0.000548	0.003551	-0.001070	-0.000549
-0.001068	0.003742	-0.001091	-0.001070	0.003747	-0.001093
-0.000548	-0.001091	0.003484	-0.000549	-0.001093	0.003489

Table 4. Summary of comparative results for all Tests

Test Description	Maximum Percentage Difference (%)		
	Series Resistance Matrix	Series Reactance matrix	Shunt Admittance Matrix
Test 1 - Single Circuit, Single Conductor	0.0448	0.0117	0.1873
Test 2 – Single Circuit, Bundled Conductor	0.0010	0.0007	**0.7246**
Test 3 – Double Circuit, Single Conductor	0.0020	0.0028	0.2890
Test 4 – Double Circuit, Bundled Conductor	**0.0465**	**0.0135**	0.3922

6. Discussion

The results show a variation of less than 0.05% and 0.02% in the Series Resistance and Series Reactance Matrices, while the Shunt Admittance Matrix shows a variation of less than 0.8%. These variations are primarily attributed to the fact that there are significantly less user approximations made by the TLPC in the initial calculations, as compared to that of the LPARA System. These percentages are manifested as a small change in the output line parameters, which usually, is still within the range of tolerable values used in the utility engineer's power system design/study.

These results demonstrate functionality and accuracy of the TLPC. Major contributions of this reengineered TLPC are not necessarily its accuracy and functionality, but rather its:

- speed of line parameter calculation
- automation of calculation processes
- user friendliness
- flexibility in terms of saving and loading custom built structure types and conductors
- decreased susceptibility to human input errors when compared to its LPARA predecessor
- ability to integrate seamlessly with T&TEC's existing procedures
- economical advantage when compared to commercially available software.

6.1 Software Implementation and Requirements

To implement the TLPC it is important to note that it has been designed and built in Matlab version R2010b, (7.11.0.584) and was subsequently complied into a 32-bit standalone executable. This .exe requires Matlab or Matlab MCR (Matlab Complier Runtime Version 7.14) to be launched, but this does not mean that Matlab is needed. The MCR is included in the program files of the

TLPC and this allows any electric utility to use the calculator without the entire Matlab Environment. Other requirements for program operation include:

- Microsoft Windows XP or later version Operating System,
- 1920 x 1080 screen resolution, display font size set to medium,
- at least 512MB of RAM, and
- Microsoft Excel 2010 or later.

6.2 Assumptions and Considerations for Program Operation

Several assumptions are also made by the TLPC and the most important of these is that the TLPC was developed to calculate the line parameters for *overhead transmission lines* only. As such, there are critical expectations and criteria which must be satisfied for the program to function accurately. These include;

1) Single Circuits MUST consist of an aerial conductor as well as the three other line conductors required for three phase power transmission.
2) Double Circuits MUST consist of either one or two aerial conductors as well as the six other line conductors required for two three phase circuits.
3) The transmission circuit is balanced and each of the three phases is identically loaded. This implies that a) each phase is made of the same type of conductor with identical conductor parameters; and b) in the case of bundled conductors, each phase consists of the same number of conductors in each bundle of the circuit.
4) For Double Circuits, the span length between two support structures is the same for both circuits of the double circuit.
5) The temperature coefficient of a conductor is constant at any given temperature of operation.
6) It is a good approximation to model bundled conductors as a set of single conductors connected in parallel.

6.3 Limitations of the Calculator

The TLPC is limited to a maximum topology of a double circuit bundled conductor configuration, consisting of up to four conductors for any of the aerial or phase conductors. Every Single Circuit computation which is done using TLPC must consist of;

- One, aerial conductor (up to a four conductor bundle), and
- Three, phase conductors (each phase consists of a maximum of a four conductor bundle).

Every Double Circuit computation must consist of;

- One, or two aerial lines (up to four conductor bundle), and
- Two sets of three, phase conductors, for each of the two circuits in the double circuit (each phase consists of a maximum of a four conductor bundle).

The GMD calculation for a three conductor bundle is restricted to that of an equilateral triangular configuration, while that of a four conductor bundle is to a square configuration. The program is developed with a set of conductors and support structures which are preloaded into the existing database; however the option exists for the user to populate the database by defining their own conductors and support structures.

7. Conclusion

This paper highlighted the theory, operation and advantages of the reengineered Matlab based TLPC over the existing LPARA software. The inputs and outputs of the software are explicitly defined; and testing and verification of the functionality and accuracy of the TLPC has been performed and documented. The TLPC has been validated with the existing LPARA software yielding a maximum percentage variation of 0.05% and 0.02% in the Series Resistance and Series Reactance Matrices, while the Shunt Admittance Matrix yielded a variation of less than 0.8%.

This variation has been accounted for and numerous tests have shown consistency between both programs. Other advantages of the TLPC over LPARA have been observed and these include an improvement in the speed of performing a line calculation, the ease of use with the TLPC, as well as an improvement in the susceptibility to human errors. These characteristics of the TLPC make it a viable software option for almost any utility to calculate short and medium length transmission line model parameters.

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Appendix 1:

This is the theory used within the LPARA, and it is also used as the foundation of the reengineered TLPC. This method, published by Galloway et al. (1964), utilises Carson's solution and as a result, regards the earth as a plane, homogeneous, semi-infinite solid with constant resistivity. Furthermore, the axial displacement of currents in the air and the earth as well as the effect of the earth return path on the shunt admittance is neglected. The work is divided into three sections: Shunt Admittance, Series Resistance and Series Reactance Matrices.

Development of Shunt Admittance Matrix

The shunt admittance matrix, ***Y***, is a function only of the physical geometry of the conductors relative to the earth plane, and it is an imaginary matrix since the conductance of the air path to ground is negligible. The physical location of the conductors is defined with respect to a coordinate system, with the earth plane as horizontal reference axis and the axis of symmetry of the tower as vertical reference. This allows all conductors on a support structure to be referenced using x and y coordinates. Using these coordinates of the conductors and the conductor radii, elements of charge coefficient matrix ***B*** can then be calculated where the i,j^{th} element is defined as

<u>Equation 4</u>

$$B_{i,j^{th}} = \ln\left(\frac{D_{ij}}{d_{ij}}\right)$$

As used in Equation 4,

D_{ij} = distance between i^{th} conductor and the image of the j^{th} conductor
d_{ij} = distance between i^{th} conductor and the image of the j^{th} conductor for $i \neq j$ (of diagonal)
= radius of ith conductor for $I = j$ (diagonal)

These quantities are shown schematically in Figure 4. The ***B*** matrix has order $3p + q$ where p represents the number of circuits and q, the number of earth wires.

If the charge matrix is represented by ψ and the voltage matrix by ***V***, then using Maxwell's equations,

$$V = \left(\frac{1}{2\pi\epsilon}\right).(B\psi)$$

And it follows that

$$\psi = 2\pi\epsilon B^{-1}V$$

However, ***V*** is a column matrix whose last q elements are zero (the voltage of the earthed or neutral wires), so that the last q columns of $\boldsymbol{B^{-1}}$ can be discarded. The last q rows of $\boldsymbol{B^{-1}}$ give the earth wire charges, and, as these are not generally required, these q rows are also discarded. The matrix obtained by discarding the last q rows and columns of $\boldsymbol{B^{-1}}$ is $\boldsymbol{B_A^{-1}}$ and has order $3p$.

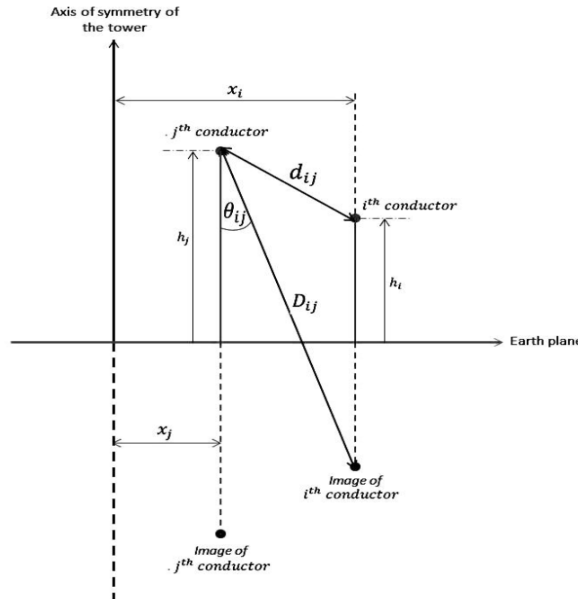

Figure 4. Schematic layout of two conductors

The shunt admittance matrix ***Y*** is defined by the general equation

$$I = YV$$

And since,

$$\Psi = 2\pi\epsilon B_A{}^{-1}V$$

$$I = \frac{d\Psi}{dt} = j\omega\Psi;$$

Then,

$$I = j2\pi\omega\epsilon B_A{}^{-1}V$$

And it follows that,

<u>Equation 5</u>

$$Y = j2\pi\omega\epsilon B_A{}^{-1}$$

Where ***Y*** includes for the effect of the earth wires

Development of the Impedance Matrix

The impedance matrix Z' consists of five components and is of the form

$$Z' = R_e + R_c + j(X_a + X_e + X_c)$$

Where the subscripts have the following significance

g = the contribution of reactance due to the physical geometry of the conductors
c = the contribution of the conductor
e = the contribution of the earth path

Reactance due to physical geometry

The reactance due to the geometry of the conductors is calculated directly from the charge coefficient matrix and is given by

$$X_g = \frac{\omega\mu B}{2\pi}$$

Where ***B*** is the identical matrix as derived for the ***Y*** matrix and is of order $3p + q$.

Determining the effect of the earth return path

The contribution of resistance and reactance, $\boldsymbol{R_e}$ and $\boldsymbol{X_e}$ due to the earth return path is calculated by using an infinite series developed by Carson. Real and imaginary correction component matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, respectively are calculated in terms if $\boldsymbol{r}$ and $\boldsymbol{\theta}$, two abstract parameters such that

Equation 6

$$r_{ij} = \sqrt{\left(\frac{\omega\mu}{\rho}\right).D_{ij}}$$

And θ_{ij} is the angle subtended at the i^{th} conductor, by the i^{th} image and the j^{th} image as illustrated in Figure 4. Galloway's work then involved rearranging Carson's formulas to suit computation as.

Equation 7

$$R_e = \frac{2\boldsymbol{P}\omega\mu}{2\pi} \text{ and } X_e = \frac{2\boldsymbol{Q}\omega\mu}{2\pi}$$

Where, for $r_{ij} \leq 5$,

Equation Set 1

$$P_{ij} = \frac{\pi}{8}(1-S_4) + \frac{1}{2}\log\left(\frac{2}{\gamma r}\right)S_2 + \frac{1}{2}\theta S_2' - \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{2} + \frac{\sigma_3}{\sqrt{2}} \quad \text{and}$$

$$Q_{ij} = \frac{1}{4} + \frac{\log\left(\frac{2}{\gamma r}\right)(1-S_4)}{2} - \frac{\theta S_4'}{2} + \frac{\sigma_1}{\sqrt{2}} - \frac{\pi S_2}{8} + \frac{\sigma_3}{\sqrt{2}} - \frac{\sigma_4}{2}$$

As shown in Equation Set 1, γ = Euler's constant ≈ 1.7811 and $S_2, S_2', S_4, S_4', \sigma_1, \sigma_2, \sigma_3, \sigma_4$ are the infinite series which are defined in Equation Set 2.

Equation Set 2

$$S_2 = \sum_0^\infty a_n \cos(4n+2)\theta$$

$$S_2' = \sum_0^\infty a_n \sin(4n+2)\theta$$

$$S_4 = \sum_0^\infty c_n \cos(4n+4)\theta$$

$$S_4' = \sum_0^\infty c_n \sin(4n+4)\theta$$

$$\sigma_1 = \sum_0^\infty e_n \cos(4n+1)\theta$$

$$\sigma_2 = \sum_0^\infty g_n (S_2)$$

$$\sigma_3 = \sum_0^\infty f_n \cos(4n+3)\theta$$

$$\sigma_4 = \sum_0^\infty h_n (S_4)$$

$a_n, c_n, e_n, f_n, g_n, h_n$ are as defined in Equation Set 3

Equation Set 3

$$a_n = \frac{-a_{n-1}}{2n(2n+1)^2(2n+2)}\left(\frac{r}{2}\right)^4, \qquad a_0 = \frac{r^2}{8}$$

$$c_n = \frac{-c_{n-1}}{(2n+1)(2n+2)^2(2n+3)}\left(\frac{r}{2}\right)^4, \qquad c_0 = \frac{r^4}{192}$$

$$e_n = \frac{-e_{n-1}}{(4n-1)(4n+1)^2(4n+3)}r^4, \qquad e_0 = \frac{r}{3}$$

$$f_n = \frac{-f_{n-1}}{(4n+1)(4n+3)^2(4n+5)}r^4, \qquad f_0 = \frac{r^3}{45}$$

$$g_n = g_{n-1} + \frac{1}{4n} + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{4n+4}, \qquad g_0 = \frac{5}{4}$$

$$h_n = h_{n-1} + \frac{1}{4n+2} + \frac{1}{2n+2} + \frac{1}{2n+3} - \frac{1}{4n+6}, \qquad h_0 = \frac{5}{3}$$

For $r_{ij} > 5$, Equation Set 4 is used to calculate P_{ij} and Q_{ij};

Equation Set 4

$$P_{ij} = \frac{\cos\theta}{\sqrt{(2)}.r} - \frac{\cos 2\theta}{r^2} + \frac{\cos 3\theta}{\sqrt{(2)}.r^3} + \frac{3\cos 5\theta}{\sqrt{(2)}.r^5}$$

$$Q_{ij} = \frac{\cos\theta}{\sqrt{(2)}.r} - \frac{\cos 3\theta}{\sqrt{(2)}.r^3} + \frac{3\cos 5\theta}{\sqrt{(2)}.r^5}$$

Determining the internal impedance of conductor

At power frequency, if the skin effect for a conductor is negligible, then the resistance per unit length of the conductor, R_c is assumed to be equal to the d.c resistance per unit length. This is the case for most overhead conductors, and the d.c. resistance per unit length can be directly obtained from the cable manufacturer specification sheet. However if the skin effect is significant at power frequency, then the manufacturer's power-frequency value will be detailed in the conductor specification sheet and this value would be used as the resistance per unit length instead.

The internal inductance and hence reactance X_c is calculated by the standard concept of geometric mean radius (GMR) and geometric mean distance (GMD). That is X_c is given as:

$$X_c = 2\pi f L$$

Where L is given by

$$L = 2 \times 10^{-7} \ln\left(\frac{GMD}{GMR}\right) \; Hm^{-1}$$

And thus,

Equation 8

$$X_c = 4\pi f \times 10^{-7} \ln\left(\frac{GMD}{GMR}\right) \; \Omega$$

In the case of bundled conductors, the number of conductors and the distance between each conductor of a particular phase is used to determine the GMR of the conductor.

Effect of earth wires in Z matrix

In general, the Z' matrix calculated as detailed in Appendix 1, (Subsection Development of Z Matrix), will have order $3p + q$ where p is the number of circuits and q is the number of earth wires.

The equation relating series voltage drop and current is;

$$V = Z'I \quad or \quad I = Z'^{-1}V$$

As in the case of the ***Y*** matrix, the last q rows and columns of Z'^{-1} are discarded, and the modified matrix of order $3p$ is re-inverted to give the corrected Z matrix which allows for the effect of the earth wires.

Developing derived line parameters

This subsection demonstrates how the admittance and impedance matrices, previously defined, may be manipulated to calculate symmetrical component parameters (Impedance and Capacitance) at the power frequency.

Equation Set 5

Positive Sequence Impedance

For a single circuit network, the positive sequence impedance z_1 (and also the negative sequence z_2) is given by;

$$z_1 = \frac{(z_{11} + z_{22} + z_{33} - z_{12} - z_{13} - z_{23})}{3}$$

While for a double circuit network, the positive sequence impedance is given by;

Positive Sequence Impedance for Circuit 1

$$z_{11} = \frac{(z_{11} + z_{22} + z_{33} - z_{12} - z_{13} - z_{23})}{3}$$

Positive Sequence Impedance for Circuit 2

$$z_{12} = \frac{(z_{44} + z_{55} + z_{66} - z_{45} - z_{46} - z_{56})}{3}$$

Zero Sequence Impedance

For a single circuit network, the zero sequence impedance z_0 is given by;

$$z_0 = \frac{(z_{11} + z_{22} + z_{33} + 2z_{12} + 2z_{13} + 2z_{23})}{3}$$

While for a double circuit network, the zero sequence impedance is given by;

Zero Sequence Impedance for Circuit 1

$$z_{01} = \frac{(z_{11} + z_{22} + z_{33} + 2z_{12} + 2z_{13} + 2z_{23})}{3}$$

Zero Sequence Impedance for Circuit 2

$$z_{02} = \frac{(z_{44} + z_{55} + z_{66} + 2z_{45} + 2z_{46} + 2z_{56})}{3}$$

Zero Sequence Mutual Impedance *

The zero sequence mutual impedance z_{00} for a double circuit configuration is given by;

$$z_{00} = \frac{(z_{14} + z_{15} + z_{16} + z_{24} + z_{25} + z_{26} + z_{34} + z_{35} + z_{36})}{3}$$

Interphase Mutual Impedance

The interphase mutual impedance z_{pp} for a circuit is given by

$$z_{pp} = \frac{(z_0 - z_1)}{3}$$

Where ed in Equation Set 5 and consistent with e r a double circuit configuration.

Earth-Loop impedance

The earth loop impedance z_p for either single or double circuit is defined as

$$z_p = z_1 + z_{pp}$$

Where z_1 and z_{pp} are consistent with either a single or double circuit configuration.

Inter-circuit mutual impedance *

The inter-circuit mutual impedance z_{cc} in a double circuit configuration is given as;

$$z_{cc} = \frac{z_{00}}{3}$$

The following demonstrates how the shunt susceptance parameters are developed.

Let $A = Y^{-1}$ where A has elements a_{ij}, then;

Positive-sequence capacitance

The positive sequence capacitance c_1 is given by

$$c_1 = \frac{3}{\omega(a_{11} + a_{22} + a_{33} - a_{12} - a_{23} - a_{13})}$$

Zero-sequence capacitance

The zero sequence capacitance c_0 is given by

$$c_0 = \frac{3}{\omega(a_{11} + a_{22} + a_{33} + 2a_{12} + 2a_{23} + 2a_{13})}$$

Zero-sequence mutual capacitance *

The zero sequence mutual capacitance c_{00} is given by

$$c_{00} = \frac{3}{\omega(a_{14} + a_{15} + a_{16} + a_{24} + a_{25} + a_{26} + a_{34} + a_{35} + a_{36})}$$

Interphase mutual capacitance

The interphase mutual capacitance c_{pp} is given by

$$c_{pp} = \frac{3}{\frac{1}{c_0} - \frac{1}{c_1}}$$

Earth loop capacitance

The earth loop capacitance c_p is given by

$$c_p = \frac{1}{\frac{1}{c_1} - \frac{1}{c_{pp}}}$$

Inter-circuit mutual capacitance *

The inter-circuit mutual capacitance c_{cc} is given by

$$c_{cc} = 3c_{00}$$

* - Only applicable to double circuit topologies.

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