Factor Proportions and the Heckscher-Ohlin Theorem

1. Recent contributions to the pure theory of international trade have relied heavily on the variable-proportions account of trade developed by Eli Heckscher and Bertil Ohlin, who linked export-import patterns to factor endowments and methods of production. Their hypothesis is expressed in summary form in the Heckscher-Ohlin Theorem: A country exports those commodities produced with relatively large quantities of the country's relatively abundant factor. Perhaps the most controversial application of this theorem to appear recently is that based on Leontief's input-output studies of the American economy. According to Leontief, America is not capital-rich compared with the rest of the world, as popularly supposed. His data suggest that American exports require a higher proportion of labor to capital than do American imports (or rather, American import-competing commodities). Leontief's conclusions then follow from his unqualified support of the Heckscher-Ohlin dictum.

However, there are significant exceptions to the Heckscher-Ohlin theorem. The theorem rests upon both a concept of factor abundance related to pre-trade factor price ratios and special shapes for the production functions. Should these strict conditions be modified, the Heckscher-Ohlin theorem in some cases no longer holds, in others becomes meaningless. The purpose of this article is to clarify the meaning of the Heckscher-Ohlin theorem and to demonstrate that there is no a priori basis for accepting the Leontief conclusions; his data are not inconsistent with the commonly-held notion of American factor endowments.

2. The two-country, two-factor, two-commodity framework customary in the variable proportions account of trade is retained in what follows unless explicitly altered. That is, each commodity in the two countries is sold in a purely competitive market and produced under constant returns to scale. No transfer costs of any kind exist, so that the prices of internationally traded commodities are equalised in the two countries. Fixed endowment quantities of two homogeneous factors of production, capital and labor, are fully employed in each country. The quality (but not quantity) of each factor in the two countries is identical, as are the production functions. This assumption is made so that differences in trading patterns and pre-trade price ratios are ascribable to differences in factor endowments on the supply side and/or to differences in demand conditions, rather than to differences in technology.

---

1 In preparing this article I have benefited greatly by conversations with Professor Robert Solow and my colleagues, Louis Lefeber and Michael Moss.


Within the framework set out by these assumptions the concepts of relative factor abundance and factor intensity each admit of more than one interpretation. The concept of relative factor abundance is examined first; this part of the discussion is based on the strong Samuelson notion of factor intensity: For any set of factor price ratios optimal resource allocation in each country entails that one commodity \((X)\) is always produced with a greater amount of capital per unit of labor \((p_x)\) than the capital/labor ratio \((p_y)\) employed in the other commodity \((Y)\).\(^1\)

In addition, initially it is assumed that trade does not lead to complete specialization in either country.

3. Ohlin’s definition of relative factor abundance rests on the pre-trade ratio of factor prices in the two countries. Denoting the countries by the subscripts 1 and 2 and capital and labor by \(C\) and \(L\) respectively, country 1 is relatively capital abundant in the Ohlin sense if before trade is initiated:

\[
\left(\frac{P_C}{P_L}\right)_1 < \left(\frac{P_C}{P_L}\right)_2
\]

By definition capital is relatively cheaper in the capital-rich country before trade. As suggested by Samuelson in his factor-price equalisation articles, if the strong factor intensity property holds, the assumptions underlying the Ohlin theory imply a unique relationship between factor price ratios and commodity price ratios. This relationship is identical for the two countries. Equality of commodity prices with trade must involve an equalisation of the returns to the same factor in each country. Furthermore, any given factor-price ratio is associated uniquely with a ratio of commodity prices, as illustrated in Figure 1. The fact that capital is relatively cheaper in country 1 before trade implies that the capital intensive commodity \((X)\) is relatively less expensive in that country. Thus with trade the capital-rich country must export the capital intensive commodity. With Ohlin’s definition of factor abundance and the strong assumptions of section 2 the Heckscher-Ohlin theorem, although true, is trivial. The converse of the theorem also holds: if a country exports

\[^1\] Samuelson, P. A. : footnote on p. 165 of the 1948 article, op. cit. and p. 182 of the 1949 article, op. cit.
the capital intensive commodity, capital must have been the relatively cheap factor of production in that country before trade.

However, this seems not to be the notion of factor abundance employed by Leontief and most others who discuss this question. Applications of the Ohlin dictum hint at a relationship between physical factor endowments and patterns of trade, rather than the latter and the pre-trade constellation of factor prices. Indeed, Ohlin, himself, makes this point succinctly:

...the real problem is to demonstrate what lies behind such inequality in prices, or, more precisely, to show in what way differences in equipment come to be expressed in differences in money costs and prices.¹

4. An alternative definition of relative factor abundance goes back to physical factor endowments. According to this definition a country is relatively capital abundant if and only if it is endowed with a higher proportion of capital to labor than the other country. Thus country 1 is relatively capital-rich if:

\[ \frac{C_1}{L_1} > \frac{C_2}{L_2} \]

where the bars denote the fixed factor quantities available in each country. This definition and the Ohlin (1) definition are certainly not equivalent. Pre-trade factor price ratios are determined both by conditions of supply and demand and are uniquely related to pre-trade commodity price ratios, whereas factor endowment proportions may be outweighed by dissimilarities in demand in influencing price ratios.² Differences in relative endowments impart a bias on the supply side, but not sufficient to insure validity of the Heckscher-Ohlin theorem if this new definition (2) of factor abundance is used.

The nature of the bias introduced by differences in factor endowments is most clearly seen in the relationship between the transformation schedules of the two countries. If both countries were endowed with the same factor proportions, the constant-returns-to-scale assumption would be sufficient to insure that the transformation schedules of the two countries, if not identical, would be radial blowups of each other. Differences in relative factor endowments are reflected by transformation schedules differing in shape. Suppose that country 1 is relatively capital abundant by the non-Ohlin definition (2). The transformation schedule of the capital-rich country must be flatter than that of the other country along any ray from the origin, as illustrated in Figure 2. That is, if the output of the two commodities is in the same proportion in both countries, the relatively capital abundant country will be able to expand its production of the capital intensive commodity at a lower opportunity cost than the other country. In this sense the relatively capital abundant country has a production bias in favor of the capital intensive commodity.³

That this production bias may not be sufficient to insure that before trade capital and the capital intensive commodity are relatively cheaper in country 1 than in country 2 is seen in Figure 2.⁴ The pre-trade price ratio, \( P_X/P_Y \), is greater in country 1 at \( b \) than in

¹ Ohlin, op. cit., p. 13. Italics mine.
² In an original version of this article I made use of the production box diagrams of each country to prove this relationship between the slopes of the two transformation schedules. Since then, T. Rybczynski, in his "Factor Endowment and Relative Commodity Prices," Economica (New Series), Volume XXII, No. 88, Nov., 1955, pp. 336-341, has demonstrated the equivalent proposition that should the supply of one factor (capital) in a country be augmented, the transformation schedule becomes flatter.
³ For example, cf. James & Pearce, op. cit., p. 113.
⁴ Two sets of community indifference curves are drawn in Figure 2. Essentially this amounts to assuming that each country behaves like a single rational consumer in a competitive market. For a demonstration of the graphical procedure, cf. Leontief, W.: "The Use of Indifference Curves in the Analysis of Foreign Trade," the Quarterly Journal of Economics, Volume XLVII (May, 1933), pp. 493-503, and reprinted as Chapter 10 in Readings in the Theory of International Trade.
country 2 at \( a \), despite the production bias. Thus with trade the capital-rich country concentrates on the production of the labor intensive commodity in order to export \( e_l \) of this commodity \( (Y) \) in exchange for imports \( (i_g) \) of the capital intensive commodity. In this case differences in tastes outweigh differences in factor endowments and the Heckscher-Ohlin theorem (using definition (2) of factor abundance) does not hold. Conversely, without specifying demand conditions it is invalid to infer factor endowment proportions from the pattern of trade.\(^1\)

5. The previous comments may be used to examine the meaning of the term, "comparative advantage." In a Ricardian, one-factor, constant-returns world, technology alone determines which commodity is exported from each country. Furthermore, each country is said to possess a comparative advantage in producing that commodity. Thus the concept of comparative advantage satisfied the following two criteria: (i) comparative advantage is determined by production conditions alone, and (ii) each country exports that commodity in which it enjoys a comparative advantage.

However, in a two-factor, variable proportions model of trade, the concept of comparative advantage does not always satisfy these criteria, so some sacrifice in the meaning of the term must be made. Under the assumptions of Section 2, and employing the Ohlin definition of factor abundance in terms of pre-trade factor prices, it is possible to define the capital-rich country to have a comparative advantage in producing the capital intensive commodity. It is then true, but trivial, to state that each country exports that commodity in which it enjoys a comparative advantage. However, the first criterion is not satisfied, for a knowledge of factor endowment proportions alone is not sufficient, with Ohlin’s definition, to determine the “comparative advantage” commodity, so defined. Demand conditions must be considered as well.

An alternative approach consists in selecting the “comparative advantage” commodity for each country merely from a knowledge of factor endowments. Thus, using the definition of factor abundance related to endowment proportions rather than to factor prices, state that the capital abundant country has a comparative advantage in producing the capital intensive commodity. Unlike the Ohlin definition, the alternative definition satisfies the first criterion, although demand conditions may invalidate the second.

\(^1\) Even the assumption of identical taste patterns in the two countries is not sufficient to assure validity of the Heckscher-Ohlin theorem, as there is no necessary relationship among the slopes of different indifference curves along any ray. It is easy to demonstrate, however, that if both countries’ taste patterns are identical and homothetic (implying unitary income elasticities), the Heckscher-Ohlin theorem must hold with either definition of factor abundance.
The meaning of comparative advantage in production is reflected in the production bias arising from differences in factor endowments. Whatever the conditions of demand and patterns of consumption, with free trade there is a unique relationship between the ratios of commodities produced in each country. In the country possessing a greater relative supply of capital, the ratio of the quantity of the capital intensive commodity produced to the production of the labor intensive commodity must always be larger than in the other country. Retaining the assumptions in section 4, the ratio \( X/Y \) in country 1 must, in free trade, always be larger than the \( X/Y \) ratio of production in country 2. In this sense it is meaningful to state that the capital-rich country has a comparative advantage in producing the capital intensive commodity.

6. As suggested in the previous sections, the relationship between factor price and commodity price ratios is unique granted constant returns to scale and the strong definition of factor intensity. This is clearly demonstrated if the analysis is extended to include more than two commodities produced in each country. Suppose each country produces three commodities, \( X, Y, Z \), each subject to constant returns to scale. Furthermore, suppose that for any given factor price ratio commodity \( X \) always employs a higher capital/labor ratio than \( Y \), which employs a higher capital/labor ratio than \( Z \). That is, let :

\[
\rho_x > \rho_y > \rho_z.
\]

Regardless of demand conditions in the two countries, if commodity \( X \) is cheaper relative to \( Y \) in country 1 than in country 2, so also must \( Y \) be cheaper relative to \( Z \) in country 1.

This binding relationship among commodity price ratios may be proved in straightforward fashion. Since each commodity is produced subject to constant returns :

\[
X = L_x f(\rho_x) \quad Y = L_y g(\rho_y) \quad Z = L_z h(\rho_z).
\]

Optimal resource allocation requires the ratio of factor prices to be equal to the ratio of the marginal physical products of labor to capital in each commodity :

\[
\frac{P_x}{P_c} = \frac{f(\rho_x) - \rho_x f'(\rho_x)}{f'(\rho_x)} \quad \frac{g(\rho_y) - \rho_y g'(\rho_y)}{g'(\rho_y)} = \frac{h(\rho_z) - \rho_z h'(\rho_z)}{h'(\rho_z)}.
\]

Pick any two commodities, e.g., \( Y \) and \( Z \). The ratio of the prices of \( Y \) and \( Z \) in equilibrium is given by the ratio of the marginal physical products of either factor in \( Z \) and \( Y \) :

\[
\frac{P_Y}{P_Z} = \frac{h(\rho_z)}{g'(\rho_y)}.
\]

This price ratio varies inversely with the capital/labour ratio employed in any commodity. For :

\[
\frac{d(P_Y/P_Z)}{d\rho_y} = \frac{g' h'' d\rho_z/d\rho_y - h' g''}{(g')^2}.
\]

Differentiating equation (5), solving for \( d\rho_z/d\rho_y \) and substituting into (7) yields :

\[
\frac{d\left( \frac{P_Y}{P_Z} \right)}{d\rho_y} = \frac{g''(h')^2 g}{h g'} - h' g''.
\]

But equation (5) may be solved for \( g/g' \):

\[
\frac{g}{g'} = \frac{h}{h'} + \rho_x - \rho_z.
\]
Substituting (9) into (8) yields:

\[ \frac{d(P_Y/P_Z)}{d\varphi_y} = \frac{g''(h)}{h(g')^2} (\varphi_y - \varphi_z). \]  

Since \( g'' \) is negative (reflecting a falling marginal physical product of capital) and since commodity \( Y \) has been assumed more capital intensive than \( Z \) for any common factor price ratio, \( d(P_Y/P_Z) / d\varphi_y \) must be negative.\(^1\)

Commodities \( Y \) and \( Z \) were picked arbitrarily. With any number of commodities, \((1, \ldots, n)\),

\[ \frac{d(P_i/P_j)}{d\varphi_i} < 0 \quad \text{if} \quad \varphi_i > \varphi_j. \]  

Therefore an \( X \) to \( Y \) commodity price ratio in country 1 lower than in country 2 must reflect a capital to labor ratio employed in any commodity in country 1 that is higher than that found in the production of the corresponding commodity in country 2. This latter relationship, and equation (11), implies a lower \( Y \) to \( Z \) commodity price ratio in country 1 as well, regardless of demand conditions.\(^2\)

This strong relationship among commodity price ratios has implications for the concept of comparative advantage as well. Ordering the commodities with respect to the capital/labor ratios employed in production is to rank them in order of comparative advantage. Demand conditions merely determine the dividing line between exports and imports; it is not possible to break the chain of comparative advantage by exporting, say, the third and fifth commodities and importing the fourth when they are ranked by factor intensity.\(^3\)

The preceding analysis seems unduly restricted by the strong factor-intensity criterion suggested by Samuelson. That is, previously technology has assumed \( X \) to require a higher proportion of capital than \( Y \) for any set of factor prices common to both commodities. By using an alternative concept of factor intensity it is possible to include cases heretofore ruled out; these cases not only qualify the meaning of the Heckscher-Ohlin theorem but may also invalidate the Leontief procedure.

The alternative notion of factor intensity has meaning only when factor endowments are specified. Given the initial level of resources, it is possible to define \( X \) as capital intensive if \( \varphi_x > \varphi_y \) for any equilibrium combination of output.\(^4\) Alternately, \( X \) is capital intensive if \( \varphi_x \) is greater than the endowment ratio, \( C/L \). Stated in terms of common factor price ratios, commodity \( X \) is capital intensive if \( \varphi_x > \varphi_y \) for all common possible equilibrium factor price ratios once a set of factor endowments is specified.

The relationship between this definition of factor intensity and the Samuelson definition is clear: to be labelled capital intensive \( \text{à la} \) Samuelson, commodity \( X \) must require a higher capital/labor ratio than \( Y \) for any common factor price ratio, whether or not this ratio can exist in equilibrium once factor endowments are specified. This requirement is not satisfied by all pairs of constant-returns production functions. On the other

\(^1\) I wish to thank Professor Harry Johnson for recommending a more simple form of the original proof. The relationship between factor intensities and commodity price ratios proved here has also been demonstrated in slightly different fashion by James Meade, \textit{Trade and Welfare} (Oxford University Press, 1955), pp. 600-601.

\(^2\) Although demand conditions are important in determining the relationship between \( p_i \) in one country and \( p_i \) in the other. Note that the relationships proved here apply as well in the two-commodity case and serve as verification of the previous remarks.

\(^3\) This is similar to the relationships that hold in the one-factor case. Cf. Harberler, G.: \textit{The Theory of International Trade} (London: Wm. Hodge and Co., Ltd., 1936), p. 137.

\(^4\) Constant returns to scale in each commodity insures that a greater \( \varphi_x \) than \( \varphi_y \) at one point along the contract curve is sufficient for \( \varphi_x > \varphi_y \) everywhere on the contract curve.
hand, any pair of linear homogeneous production functions may be subjected to the suggested alternative criterion.\(^1\)

The significance of this different interpretation is clearly seen if two production functions that do not satisfy the strong Samuelson condition are examined. By continuity, if \(p_x > p_y\) for some set of common factor price ratios and \(p_x < p_y\) for some other set, the capital to labor ratios employed in the production of the two commodities must be equal for some factor price ratio.\(^2\) In Figure 3 a member of each isoquant family is depicted.\(^3\)

If the endowment ratio in country 1, \(C_1/L_1\), is greater than the \(\lambda\)-ratio, while the endowment ratio in country 2, \(C_2/L_2\), is less than the \(\lambda\)-ratio, \(X\) must be the labor intensive commodity in country 1 and the capital intensive commodity in country 2.\(^4\)

Should the endowment proportions in both countries lie on the same side of the \(\lambda\)-ratio, all the previous conclusions of this paper hold, despite the fact that the Samuelson criterion is not satisfied (except in the special case of complete specialization, to be discussed in section 9).\(^5\)

\(^1\) Except in the singular case in which the contract curve and the diagonal of the production box are identical.

\(^2\) Assume for simplicity the existence of only one such critical factor price ratio. This excludes the main case considered by James and Pearce, op. cit. In their excellent demonstration of the fact that capital may be relatively cheaper in the labor abundant country before trade and that factor prices may diverge rather than be equalized once trade is initiated, they concentrate on the case in which two critical factor price ratios exist (although their terminology and graphical demonstration differ substantially from that employed in this paper). Both ratios lie between the endowment proportions of the two countries; thus the same commodity is capital intensive in both countries. However, our primary concern here is not the effect of trade on factor prices but rather the relationship between pre-trade factor price ratios in the two countries and the pattern of trade. Suffice it to say that in the James and Pearce case, as well as in our one-critical-factor-price-ratio case, it is possible for the capital abundant country to export the labor intensive commodity.

\(^3\) The pioneering examination of these cases is that of Lerner, A.: "Factor Prices and International Trade," Economica (New Series), Volume XIX, No. 73, February, 1952, pp. 1-15.

\(^4\) A heuristic proof, which can be made quite rigorous, is sketched here. As in Figure 3, assume the \(X\)-isoquant to be more concave than the \(Y\)-isoquant, reflecting the fact that there is less possibility for factor substitution in \(X\)-production than in \(Y\)-production. That is, for any factor price ratio common to both industries (excluding the price ratio at \(A\)), the critical \(\lambda\)-capital/labor ratio must more closely correspond to \(p_x\) than to \(p_y\). In country 1 the \(\lambda\)-ratio is "labor-intensive" relative to factor endowments. Therefore \(p_x\) must be less than \(p_y\) at any point on the contract curve. In country 2, \(p_x > p_y\) at all points.

\(^5\) In the general case of more than one \(\lambda\)-ratio the previous conclusions of this paper hold if factor endowments for both countries lie between the same two consecutive critical factor ratios, \(\lambda_r\) and \(\lambda_{r+1}\). As has been pointed out to me by Professor Pearce, the above conditions, although sufficient, are not necessary for the previous conclusions. For if factor endowments in the two countries are separated by an even number of \(\lambda\)-ratios the same commodity is capital intensive in both countries and the capital abundant country may export the capital intensive commodity (although not in all cases, cf. n. 2 above). The case cited in the text is the special case in which only one \(\lambda\)-ratio exists; if both sets of factor endowments lie on the same side of this \(\lambda\)-ratio, its existence creates no new problems and all previous conclusions hold.
8. If the factor endowment ratios in each country lie on opposite sides of the $\lambda$-ratio, as suggested above, $\rho_x < \rho_y$ in country 1, and $\rho_x > \rho_y$ in country 2. But the literal meaning of the Heckscher-Ohlin theorem is, in this case, open to question, as is Leontief's procedure of comparing capital/labor ratios in exports and import-competing industries.

The contract curves in the two countries are shown in Figure 4. Since $\rho_x < \rho_y$ in country 1, $P_X/P_Y$ is an increasing function of $\rho_x$ (cf. section 6). In country 2, $P_X/P_Y$ is a decreasing function of $\rho_x$. But, as is seen in Figure 4, $\rho_x$ and $\rho_y$ in country 1 are both greater than either $\rho_x$ or $\rho_y$ in country 2; these sets of ratios are separated by the critical $\lambda$-ratio. The relationship between commodity and factor price ratios and $\rho_x$ in each country is shown in Figure 5.

Country 1 is relatively capital abundant; not only is $C_1/L_1$ greater than $C_2/L_2$ but, as Figure 4 demonstrates, $\rho_x$ in country 1 must always be higher than in country 2, which implies (cf. Figure 5) that capital must be relatively cheaper in country 1, regardless of demand conditions. Commodity $X$, however, need not be relatively cheaper in country 1 before trade, as shown by points A and B in Figure 5.† It may be the case that the relatively capital abundant country exports its labor intensive commodity. America may be capital-rich, either by the Ohlin definition or in terms of factor endowment proportions, and export commodities employing relatively less capital than in its import-competing industries.

Closer inspection reveals that, in a formal sense, the Heckscher-Ohlin theorem as applied to both countries in this case cannot possibly hold. For the exports of each country are either both capital intensive (relative to the other commodity produced in the country) or labor intensive. It is impossible for both countries to export that commodity requiring a relatively high quantity of the factor in which the country is relatively well endowed.

This is only a formal objection to the Heckscher-Ohlin theorem, however. For no matter which commodity is exported from the capital-rich country, it must embody a higher proportion of capital than either commodity produced in the other country. Although validated in this sense, the Heckscher-Ohlin theorem cannot be put into reverse in the Leontief manner. In concluding that America must be labor abundant, Leontief compared the capital/labor ratios in American exports and American import-competing industries; no comparison was made with factor proportions abroad. If both American

† Note also that in Figure 5 an equality of commodity prices with free trade could never result in factor price equalisation.
exports and import-competing products are produced with more capital intensive methods than abroad, Leontief's paradoxical conclusion is no longer valid. In this case, and there is no a priori reason to believe this case is more unlikely than any other, Leontief's method tells us nothing about American factor endowments relative to the rest of the world.

9. The case of complete specialization offers the best support to the Heckscher-Ohlin theorem and Ohlin's factor-price equalisation views. Suppose commodity $X$ is produced by capital intensive methods in both countries, country 1 is relatively capital abundant by the definition that compares endowment proportions, and its transformation schedule is everywhere flatter than country 2's transformation schedule. In this case before trade (and after trade) capital must be relatively cheaper in country 1 than in country 2, and one

$^1$ Assume the existence of no more than one critical $\lambda$-ratio.
country must specialize completely in its export commodity. The Heckscher-Ohlin theorem must hold and Ohlin is justified in his supposition that trade brings about a tendency, necessarily incomplete, towards factor price equalisation.¹

10. Thus in the two-commodity model, with identical constant-returns-to-scale technology in the two countries, the Heckscher-Ohlin theorem may not hold in all cases. If production functions are of the strict Samuelson type, the Heckscher-Ohlin theorem must hold if relative factor abundance is tied to pre-trade factor prices. However, it may not be valid to use the Heckscher-Ohlin theorem to infer relative endowment proportions from existing trading patterns if demand conditions in the two countries are quite dissimilar.

If the Samuelson restriction is lifted, strict application of the Heckscher-Ohlin theorem may not be valid if factor endowments differ considerably. And the Leontief procedure of comparing factor intensities in only one country may be invalid.

As suggested in section six, similar binding relationships exist in a three-or-more commodity world. However, lifting the two-factor assumption poses problems for analysis; unless two or more factors are completely substitutable, the concept of factor intensity, upon which, in part, the Heckscher-Ohlin theorem rests, loses some of its meaning.² Finally taking into account differences in technology or quality of the factors of production as well as endowment proportions makes for sterile analysis, as trading patterns could then be explained by differences in factor endowments, or in the quality of the factors, or in technology, or in conditions of demand.

Swarthmore, Pa. 

R. W. Jones.

¹ As pointed out by Professor Pearce, an incomplete tendency towards factor price equalisation also follows if factor endowments are separated by one or more λ-ratios, even if specialization is incomplete.
² The case in which there are more factors than commodities, and vice versa, and implications for factor price equalisation have been handled by James and Pearce, op. cit. and by Samuelson, "Prices of Factors and Goods in General Equilibrium," the Review of Economic Studies, 1953-54, Vol. XXI (1), No. 54.